1998 AP Calculus AB Free-Response Questions

CALCULUS AB

Section II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

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1. Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
   (a) Find the area of the region R.
   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
   (c) Find the volume of the solid generated when R is revolved about the x–axis.
   (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

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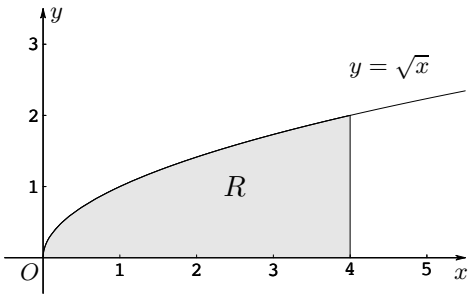
1. Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
   (a) Find the area of the region R.
   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
   (c) Find the volume of the solid generated when R is revolved about the x–axis.
   (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

(a)



$A = \displaystyle\int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{16}{3}$ or 5.333

2 { 1: $A = \displaystyle\int_0^4 \sqrt{x}\,dx$; 1: answer }

(b) $\displaystyle\int_0^h \sqrt{x}\,dx = \frac{8}{3}$ –or– $\displaystyle\int_0^h \sqrt{x}\,dx = \int_h^4 \sqrt{x}\,dx$

$\displaystyle\frac{2}{3}h^{3/2} = \frac{8}{3}$ –or– $\displaystyle\frac{2}{3}h^{3/2} = \frac{16}{3} - \frac{2}{3}h^{3/2}$

$h = \sqrt[3]{16}$ or 2.520 or 2.519

2 { 1: equation in h ; 1: answer }

(c) $V = \pi \displaystyle\int_0^4 (\sqrt{x})^2\,dx = \pi\frac{x^2}{2}\Big|_0^4 = 8\pi$

or 25.133 or 25.132

3 { 1: limits and constant ; 1: integrand ; 1: answer }

(d) $\pi \displaystyle\int_0^k (\sqrt{x})^2\,dx = 4\pi$ –or– $\pi \displaystyle\int_0^k (\sqrt{x})^2\,dx = \pi\int_k^4 (\sqrt{x})^2\,dx$

$\displaystyle\pi\frac{k^2}{2} = 4\pi$ –or– $\displaystyle\pi\frac{k^2}{2} = 8\pi - \pi\frac{k^2}{2}$

$k = \sqrt{8}$ or 2.828

2 { 1: equation in k ; 1: answer }

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2. Let f be the function given by $f(x) = 2xe^{2x}$.
   (a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.
   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   (c) What is the range of f?
   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

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2. Let f be the function given by $f(x) = 2xe^{2x}$.
   (a) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.
   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   (c) What is the range of f?
   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

(a) $\lim_{x\to-\infty} 2xe^{2x} = 0$

$\lim_{x\to\infty} 2xe^{2x} = \infty$ or DNE

2 { 1: 0 as $x \to -\infty$; 1: ∞ or DNE as $x \to \infty$

(b) $f'(x) = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} = 2e^{2x}(1 + 2x) = 0$

if $x = -1/2$

$f(-1/2) = -1/e$ or -0.368 or -0.367

$-1/e$ is an absolute minimum value because:

(i) $f'(x) < 0$ for all $x < -1/2$ and
$f'(x) > 0$ for all $x > -1/2$

–or–

(ii)

x		$-1/2$	
$f'(x)$	$-$	0	$+$

and $x = -1/2$ is the only critical number

3 { 1: solves $f'(x) = 0$; 1: evaluates f at student's critical point; 0/1 if not local minimum from student's derivative ; 1: justifies absolute minimum value; 0/1 for a local argument; 0/1 without explicit symbolic derivative

Note: 0/3 if no absolute minimum based on student's derivative

(c) Range of $f = [-1/e, \infty)$
or $[-0.367, \infty)$
or $[-0.368, \infty)$

1: answer

Note: must include the left–hand endpoint; exclude the right–hand "endpoint"

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$

if $x = -1/b$

At $x = -1/b$, $y = -1/e$

y has an absolute minimum value of $-1/e$ for all nonzero b

3 { 1: sets $y' = be^{bx}(1 + bx) = 0$; 1: solves student's $y' = 0$; 1: evaluates y at a critical number and gets a value independent of b

Note: 0/3 if only considering specific values of b

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AB 2 Board Note # 1

Part (d)

3/3 Argument with the following three ingredients:

1. The graph of $y = bxe^{bx}$ is a horizontal compression or expansion (with a reflection across the y–axis if $b < 0$) of the graph of $y = xe^x$.
2. The range of $y = bxe^{bx}$ is therefore the same as the range of $y = xe^x$.
3. Therefore the absolute minimum value of $y = bxe^{bx}$ is the same for all (non–zero) values of b.

0/3 Analyzing the horizontal compression/expansion of graphs of $y = bxe^{bx}$ for specific values of b.

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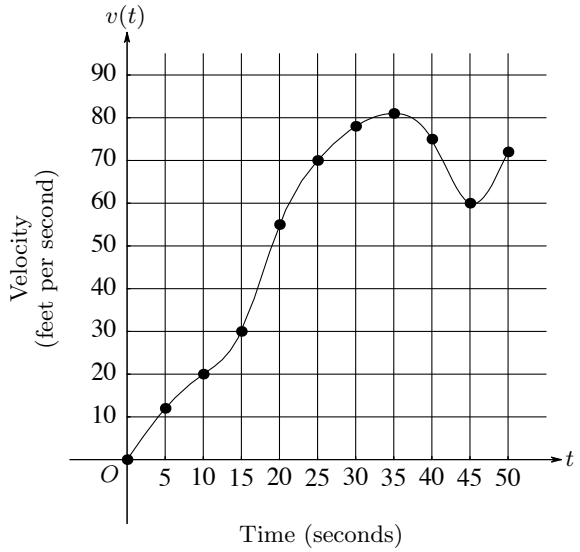

t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.
   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.
   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.
   (d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

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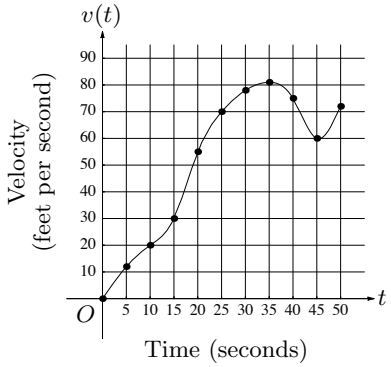

t (seconds)	$v(t)$ (feet per second)
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50	72

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   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.
   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.
   (d) Approximate $\displaystyle\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

(a) Acceleration is positive on $(0, 35)$ and $(45, 50)$ because the velocity $v(t)$ is increasing on $[0, 35]$ and $[45, 50]$

3 { 1: $(0, 35)$; 1: $(45, 50)$; 1: reason

Note: ignore inclusion of endpoints

(b) Avg. Acc. $= \dfrac{v(50) - v(0)}{50 - 0} = \dfrac{72 - 0}{50} = \dfrac{72}{50}$

or $\quad 1.44$ ft/sec^2

1: answer

(c) Difference quotient; e.g.

$\dfrac{v(45) - v(40)}{5} = \dfrac{60 - 75}{5} = -3$ ft/sec^2 or

$\dfrac{v(40) - v(35)}{5} = \dfrac{75 - 81}{5} = -\dfrac{6}{5}$ ft/sec^2 or

$\dfrac{v(45) - v(35)}{10} = \dfrac{60 - 81}{10} = -\dfrac{21}{10}$ ft/sec^2

–or–

Slope of tangent line, e.g.

through $(35, 90)$ and $(40, 75)$: $\dfrac{90 - 75}{35 - 40} = -3$ ft/sec^2

2 { 1: method ; 1: answer

Note: 0/2 if first point not earned

(d) $\displaystyle\int_0^{50} v(t)\,dt$

$\approx 10[v(5) + v(15) + v(25) + v(35) + v(45)]$

$= 10(12 + 30 + 70 + 81 + 60)$

$= 2530$ feet

This integral is the total distance traveled in feet over the time 0 to 50 seconds.

3 { 1: midpoint Riemann sum ; 1: answer ; 1: meaning of integral

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4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2 + 1}{2y}$.
   (a) Find the slope of the graph of f at the point where $x = 1$.
   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.
   (c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.
   (d) Use your solution from part (c) to find $f(1.2)$.

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   (d) Use your solution from part (c) to find $f(1.2)$.

(a) $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$

$\left.\dfrac{dy}{dx}\right|_{\substack{x=1 \\ y=4}} = \dfrac{3+1}{2 \cdot 4} = \dfrac{4}{8} = \dfrac{1}{2}$

1: answer

(b) $y - 4 = \dfrac{1}{2}(x - 1)$

$f(1.2) - 4 \approx \dfrac{1}{2}(1.2 - 1)$

$f(1.2) \approx 0.1 + 4 = 4.1$

2
- 1: equation of tangent line
- 1: uses equation to approximate $f(1.2)$

(c) $2y\,dy = (3x^2 + 1)\,dx$

$\displaystyle\int 2y\,dy = \int (3x^2 + 1)\,dx$

$y^2 = x^3 + x + C$

$4^2 = 1 + 1 + C$

$14 = C$

$y^2 = x^3 + x + 14$

$y = \sqrt{x^3 + x + 14}$ is branch with point $(1, 4)$

$f(x) = \sqrt{x^3 + x + 14}$

5
- 1: separates variables
- 1: antiderivative of dy term
- 1: antiderivative of dx term
- 1: uses $y = 4$ when $x = 1$ to pick one function out of a family of functions
- 1: solves for y

0/1 if solving a linear equation in y
0/1 if no constant of integration

Note: max 0/5 if no separation of variables

Note: max 1/5 [1-0-0-0-0] if substitutes value(s) for x, y, or dy/dx before antidifferentiation

(d) $f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$

1: answer, from student's solution to the given differential equation in (c)

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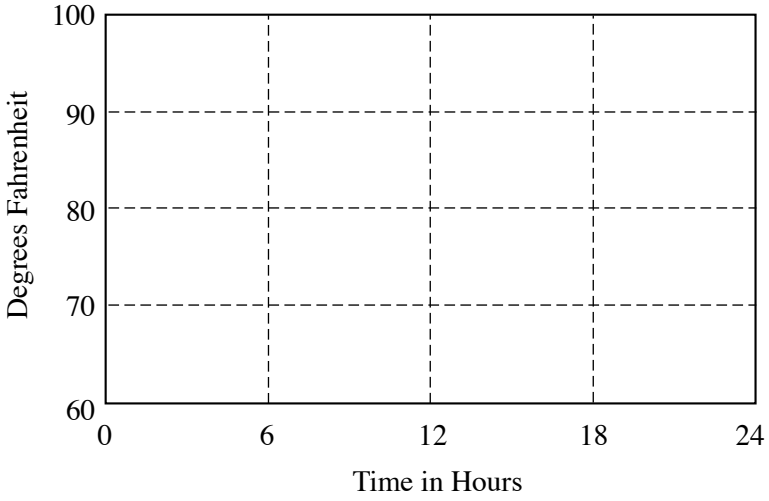
5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right),\ 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Sketch the graph of F on the grid below.



(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24–hour period?

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where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Sketch the graph of F on the grid below.

(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

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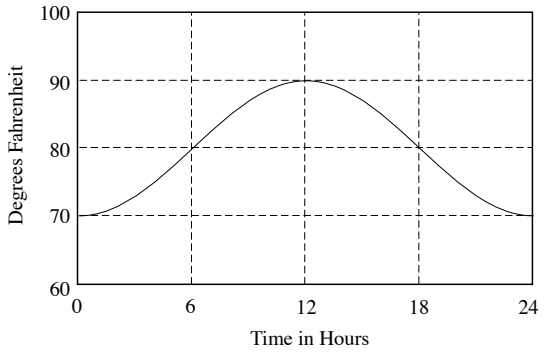
(a)



1: bell–shaped graph
minimum 70 at $t = 0$, $t = 24$ only
maximum 90 at $t = 12$ only

(b) $$\text{Avg.} = \frac{1}{14-6}\int_6^{14}\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right]dt$$

$$= \frac{1}{8}(697.2957795)$$

$$= 87.162 \text{ or } 87.161$$

$$\approx 87^\circ \text{ F}$$

3
- 2: integral
  - 1: limits and $1/(14-6)$
  - 1: integrand
- 1: answer
- 0/1 if integral not of the form $\frac{1}{b-a}\int_a^b F(t)\,dt$

(c) $$\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right] - 78 \ge 0$$

$$2 - 10\cos\left(\frac{\pi t}{12}\right) \ge 0$$

$$\left.\begin{matrix}5.230\\ \text{or}\\ 5.231\end{matrix}\right\} \le t \le \left\{\begin{matrix}18.769\\ \text{or}\\ 18.770\end{matrix}\right.$$

2
- 1: inequality or equation
- 1: solutions with interval

(d) $$C = 0.05\int_{\substack{5.231\\ \text{or}\\ 5.230}}^{\substack{18.769\\ \text{or}\\ 18.770}}\left(\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right] - 78\right)dt$$

$$= 0.05(101.92741) = 5.096 \approx \$5.10$$

3
- 2: integral
  - 1: limits and 0.05
  - 1: integrand
- 1: answer
- 0/1 if integral not of the form $k\int_a^b (F(t) - 78)\,dt$

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6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.
   (a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.
   (b) Write an equation of each horizontal tangent line to the curve.
   (c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x– and y–coordinates of point P.

END OF EXAMINATION

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6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.
   (a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.
   (b) Write an equation of each horizontal tangent line to the curve.
   (c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x– and y–coordinates of point P.

(a) $6y^2\dfrac{dy}{dx} + 6x^2\dfrac{dy}{dx} + 12xy - 24x + 6\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$

$\dfrac{dy}{dx} = \dfrac{24x - 12xy}{6x^2 + 6y^2 + 6} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$

2 { 1: implicit differentiation; 1: verifies expression for $\dfrac{dy}{dx}$

(b) $\dfrac{dy}{dx} = 0$

$4x - 2xy = 2x(2 - y) = 0$

$x = 0$ or $y = 2$

When $x = 0$, $2y^3 + 6y = 1$; $y = 0.165$

There is no point on the curve with y coordinate of 2.

$y = 0.165$ is the equation of the only horizontal tangent line.

4 { 1: sets $\dfrac{dy}{dx} = 0$; 1: solves $\dfrac{dy}{dx} = 0$; 1: uses solutions for x to find equations of horizontal tangent lines; 1: verifies which solutions for y yield equations of horizontal tangent lines

Note: max 1/4 [1-0-0-0] if $dy/dx = 0$ is not of the form $g(x, y)/h(x, y) = 0$ with solutions for both x and y

(c) $y = -x$ is equation of the line.

$2(-x)^3 + 6x^2(-x) - 12x^2 + 6(-x) = 1$

$-8x^3 - 12x^2 - 6x - 1 = 0$

$x = -1/2\,, \quad y = 1/2$

–or–

$\dfrac{dy}{dx} = -1$

$4x - 2xy = -x^2 - y^2 - 1$

$4x + 2x^2 = -x^2 - x^2 - 1$

$4x^2 + 4x + 1 = 0$

$x = -1/2\,, \quad y = 1/2$

3 { 1: $y = -x$; 1: substitutes $y = -x$ into equation of curve; 1: solves for x and y

–or–

3 { 1: sets $\dfrac{dy}{dx} = -1$; 1: substitutes $y = -x$ into $\dfrac{dy}{dx}$; 1: solves for x and y

Note: max 2/3 [1-1-0] if importing incorrect derivative from part (a)

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Advanced Placement Examination

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

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1. A particle moves along the y-axis with velocity given by $v(t) = t \sin(t^2)$ for $t \geq 0$.

   (a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

   (b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

   (c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

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   (c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.
   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

(a) $v(1.5) = 1.5\sin(1.5^2) = 1.167$

Up, because $v(1.5) > 0$

1: answer and reason

(b) $a(t) = v'(t) = \sin t^2 + 2t^2\cos t^2$

$a(1.5) = v'(1.5) = -2.048$ or -2.049

No; v is decreasing at 1.5 because $v'(1.5) < 0$

2
- 1: $a(1.5)$
- 1: conclusion and reason

(c) $y(t) = \int v(t)\,dt$

$= \int t\sin t^2\,dt = -\dfrac{\cos t^2}{2} + C$

$y(0) = 3 = -\dfrac{1}{2} + C \implies C = \dfrac{7}{2}$

$y(t) = -\dfrac{1}{2}\cos t^2 + \dfrac{7}{2}$

$y(2) = -\dfrac{1}{2}\cos 4 + \dfrac{7}{2} = 3.826$ or 3.827

3
- 1: $y(t) = \int v(t)\,dt$
- 1: $y(t) = -\dfrac{1}{2}\cos t^2 + C$
- 1: $y(2)$

(d) distance $= \displaystyle\int_0^2 |v(t)|\,dt = 1.173$

or

$v(t) = t\sin t^2 = 0$

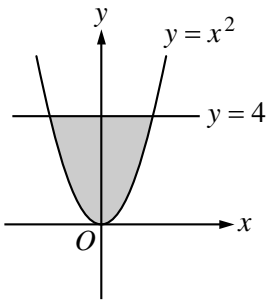
$t = 0$ or $t = \sqrt{\pi} \approx 1.772$

$y(0) = 3$; $y(\sqrt{\pi}) = 4$; $y(2) = 3.826$ or 3.827

$[y(\sqrt{\pi}) - y(0)] + [y(\sqrt{\pi}) - y(2)]$

$= 1.173$ or 1.174

3
- 1: limits of 0 and 2 on an integral of $v(t)$ or $|v(t)|$
  or
  uses $y(0)$ and $y(2)$ to compute distance
- 1: handles change of direction at student's turning point
- 1: answer
  0/1 if incorrect turning point



2. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated by revolving R about the x-axis.

   (c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

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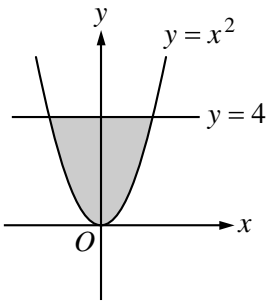
2. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated by revolving R about the x–axis.
   (c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.



(a) Area $= \displaystyle\int_{-2}^{2} (4 - x^2)\,dx$

$= 2\displaystyle\int_{0}^{2} (4 - x^2)\,dx$

$= 2\left[4x - \dfrac{x^3}{3}\right]_0^2$

$= \dfrac{32}{3} = 10.666$ or 10.667

2 { 1: integral; 1: answer }

(b) Volume $= \pi\displaystyle\int_{-2}^{2} \left(4^2 - (x^2)^2\right) dx$

$= 2\pi\displaystyle\int_{0}^{2} (16 - x^4)\,dx$

$= 2\pi\left[16x - \dfrac{x^5}{5}\right]_0^2$

$= \dfrac{256\pi}{5} = 160.849$ or 160.850

3 { 1: limits and constant; 1: integrand; 1: answer }

(c) $\pi\displaystyle\int_{-2}^{2} \left[(k - x^2)^2 - (k - 4)^2\right] dx = \dfrac{256\pi}{5}$

4 { 1: limits and constant; 2: integrand <−1> each error; 1: equation }

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$. Using correct units, explain the meaning of your answer in terms of water flow.

   (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

   (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$.
   Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period.
   Indicate units of measure.

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3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24–hour period.

t (hours)	$R(t)$ (gallons per hour)
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3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

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   (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24–hour time period. Indicate units of measure.

(a) $\int_0^{24} R(t)\,dt \approx 6[R(3) + R(9) + R(15) + R(21)]$
$= 6[10.4 + 11.2 + 11.3 + 10.2]$
$= 258.6$ gallons

This is an approximation to the total flow in gallons of water from the pipe in the 24–hour period.

3 { 1: $R(3) + R(9) + R(15) + R(21)$; 1: answer ; 1: explanation }

(b) Yes;

Since $R(0) = R(24) = 9.6$, the Mean Value Theorem guarantees that there is a t, $0 < t < 24$, such that $R'(t) = 0$.

2 { 1: answer ; 1: MVT or equivalent }

(c) Average rate of flow
$\approx$ average value of $Q(t)$
$= \frac{1}{24}\int_0^{24} \frac{1}{79}(768 + 23t - t^2)\,dt$
$= 10.785$ gal/hr or 10.784 gal/hr

3 { 1: limits and average value constant ; 1: $Q(t)$ as integrand ; 1: answer }

(units) Gallons in part (a) and gallons/hr in part (c), or equivalent.

1: units

4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

   (a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

   (b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

   (c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

   (d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

GO ON TO THE NEXT PAGE

4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.
   (a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.
   (b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.
   (c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.
   (d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

(a) Slope at $x = 0$ is $f'(0) = -3$

At $x = 0$, $y = 2$

$y - 2 = -3(x - 0)$

1: equation

(b) No. Whether $f''(x)$ changes sign at $x = 0$ is unknown. The only given value of $f''(x)$ is $f''(0) = 0$.

2 { 1: answer; 1: explanation }

(c) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$g'(0) = e^0(3f(0) + 2f'(0))$

$= 3(2) + 2(-3) = 0$

$y - 4 = 0(x - 0)$

$y = 4$

2 { 1: $g'(0)$; 1: equation }

(d) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

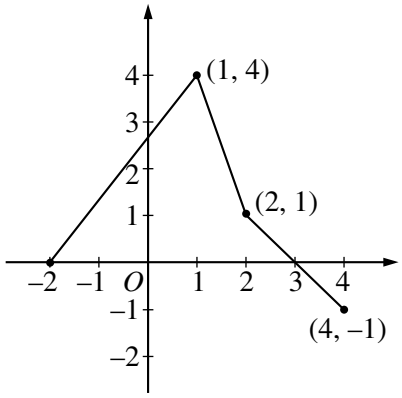
$g''(x) = (-2e^{-2x})(3f(x) + 2f'(x)) + e^{-2x}(3f'(x) + 2f''(x))$

$= e^{-2x}(-6f(x) - f'(x) + 2f''(x))$

$g''(0) = e^0[(-6)(2) - (-3) + 2(0)] = -9$

Since $g'(0) = 0$ and $g''(0) < 0$, g does have a local maximum at $x = 0$.

4 { 2: verify derivative; 0/2 product or chain rule error; <−1> algebra errors; 1: $g'(0) = 0$ and $g''(0)$; 1: answer and reasoning }



5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

   (a) Compute $g(4)$ and $g(-2)$.

   (b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

   (c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

   (d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

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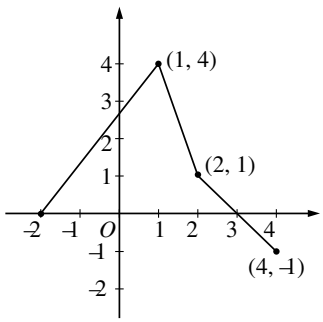
5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.



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   (b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

   (c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

   (d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x–coordinates of points of inflection of the graph of g? Justify your answer.

(a) $g(4) = \int_1^4 f(t)\,dt = \frac{3}{2} + 1 + \frac{1}{2} - \frac{1}{2} = \frac{5}{2}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\frac{1}{2}(12) = -6$

2 { 1: $g(4)$; 1: $g(-2)$

(b) $g'(1) = f(1) = 4$

1: answer

(c) g is increasing on $[-2, 3]$ and decreasing on $[3, 4]$.

Therefore, g has absolute minimum at an endpoint of $[-2, 4]$.

Since $g(-2) = -6$ and $g(4) = \frac{5}{2}$,

the absolute minimum value is -6.

3 { 1: interior analysis ; 1: endpoint analysis ; 1: answer

(d) One; $x = 1$

On $(-2, 1)$, $g''(x) = f'(x) > 0$

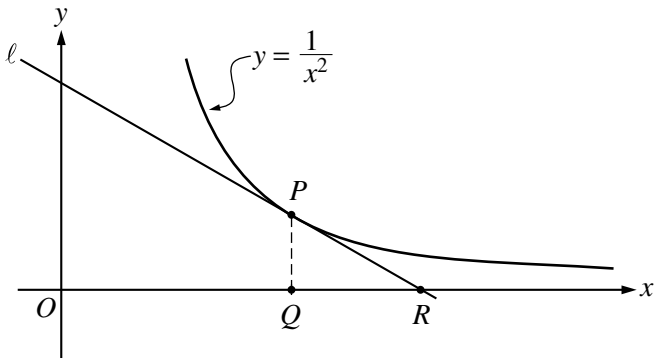
On $(1, 2)$, $g''(x) = f'(x) < 0$

On $(2, 4)$, $g''(x) = f'(x) < 0$

Therefore $(1, g(1))$ is a point of inflection and $(2, g(2))$ is not.

3 { 1: choice of $x = 1$ only ; 1: show $(1, g(1))$ is a point of inflection ; 1: show $(2, g(2))$ is not a point of inflection

1999 CALCULUS AB



6. In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P, with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x-axis at point R, with coordinates $(k, 0)$.

   (a) Find the value of k when $w = 3$.

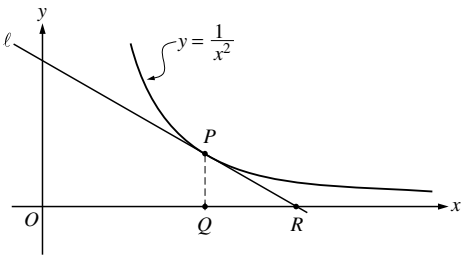
   (b) For all $w > 0$, find k in terms of w.

   (c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

   (d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of n PQR with respect to time? Determine whether the area is increasing or decreasing at this instant.

END OF EXAMINATION

6. In the figure above, line ℓ is tangent to the graph of $y = \dfrac{1}{x^2}$ at point P, with coordinates $\left(w, \dfrac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x–axis at the point R, with coordinates $(k, 0)$.



(a) Find the value of k when $w = 3$.

(b) For all $w > 0$, find k in terms of w.

(c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

(d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

(a) $\dfrac{dy}{dx} = -\dfrac{2}{x^3}$; $\left.\dfrac{dy}{dx}\right|_{x=3} = -\dfrac{2}{27}$

Line ℓ through $\left(3, \dfrac{1}{9}\right)$ and $(k, 0)$ has slope $-\dfrac{2}{27}$.

Therefore, $\dfrac{0 - \frac{1}{9}}{k - 3} = -\dfrac{2}{27}$ or $0 - \dfrac{1}{9} = -\dfrac{2}{27}(k - 3)$

$k = \dfrac{9}{2}$

2 { 1: $\left.\dfrac{dy}{dx}\right|_{x=3}$; 1: answer }

(b) Line ℓ through $\left(w, \dfrac{1}{w^2}\right)$ and $(k, 0)$ has slope $-\dfrac{2}{w^3}$.

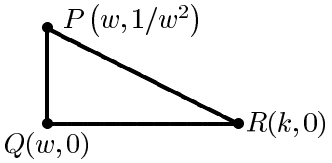
Therefore, $\dfrac{0 - \frac{1}{w^2}}{k - w} = -\dfrac{2}{w^3}$ or $0 - \dfrac{1}{w^2} = -\dfrac{2}{w^3}(k - w)$

$k = \dfrac{3}{2}w$

2 { 1: equation relating w and k, using slopes ; 1: answer }

(c) $\dfrac{dk}{dt} = \dfrac{3}{2}\dfrac{dw}{dt} = \dfrac{3}{2} \cdot 7 = \dfrac{21}{2}$; $\left.\dfrac{dk}{dt}\right|_{w=5} = \dfrac{21}{2}$

1: answer using $\dfrac{dw}{dt} = 7$

(d) 

$A = \dfrac{1}{2}(k - w)\dfrac{1}{w^2} = \dfrac{1}{2}\left(\dfrac{3}{2}w - w\right)\dfrac{1}{w^2} = \dfrac{1}{4w}$

$\dfrac{dA}{dt} = -\dfrac{1}{4w^2}\dfrac{dw}{dt}$

$\left.\dfrac{dA}{dt}\right|_{w=5} = -\dfrac{1}{100} \cdot 7 = -0.07$

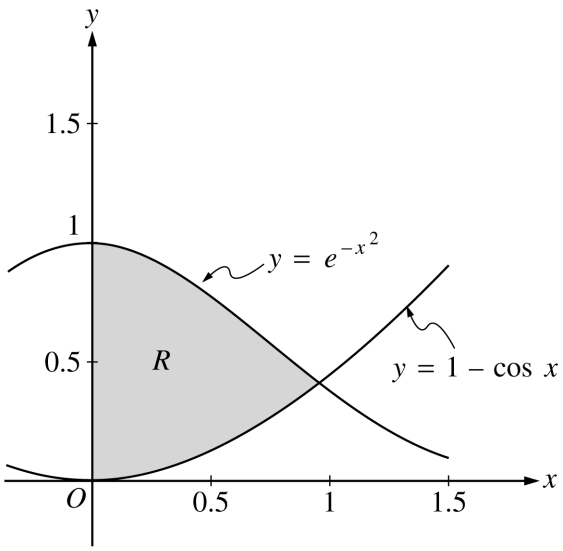
Therefore, area is decreasing.

4 { 1: area in terms of w and/or k ; 1: $\dfrac{dA}{dt}$ implicitly ; 1: $\left.\dfrac{dA}{dt}\right|_{w=5}$ using $\dfrac{dw}{dt} = 7$; 1: conclusion }

Note: 0/4 if A constant

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

   (a) Find the area of the region R.

   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

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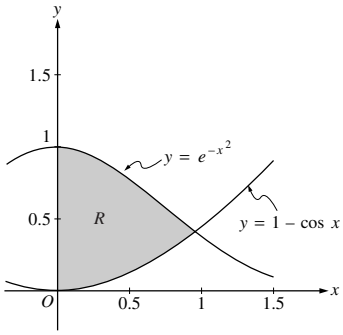
(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



Region R

$e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$

(a) Area $= \int_0^A (e^{-x^2} - (1 - \cos x))\,dx$

$= 0.590$ or 0.591

2 { 1 : integrand; 1 : answer }

(b) Volume $= \pi \int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2 \right) dx$

$= 0.55596\pi = 1.746$ or 1.747

3 { 2 : integrand and constant; $< -1 >$ each error; 1 : answer }

(c) Volume $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$

$= 0.461$

3 { 2 : integrand; $< -1 >$ each error; Note: 0/2 if not of the form $k\int_c^d (f(x) - g(x))^2\,dx$; 1 : answer }

1 : Correct limits in an integral in (a), (b), or (c).

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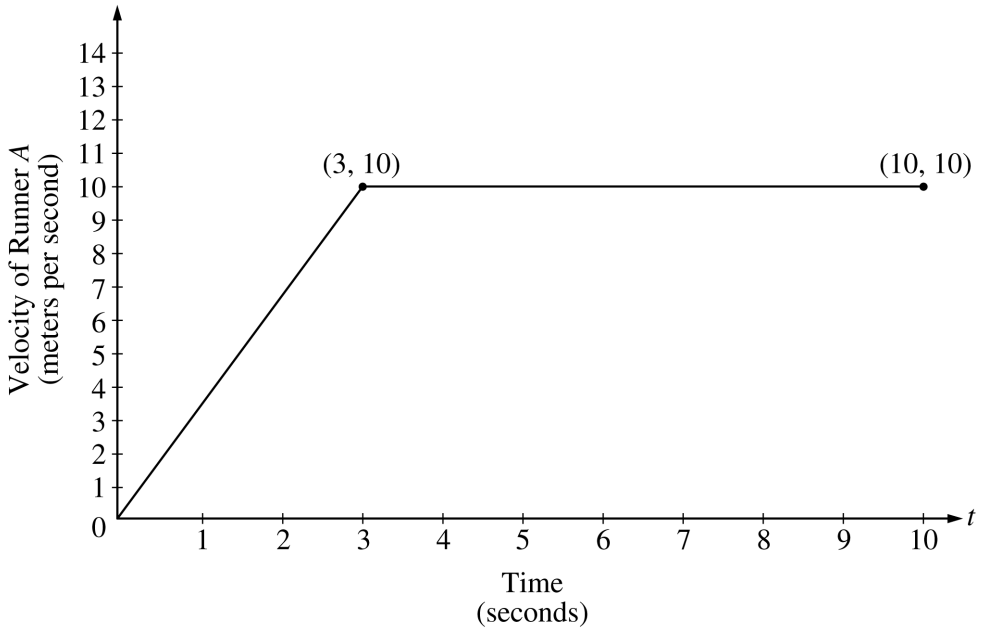

2. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t + 3}$.

   (a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

   (b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

   (c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

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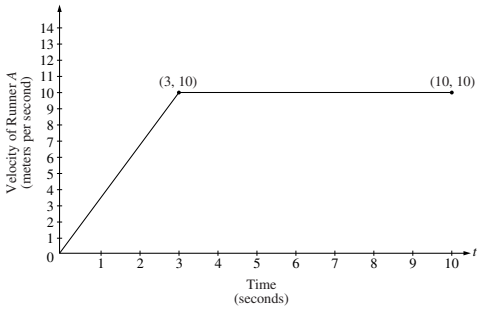

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(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

(a) Runner A: velocity $= \dfrac{10}{3} \cdot 2 = \dfrac{20}{3}$

$= 6.666$ or 6.667 meters/sec

Runner B: $v(2) = \dfrac{48}{7} = 6.857$ meters/sec

2 { 1 : velocity for Runner A ; 1 : velocity for Runner B }

(b) Runner A: acceleration $= \dfrac{10}{3} = 3.333$ meters/sec^2

Runner B: $a(2) = v'(2) = \left.\dfrac{72}{(2t+3)^2}\right|_{t=2}$

$= \dfrac{72}{49} = 1.469$ meters/sec^2

2 { 1 : acceleration for Runner A ; 1 : acceleration for Runner B }

(c) Runner A: distance $= \dfrac{1}{2}(3)(10) + 7(10) = 85$ meters

Runner B: distance $= \displaystyle\int_0^{10} \frac{24t}{2t+3}\,dt = 83.336$ meters

4 { 2 : distance for Runner A ; 1 : method ; 1 : answer ; 2 : distance for Runner B ; 1 : integral ; 1 : answer }

(units) meters/sec in part (a), meters/sec^2 in part (b), and meters in part (c), or equivalent.

1: units

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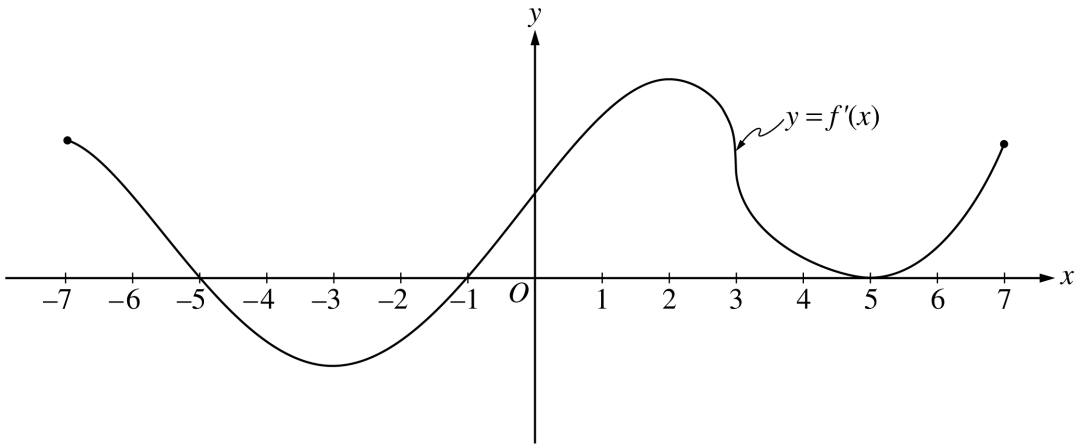

3. The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

   (a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

   (b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

   (c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

   (d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

END OF PART A OF SECTION II

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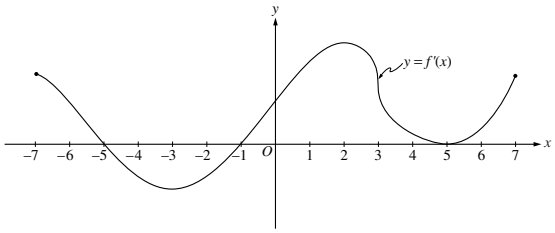

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(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

(a) $x = -1$

$f'(x)$ changes from negative to positive at $x = -1$

2 { 1 : answer; 1 : justification }

(b) $x = -5$

$f'(x)$ changes from positive to negative at $x = -5$

2 { 1 : answer; 1 : justification }

(c) $f''(x)$ exists and f' is decreasing on the intervals $(-7,-3)$, $(2,3)$, and $(3,5)$

2 { 1 : $(-7,-3)$; 1 : $(2,3) \cup (3,5)$ }

(d) $x = 7$

The absolute maximum must occur at $x = -5$ or at an endpoint.
$f(-5) > f(-7)$ because f is increasing on $(-7,-5)$

The graph of f' shows that the magnitude of the negative change in f from $x = -5$ to $x = -1$ is smaller than the positive change in f from $x = -1$ to $x = 7$. Therefore the net change in f is positive from $x = -5$ to $x = 7$, and $f(7) > f(-5)$. So $f(7)$ is the absolute maximum.

3 { 1 : answer; 1 : identifies $x = -5$ and $x = 7$ as candidates; – or – ; indicates that the graph of f increases, decreases, then increases; 1 : justifies $f(7) > f(-5)$ }

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

   (a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

   (b) How many gallons of water are in the tank at time $t = 3$ minutes?

   (c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

   (d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

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Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

(a) Method 1: $\displaystyle\int_0^3 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2}\Big|_0^3 = \frac{14}{3}$

– or –

Method 2: $L(t)$ = gallons leaked in first t minutes

$$\frac{dL}{dt} = \sqrt{t+1}; \quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$$
$$L(0) = 0; \quad C = -\frac{2}{3}$$
$$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; \quad L(3) = \frac{14}{3}$$

Method 1:

3 { 2 : definite integral; 1 : limits; 1 : integrand; 1 : answer

– or –

Method 2:

3 { 1 : antiderivative with C; 1 : solves for C using $L(0) = 0$; 1 : answer

(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

1 : answer

(c) Method 1:

$$A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right) dx$$
$$= 30 + 8t - \int_0^t \sqrt{x+1}\,dx$$

Method 1:

2 { 1 : $30 + 8t$; 1 : $-\int_0^t \sqrt{x+1}\,dx$

– or –

Method 2:

$$\frac{dA}{dt} = 8 - \sqrt{t+1}$$
$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$$
$$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C; \quad C = \frac{92}{3}$$
$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$$

– or –

Method 2:

2 { 1 : antiderivative with C; 1 : answer

(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$

$A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.

3 { 1 : sets $A'(t) = 0$; 1 : solves for t; 1 : justification

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

5. Consider the curve given by $xy^2 - x^3y = 6$.

   (a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

   (b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

   (c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

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Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a) $y^2 + 2xy\dfrac{dy}{dx} - 3x^2y - x^3\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$

$\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$

2 { 1 : implicit differentiation; 1 : verifies expression for $\dfrac{dy}{dx}$

(b) When $x = 1$, $y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y - 3)(y + 2) = 0$

$y = 3,\ y = -2$

At $(1,3)$, $\dfrac{dy}{dx} = \dfrac{9 - 9}{6 - 1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\dfrac{dy}{dx} = \dfrac{-6 - 4}{-4 - 1} = \dfrac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x - 1)$

4 { 1 : $y^2 - y = 6$; 1 : solves for y; 2 : tangent lines

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x\left(2y - x^2\right) = 0$ gives $x = 0$ or $y = \dfrac{1}{2}x^2$

There is no point on the curve with x-coordinate 0.

When $y = \dfrac{1}{2}x^2$, $\dfrac{1}{4}x^5 - \dfrac{1}{2}x^5 = 6$

$-\dfrac{1}{4}x^5 = 6$

$x = \sqrt[5]{-24}$

3 { 1 : sets denominator of $\dfrac{dy}{dx}$ equal to 0; 1 : substitutes $y = \dfrac{1}{2}x^2$ or $x = \pm\sqrt{2y}$ into the equation for the curve; 1 : solves for x-coordinate

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

6. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

   (a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

   (b) Find the domain and range of the function f found in part (a).

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Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

(a) $e^{2y}\,dy = 3x^2\,dx$

$\dfrac{1}{2}e^{2y} = x^3 + C_1$

$e^{2y} = 2x^3 + C$

$y = \dfrac{1}{2}\ln\left(2x^3 + C\right)$

$\dfrac{1}{2} = \dfrac{1}{2}\ln(0 + C); \quad C = e$

$y = \dfrac{1}{2}\ln(2x^3 + e)$

6
- 1 : separates variables
- 1 : antiderivative of dy term
- 1 : antiderivative of dx term
- 1 : constant of integration
- 1 : uses initial condition $f(0) = \dfrac{1}{2}$
- 1 : solves for y
- Note: 0/1 if y is not a logarithmic function of x

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(b) Domain: $2x^3 + e > 0$

$x^3 > -\dfrac{1}{2}e$

$x > \left(-\dfrac{1}{2}e\right)^{1/3} = -\left(\dfrac{1}{2}e\right)^{1/3}$

Range: $-\infty < y < \infty$

3
- 1 : $2x^3 + e > 0$
- 1 : domain
  - Note: 0/1 if 0 is not in the domain
- 1 : range

Note: 0/3 if y is not a logarithmic function of x

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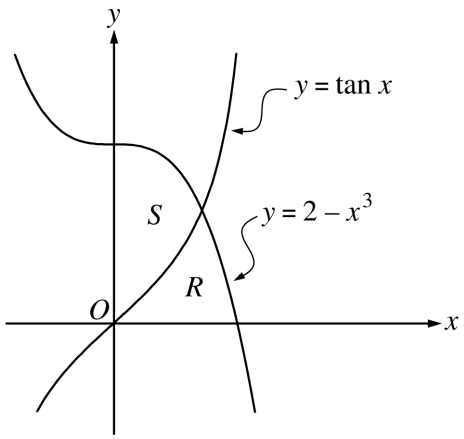
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when S is revolved about the x-axis.

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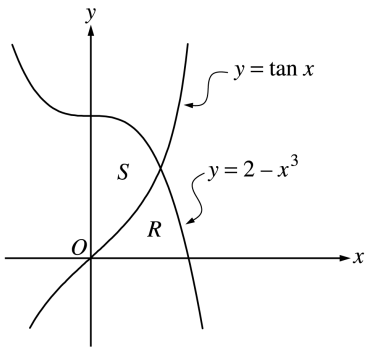
Question 1

Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.

Point of intersection

$2 - x^3 = \tan x$ at $(A, B) = (0.902155, 1.265751)$

(a) Area $R = \displaystyle\int_0^A \tan x\,dx + \int_A^{\sqrt[3]{2}} \left(2 - x^3\right) dx = 0.729$

or

Area $R = \displaystyle\int_0^B \left((2-y)^{1/3} - \tan^{-1} y\right) dy = 0.729$

or

Area $R = \displaystyle\int_0^{\sqrt[3]{2}} \left(2 - x^3\right) dx - \int_0^A \left(2 - x^3 - \tan x\right) dx = 0.729$

3 : { 1 : limits; 1 : integrand; 1 : answer }

(b) Area $S = \displaystyle\int_0^A \left(2 - x^3 - \tan x\right) dx = 1.160$ or 1.161

or

Area $S = \displaystyle\int_0^B \tan^{-1} y\,dy + \int_B^2 (2-y)^{1/3}\,dy = 1.160$ or 1.161

or

Area S

$= \displaystyle\int_0^2 (2-y)^{1/3}\,dy - \int_0^B \left((2-y)^{1/3} - \tan^{-1} y\right) dy$

$= 1.160$ or 1.161

3 : { 1 : limits; 1 : integrand; 1 : answer }

(c) Volume $= \pi \displaystyle\int_0^A \left(\left(2 - x^3\right)^2 - \tan^2 x\right) dx$

$= 2.652\pi$ or 8.331 or 8.332

3 : { 1 : limits and constant; 1 : integrand; 1 : answer }

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t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

   (a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

   (c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

   (d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

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Question 2

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

(a) Difference quotient; e.g.

$W'(12) \approx \dfrac{W(15) - W(12)}{15 - 12} = -\dfrac{1}{3}$ °C/day or

$W'(12) \approx \dfrac{W(12) - W(9)}{12 - 9} = -\dfrac{2}{3}$ °C/day or

$W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = -\dfrac{1}{2}$ °C/day

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer (with units)} \end{cases}$

(b) $\dfrac{3}{2}(20 + 2(31) + 2(28) + 2(24) + 2(22) + 21) = 376.5$

Average temperature $\approx \dfrac{1}{15}(376.5) = 25.1\,°\text{C}$

$2 : \begin{cases} 1 : \text{trapezoidal method} \\ 1 : \text{answer} \end{cases}$

(c) $P'(12) = 10e^{-t/3} - \dfrac{10}{3}te^{-t/3}\Big|_{t=12}$

$= -30e^{-4} = -0.549$ °C/day

This means that the temperature is decreasing at the rate of 0.549 °C/day when $t = 12$ days.

$2 : \begin{cases} 1 : P'(12) \text{ (with or without units)} \\ 1 : \text{interpretation} \end{cases}$

(d) $\dfrac{1}{15}\displaystyle\int_0^{15}\left(20 + 10te^{-t/3}\right)dt = 25.757\,°\text{C}$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and} \\ \quad \text{average value constant} \\ 1 : \text{answer} \end{cases}$

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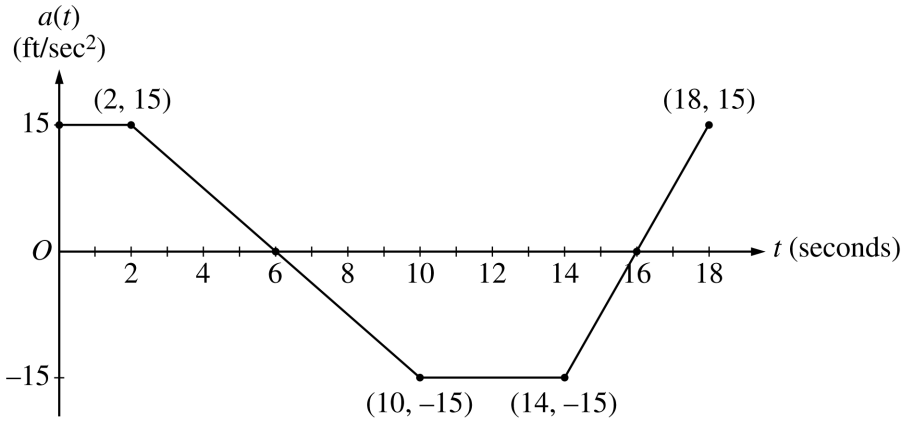

3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

   (a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

   (b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

   (c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

   (d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

END OF PART A OF SECTION II

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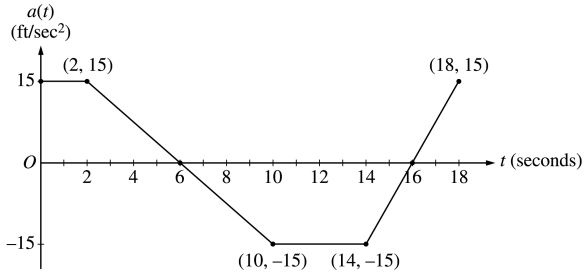
Question 3

A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.



(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

(a) Since $v'(2) = a(2)$ and $a(2) = 15 > 0$, the velocity is increasing at $t = 2$.

1 : answer and reason

(b) At time $t = 12$ because
$v(12) - v(0) = \int_0^{12} a(t)\,dt = 0$.

2 : { 1 : $t = 12$; 1 : reason }

(c) The absolute maximum velocity is 115 ft/sec at $t = 6$.
The absolute maximum must occur at $t = 6$ or at an endpoint.
$v(6) = 55 + \int_0^6 a(t)\,dt$
$= 55 + 2(15) + \frac{1}{2}(4)(15) = 115 > v(0)$
$\int_6^{18} a(t)\,dt < 0$ so $v(18) < v(6)$

4 : { 1 : $t = 6$; 1 : absolute maximum velocity; 1 : identifies $t = 6$ and $t = 18$ as candidates or indicates that v increases, decreases, then increases; 1 : eliminates $t = 18$ }

(d) The car's velocity is never equal to 0. The absolute minimum occurs at $t = 16$ where
$v(16) = 115 + \int_6^{16} a(t)\,dt = 115 - 105 = 10 > 0$.

2 : { 1 : answer; 1 : reason }

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

   (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   (b) On what intervals, if any, is the graph of h concave up? Justify your answer.

   (c) Write an equation for the line tangent to the graph of h at $x = 4$.

   (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

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Question 4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

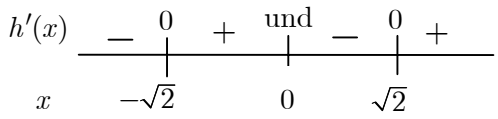
(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$



Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$

4 : { 1 : $x = \pm\sqrt{2}$; 1 : analysis; 2 : conclusions; $< -1 >$ not dealing with discontinuity at 0

(b) $h''(x) = 1 + \dfrac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of h is concave up for all $x \neq 0$.

3 : { 1 : $h''(x)$; 1 : $h''(x) > 0$; 1 : answer

(c) $h'(4) = \dfrac{16 - 2}{4} = \dfrac{7}{2}$

$y + 3 = \dfrac{7}{2}(x - 4)$

1 : tangent line equation

(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

1 : answer with reason

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k ?

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Question 5

A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k?

(a) $f'(x) = 12x^2 + 2ax + b$

$f''(x) = 24x + 2a$

$f'(-1) = 12 - 2a + b = 0$

$f''(-2) = -48 + 2a = 0$

$a = 24$

$b = -12 + 2a = 36$

$5 : \begin{cases} 1 : f'(x) \\ 1 : f''(x) \\ 1 : f'(-1) = 0 \\ 1 : f''(-2) = 0 \\ 1 : a,\ b \end{cases}$

(b) $\int_0^1 \left(4x^3 + 24x^2 + 36x + k\right) dx$

$= x^4 + 8x^3 + 18x^2 + kx\Big|_{x=0}^{x=1} = 27 + k$

$27 + k = 32$

$k = 5$

$4 : \begin{cases} 2 : \text{antidifferentiation} \\ \quad < -1 > \text{each error} \\ 1 : \text{expression in } k \\ 1 : k \end{cases}$

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

   (a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

   (b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

END OF EXAMINATION

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Question 6

The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

(a) $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}(6 - 2x) - 2y^2$

$= 2y^3(6 - 2x)^2 - 2y^2$

$\left.\frac{d^2y}{dx^2}\right|_{\left(3, \frac{1}{4}\right)} = 0 - 2\left(\frac{1}{4}\right)^2 = -\frac{1}{8}$

3 : $\begin{cases} 2 : \frac{d^2y}{dx^2} \\ \quad < -2 > \text{product rule or} \\ \quad\quad \text{chain rule error} \\ 1 : \text{value at } \left(3, \frac{1}{4}\right) \end{cases}$

(b) $\frac{1}{y^2}\,dy = (6 - 2x)\,dx$

$-\frac{1}{y} = 6x - x^2 + C$

$-4 = 18 - 9 + C = 9 + C$

$C = -13$

$y = \frac{1}{x^2 - 6x + 13}$

6 : $\begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } f(3) = \frac{1}{4} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

---

1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

   (a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

   (b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

   (c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

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AP® CALCULUS AB 2002 SCORING GUIDELINES

Question 1

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

(a) Area $= \int_{1/2}^{1} (e^x - \ln x)\,dx = 1.222$ or 1.223

(b) Volume $= \pi \int_{1/2}^{1} \left((4 - \ln x)^2 - (4 - e^x)^2 \right) dx$

$= 7.515\pi$ or 23.609

(c) $h'(x) = f'(x) - g'(x) = e^x - \frac{1}{x} = 0$

$x = 0.567143$

Absolute minimum value and absolute maximum value occur at the critical point or at the endpoints.

$h(0.567143) = 2.330$

$h(0.5) = 2.3418$

$h(1) = 2.718$

The absolute minimum is 2.330.

The absolute maximum is 2.718.

2 { 1 : integral
1 : answer

4 { 1 : limits and constant
2 : integrand
< −1 > each error
Note: 0 / 2 if not of the form $k \int_a^b \left(R(x)^2 - r(x)^2 \right) dx$
1 : answer

3 { 1 : considers $h'(x) = 0$
1 : identifies critical point and endpoints as candidates
1 : answers

Note: Errors in computation come off the third point.

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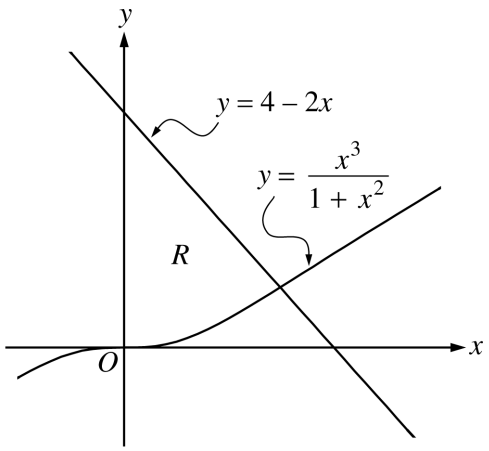
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1 + x^2}$ and $y = 4 - 2x$, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

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AP® CALCULUS AB
2002 SCORING GUIDELINES (Form B)

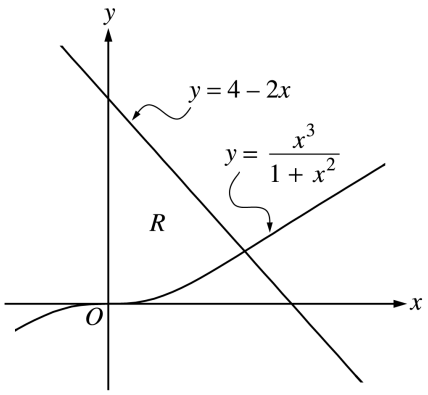
Question 1

Let R be the region bounded by the y-axis and the graphs of $y = \dfrac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



Region R

$\dfrac{x^3}{1+x^2} = 4 - 2x$ at $x = 1.487664 = A$

(a) Area $= \displaystyle\int_0^A \left(4 - 2x - \frac{x^3}{1+x^2}\right) dx$

$= 3.214$ or 3.215

2 : 1 : integrand; 1 : answer

(b) Volume

$= \pi \displaystyle\int_0^A \left((4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right) dx$

$= 31.884$ or 31.885 or 10.149π

3 : 2 : integrand and constant; $< -1 >$ each error; 1 : answer

(c) Volume $= \displaystyle\int_0^A \left(4 - 2x - \frac{x^3}{1+x^2}\right)^2 dx$

$= 8.997$

3 : 2 : integrand; $< -1 >$ each error; note: 0/2 if not of the form $k\displaystyle\int_c^d (f(x) - g(x))^2\, dx$; 1 : answer

1 : Correct limits in an integral in (a), (b), or (c).

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2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

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Question 2

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

(a) $\int_9^{17} E(t)\,dt = 6004.270$

6004 people entered the park by 5 pm.

3 { 1 : limits; 1 : integrand; 1 : answer }

(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$

The amount collected was \$104,048.

or

$\int_{17}^{23} E(t)\,dt = 1271.283$

1271 people entered the park between 5 pm and 11 pm, so the amount collected was $\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041$.

1 : setup

(c) $H'(17) = E(17) - L(17) = -380.281$

There were 3725 people in the park at $t = 17$.

The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.

3 { 1 : value of $H'(17)$; 2 : meanings { 1 : meaning of $H(17)$; 1 : meaning of $H'(17)$; $< -1 >$ if no reference to $t = 17$ } }

(d) $H'(t) = E(t) - L(t) = 0$

$t = 15.794$ or 15.795

2 { 1 : $E(t) - L(t) = 0$; 1 : answer }

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

2. The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

   (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

   (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

   (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

   (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

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GO ON TO THE NEXT PAGE.

AP® CALCULUS AB
2002 SCORING GUIDELINES (Form B)

Question 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

(a) $P'(9) = 1 - 3e^{-0.6} = -0.646 < 0$
so the amount is not increasing at this time.

1 : answer with reason

(b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$
$t = (5\ln 3)^2 = 30.174$
$P'(t)$ is negative for $0 < t < (5\ln 3)^2$ and positive for $t > (5\ln 3)^2$. Therefore there is a minimum at $t = (5\ln 3)^2$.

3 { 1 : sets $P'(t) = 0$; 1 : solves for t ; 1 : justification }

(c) $P(30.174) = 50 + \int_0^{30.174} \left(1 - 3e^{-0.2\sqrt{t}}\right) dt$
$= 35.104 < 40$, so the lake is safe.

3 { 1 : integrand ; 1 : limits ; 1 : conclusion with reason based on integral of $P'(t)$ }

(d) $P'(0) = 1 - 3 = -2$. The lake will become safe when the amount decreases by 10. A linear model predicts this will happen when $t = 5$.

2 { 1 : slope of tangent line ; 1 : answer }

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3. An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

   (a) What is the acceleration of the object at time $t = 4$?

   (b) Consider the following two statements.

   Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.
   Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

   Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

   (c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

   (d) What is the position of the object at time $t = 4$?

END OF PART A OF SECTION II

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Question 3

An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

(a) $a(4) = v'(4) = \frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right)$

$= -\frac{\pi}{6}$ or -0.523 or -0.524

1 : answer

(b) On $3 < t < 4.5$:

$a(t) = v'(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right) < 0$

Statement I is correct since $a(t) < 0$.

Statement II is correct since $v(t) < 0$ and $a(t) < 0$.

3 { 1 : I correct, with reason; 1 : II correct; 1 : reason for II }

(c) Distance $= \int_0^4 |v(t)|\,dt = 2.387$

OR

$x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$

$x(0) = 2$

$x(4) = 2 + \frac{9}{2\pi} = 3.43239$

$v(t) = 0$ when $t = 3$

$x(3) = \frac{6}{\pi} + 2 = 3.90986$

$|x(3) - x(0)| + |x(4) - x(3)| = \frac{15}{2\pi} = 2.387$

3 { 1 : { limits of 0 and 4 on an integral of $v(t)$ or $|v(t)|$ or uses $x(0)$ and $x(4)$ to compute distance }; 1 : handles change of direction at student's turning point; 1 : answer; 0/1 if incorrect turning point or no turning point }

(d) $x(4) = x(0) + \int_0^4 v(t)\,dt = 3.432$

OR

$x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$

$x(4) = 2 + \frac{9}{2\pi} = 3.432$

2 { 1 : integral; 1 : answer }

OR

2 { 1 : $x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + C$; 1 : answer; 0/1 if no constant of integration }

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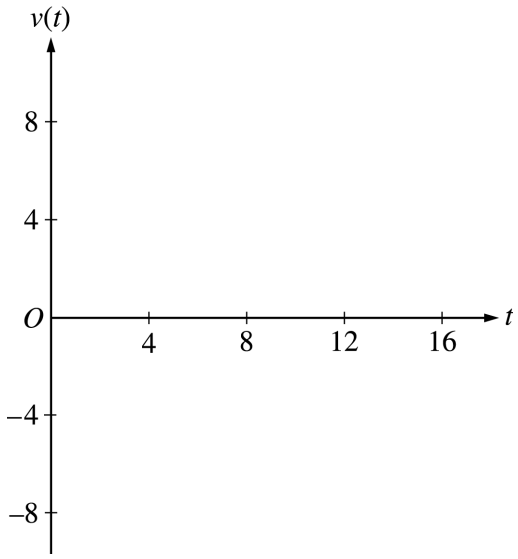
3. A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

   (a) On the axes provided, sketch the graph of $v(t)$ for $0 \le t \le 16$.
   (Note: Use the axes provided in the test booklet.)



   (b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

   (c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

   (d) Is there any time t, $0 < t \le 16$, at which the particle returns to the origin? Justify your answer.

END OF PART A OF SECTION II

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AP® CALCULUS AB
2002 SCORING GUIDELINES (Form B)

Question 3

A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

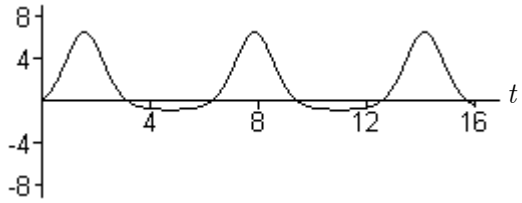
(a) On the axes provided, sketch the graph of $v(t)$ for $0 \le t \le 16$.

(b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

(d) Is there any time t, $0 < t \le 16$, at which the particle returns to the origin? Justify your answer.

(a) $v(t)$



1 : graph
three "humps"
periodic behavior
starts at origin
reasonable relative max and min values

(b) Particle is moving to the left when $v(t) < 0$, i.e. $e^{2\sin t} < 1$.

$(\pi, 2\pi)$, $(3\pi, 4\pi)$ and $(5\pi, 16]$

3 : 2 : intervals
$< -1 >$ each missing or incorrect interval
1 : reason

(c) $\int_0^4 |v(t)|\,dt = 10.542$

or

$v(t) = e^{2\sin t} - 1 = 0$

$t = 0$ or $t = \pi$

$x(\pi) = \int_0^{\pi} v(t)\,dt = 10.10656$

$x(4) = \int_0^4 v(t)\,dt = 9.67066$

$|x(\pi) - x(0)| + |x(4) - x(\pi)|$

$= 10.542$

3 : 1 : limits of 0 and 4 on an integral of
$v(t)$ or $|v(t)|$
or
uses $x(0)$ and $x(4)$ to compute distance
1 : handles change of direction at student's turning point
1 : answer
note: 0/1 if incorrect turning point

(d) There is no such time because $\int_0^T v(t)\,dt > 0$ for all $T > 0$.

2 : 1 : no such time
1 : reason

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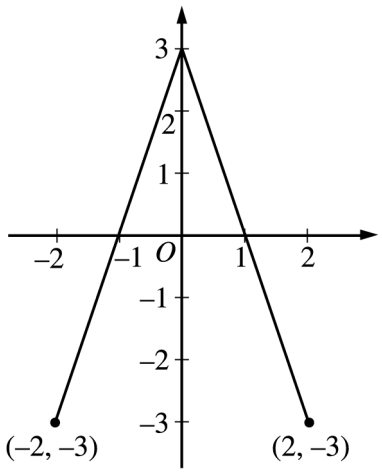
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

   (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

   (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

   (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
   (Note: The axes are provided in the pink test booklet only.)

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Question 4

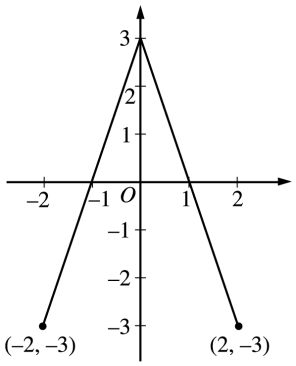
The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of x in the open interval $(-2,2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2,2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2,2]$.



Graph of f

(a) $g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = -\frac{3}{2}$

$g'(-1) = f(-1) = 0$

$g''(-1) = f'(-1) = 3$

3 { 1 : $g(-1)$; 1 : $g'(-1)$; 1 : $g''(-1)$

(b) g is increasing on $-1 < x < 1$ because $g'(x) = f(x) > 0$ on this interval.

2 { 1 : interval; 1 : reason

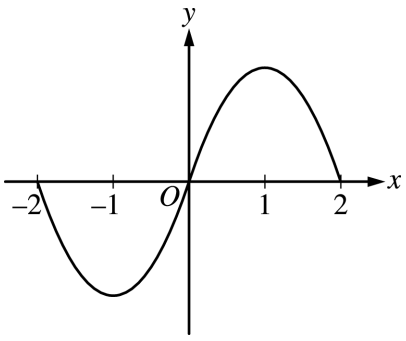
(c) The graph of g is concave down on $0 < x < 2$ because $g''(x) = f'(x) < 0$ on this interval.

or

because $g'(x) = f(x)$ is decreasing on this interval.

2 { 1 : interval; 1 : reason

(d)



2 { 1 : $g(-2) = g(0) = g(2) = 0$; 1 : appropriate increasing/decreasing and concavity behavior; $< -1 >$ vertical asymptote

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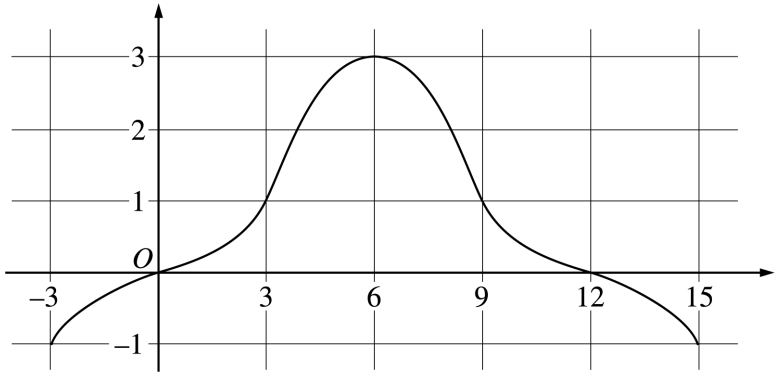
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

   (a) Find $g(6)$, $g'(6)$, and $g''(6)$.

   (b) On what intervals is g decreasing? Justify your answer.

   (c) On what intervals is the graph of g concave down? Justify your answer.

   (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

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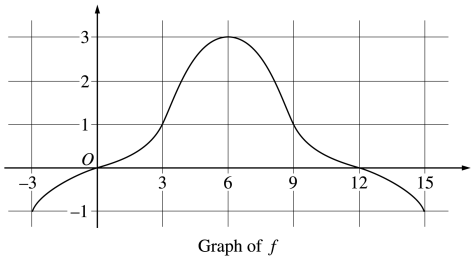
Question 4

The graph of a differentiable function f on the closed interval $[-3,15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.



Graph of f

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

(a) $g(6) = 5 + \int_6^6 f(t)\,dt = 5$

$g'(6) = f(6) = 3$

$g''(6) = f'(6) = 0$

3 { 1 : $g(6)$; 1 : $g'(6)$; 1 : $g''(6)$ }

(b) g is decreasing on $[-3,0]$ and $[12,15]$ since $g'(x) = f(x) < 0$ for $x < 0$ and $x > 12$.

3 { 1 : $[-3,0]$; 1 : $[12,15]$; 1 : justification }

(c) The graph of g is concave down on $(6,15)$ since $g' = f$ is decreasing on this interval.

2 { 1 : interval; 1 : justification }

(d) $\frac{3}{2}(-1 + 2(0 + 1 + 3 + 1 + 0) - 1)$

$= 12$

1 : trapezoidal method

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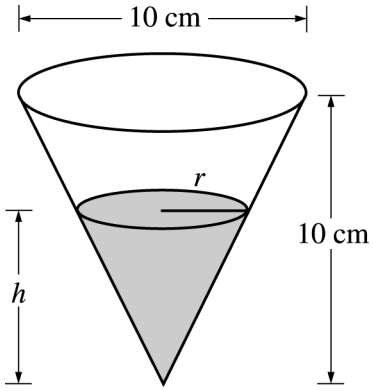

5. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

   (Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

   (a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

   (b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

   (c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

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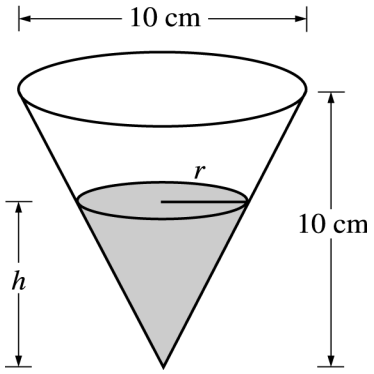
Question 5



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

(a) When $h = 5$, $r = \frac{5}{2}$; $V(5) = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 5 = \frac{125}{12}\pi$ cm^3

(b) $\frac{r}{h} = \frac{5}{10}$, so $r = \frac{1}{2}h$

$V = \frac{1}{3}\pi\left(\frac{1}{4}h^2\right)h = \frac{1}{12}\pi h^3$; $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\left.\frac{dV}{dt}\right|_{h=5} = \frac{1}{4}\pi(25)\left(-\frac{3}{10}\right) = -\frac{15}{8}\pi$ cm^3/hr

OR

$\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right)$; $\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$

$\left.\frac{dV}{dt}\right|_{h=5, r=\frac{5}{2}} = \frac{1}{3}\pi\left(\left(\frac{25}{4}\right)\left(-\frac{3}{10}\right) + 2\left(\frac{5}{2}\right)5\left(-\frac{3}{20}\right)\right)$

$= -\frac{15}{8}\pi$ cm^3/hr

(c) $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt} = -\frac{3}{40}\pi h^2$

$= -\frac{3}{40}\pi(2r)^2 = -\frac{3}{10}\pi r^2 = -\frac{3}{10} \cdot$ area

The constant of proportionality is $-\frac{3}{10}$.

5
- 1 : V when $h = 5$
- 1 : $r = \frac{1}{2}h$ in (a) or (b)
- 1 : V as a function of one variable in (a) or (b) OR $\frac{dr}{dt}$
- 2 : $\frac{dV}{dt}$ $< -2 >$ chain rule or product rule error
- 1 : evaluation at $h = 5$

2
- 1 : shows $\frac{dV}{dt} = k \cdot$ area
- 1 : identifies constant of proportionality

---

units of cm^3 in (a) and cm^3/hr in (b)

1 : correct units in (a) and (b)

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

5. Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

   (a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

   (b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

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AP® CALCULUS AB
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Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

(a) $\frac{dy}{dx} = 0$ when $x = 3$

$\left.\frac{d^2y}{dx^2}\right|_{(3,-2)} = \left.\frac{-y - y'(3-x)}{y^2}\right|_{(3,-2)} = \frac{1}{2}$,

so f has a local minimum at this point.

or

Because f is continuous for $1 < x < 5$, there is an interval containing $x = 3$ on which $y < 0$. On this interval, $\frac{dy}{dx}$ is negative to the left of $x = 3$ and $\frac{dy}{dx}$ is positive to the right of $x = 3$. Therefore f has a local minimum at $x = 3$.

3 { 1 : $x = 3$; 1 : local minimum ; 1 : justification }

(b) $y\,dy = (3-x)\,dx$

$\frac{1}{2}y^2 = 3x - \frac{1}{2}x^2 + C$

$8 = 18 - 18 + C$; $C = 8$

$y^2 = 6x - x^2 + 16$

$y = -\sqrt{6x - x^2 + 16}$

6 { 1 : separates variables ; 1 : antiderivative of dy term ; 1 : antiderivative of dx term ; 1 : constant of integration ; 1 : uses initial condition $g(6) = -4$; 1 : solves for y }

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

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x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

6. Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

   (a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

   (b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

   (c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

   (d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0 \end{cases}$.
   The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

END OF EXAMINATION

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AP® CALCULUS AB 2002 SCORING GUIDELINES

Question 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} \big(3f'(x) + 4\big)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $\big(x, f(x)\big)$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

(a) $\displaystyle\int_0^{1.5} \big(3f'(x) + 4\big)\,dx = 3\int_0^{1.5} f'(x)\,dx + \int_0^{1.5} 4\,dx$

$= 3f(x) + 4x\Big|_0^{1.5} = 3(-1 - (-7)) + 4(1.5) = 24$

2 : 1 : antiderivative; 1 : answer

(b) $y = 5(x - 1) - 4$

$f(1.2) \approx 5(0.2) - 4 = -3$

The approximation is less than $f(1.2)$ because the graph of f is concave up on the interval $1 < x < 1.2$.

3 : 1 : tangent line; 1 : computes y on tangent line at $x = 1.2$; 1 : answer with reason

(c) By the Mean Value Theorem there is a c with $0 < c < 0.5$ such that

$f''(c) = \dfrac{f'(0.5) - f'(0)}{0.5 - 0} = \dfrac{3 - 0}{0.5} = 6 = r$

2 : 1 : reference to MVT for f' (or differentiability of f'); 1 : value of r for interval $0 \le x \le 0.5$

(d) $\lim\limits_{x \to 0^-} g'(x) = \lim\limits_{x \to 0^-} (4x - 1) = -1$

$\lim\limits_{x \to 0^+} g'(x) = \lim\limits_{x \to 0^+} (4x + 1) = +1$

Thus g' is not continuous at $x = 0$, but f' is continuous at $x = 0$, so $f \ne g$.

OR

$g''(x) = 4$ for all $x \ne 0$, but it was shown in part (c) that $f''(c) = 6$ for some $c \ne 0$, so $f \ne g$.

2 : 1 : answers "no" with reference to g' or g''; 1 : correct reason

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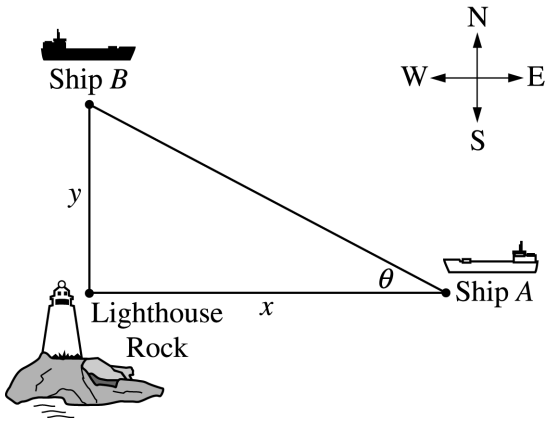

6. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

   (a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

   (b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

   (c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

END OF EXAMINATION

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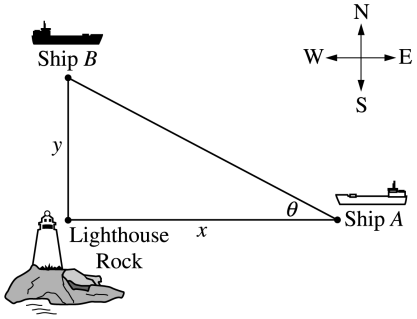
AP® CALCULUS AB
2002 SCORING GUIDELINES (Form B)

Question 6

Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.



(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

(a) Distance $= \sqrt{3^2 + 4^2} = 5$ km

1 : answer

(b) $r^2 = x^2 + y^2$

$2r\dfrac{dr}{dt} = 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt}$

or explicitly:

$r = \sqrt{x^2 + y^2}$

$\dfrac{dr}{dt} = \dfrac{1}{2\sqrt{x^2 + y^2}}\left(2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt}\right)$

At $x = 4$, $y = 3$,

$\dfrac{dr}{dt} = \dfrac{4(-15) + 3(10)}{5} = -6$ km/hr

4 : 1 : expression for distance
2 : differentiation with respect to t
$< -2 >$ chain rule error
1 : evaluation

(c) $\tan\theta = \dfrac{y}{x}$

$\sec^2\theta\,\dfrac{d\theta}{dt} = \dfrac{\dfrac{dy}{dt}x - \dfrac{dx}{dt}y}{x^2}$

At $x = 4$ and $y = 3$, $\sec\theta = \dfrac{5}{4}$

$\dfrac{d\theta}{dt} = \dfrac{16}{25}\left(\dfrac{10(4) - (-15)(3)}{16}\right)$

$= \dfrac{85}{25} = \dfrac{17}{5}$ radians/hr

4 : 1 : expression for θ in terms of x and y
2 : differentiation with respect to t
$< -2 >$ chain rule, quotient rule, or transcendental function error
note: 0/2 if no trig or inverse trig function
1 : evaluation

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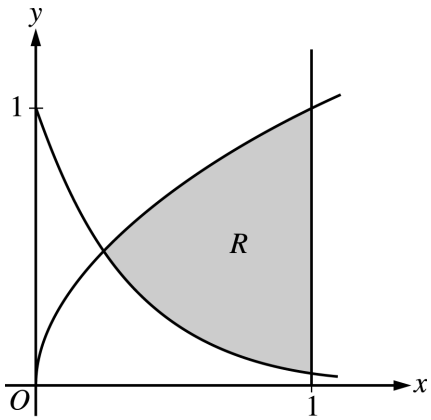
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

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AP® CALCULUS AB
2003 SCORING GUIDELINES

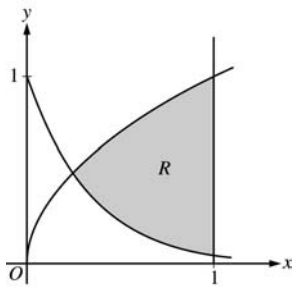
Question 1

Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.



Point of intersection
$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$

(a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx$
$= 0.442$ or 0.443

1: Correct limits in an integral in (a), (b), or (c)

2 : { 1 : integrand; 1 : answer }

(b) Volume $= \pi \int_T^1 \left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2\right) dx$
$= 0.453\pi$ or 1.423 or 1.424

3 : { 2 : integrand
$< -1 >$ reversal
$< -1 >$ error with constant
$< -1 >$ omits 1 in one radius
$< -2 >$ other errors
1 : answer }

(c) Length $= \sqrt{x} - e^{-3x}$
Height $= 5\left(\sqrt{x} - e^{-3x}\right)$

Volume $= \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$

3 : { 2 : integrand
$< -1 >$ incorrect but has $\sqrt{x} - e^{-3x}$ as a factor
1 : answer }

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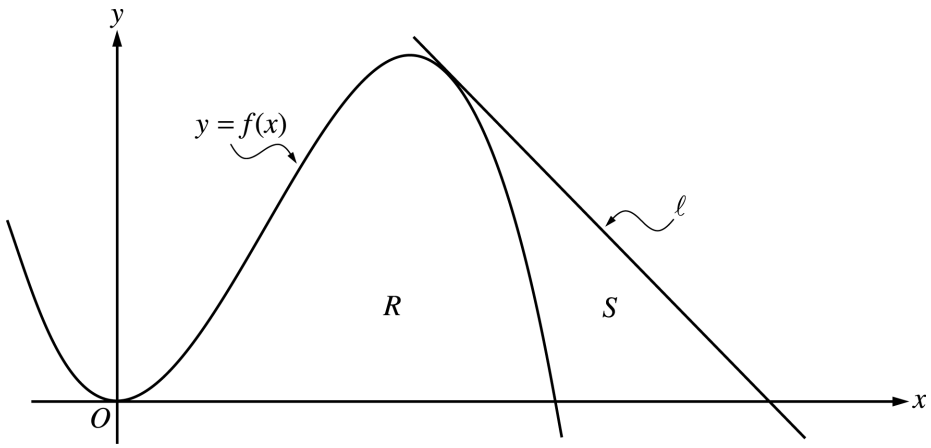
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

   (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when R is revolved about the x-axis.

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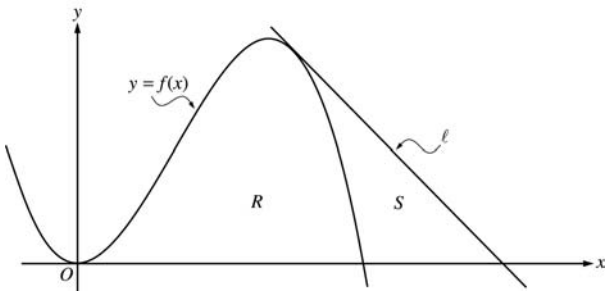
AP® CALCULUS AB
2003 SCORING GUIDELINES (Form B)

Question 1

Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.



(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.

(c) Find the volume of the solid generated when R is revolved about the x-axis.

(a) $f'(x) = 8x - 3x^2$; $f'(3) = 24 - 27 = -3$
$f(3) = 36 - 27 = 9$
Tangent line at $x = 3$ is
$y = -3(x - 3) + 9 = -3x + 18$,
which is the equation of line ℓ.

2 : { 1 : finds $f'(3)$ and $f(3)$; 1 : { finds equation of tangent line or shows (3,9) is on both the graph of f and line ℓ }

(b) $f(x) = 0$ at $x = 4$
The line intersects the x-axis at $x = 6$.

$$\text{Area} = \frac{1}{2}(3)(9) - \int_3^4 \left(4x^2 - x^3\right) dx$$
$$= 7.916 \text{ or } 7.917$$

OR

$$\text{Area} = \int_3^4 \left((18 - 3x) - \left(4x^2 - x^3\right)\right) dx + \frac{1}{2}(2)(18 - 12)$$
$$= 7.916 \text{ or } 7.917$$

4 : { 2 : integral for non-triangular region (1 : limits ; 1 : integrand) ; 1 : area of triangular region ; 1 : answer }

(c) $\text{Volume} = \pi \int_0^4 \left(4x^2 - x^3\right)^2 dx$
$= 156.038\pi$ or 490.208

3 : { 1 : limits and constant ; 1 : integrand ; 1 : answer }

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2. A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t + 1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

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Question 2

A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

(a) $a(2) = v'(2) = 1.587$ or 1.588

$v(2) = -3\sin(2) < 0$

Speed is decreasing since $a(2) > 0$ and $v(2) < 0$.

$2 : \begin{cases} 1 : a(2) \\ 1 : \text{speed decreasing with reason} \end{cases}$

(b) $v(t) = 0$ when $\dfrac{t^2}{2} = \pi$

$t = \sqrt{2\pi}$ or 2.506 or 2.507

Since $v(t) < 0$ for $0 < t < \sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi} < t < 3$, the particle changes directions at $t = \sqrt{2\pi}$.

$2 : \begin{cases} 1 : t = \sqrt{2\pi} \text{ only} \\ 1 : \text{justification} \end{cases}$

(c) Distance $= \displaystyle\int_0^3 |v(t)|\,dt = 4.333$ or 4.334

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(d) $\displaystyle\int_0^{\sqrt{2\pi}} v(t)\,dt = -3.265$

$x\left(\sqrt{2\pi}\right) = x(0) + \displaystyle\int_0^{\sqrt{2\pi}} v(t)\,dt = -2.265$

Since the total distance from $t = 0$ to $t = 3$ is 4.334, the particle is still to the left of the origin at $t = 3$. Hence the greatest distance from the origin is 2.265.

$2 : \begin{cases} 1 : \pm \text{ (distance particle travels while velocity is negative)} \\ 1 : \text{answer} \end{cases}$

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2. A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t + 1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

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2003 SCORING GUIDELINES (Form B)

Question 2

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

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$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

(a) $\int_0^{12} H(t)\,dt = 70.570$ or 70.571

2 : { 1 : integral; 1 : answer }

(b) $H(6) - R(6) = -2.924$,
so the level of heating oil is falling at $t = 6$.

1 : answer with reason

(c) $125 + \int_0^{12} (H(t) - R(t))\,dt = 122.025$ or 122.026

3 : { 1 : limits; 1 : integrand; 1 : answer }

(d) The absolute minimum occurs at a critical point or an endpoint.
$H(t) - R(t) = 0$ when $t = 4.790$ and $t = 11.318$.

The volume increases until $t = 4.790$, then decreases until $t = 11.318$, then increases, so the absolute minimum will be at $t = 0$ or at $t = 11.318$.

$125 + \int_0^{11.318} (H(t) - R(t))\,dt = 120.738$

Since the volume is 125 at $t = 0$, the volume is least at $t = 11.318$.

3 : { 1 : sets $H(t) - R(t) = 0$; 1 : volume is least at $t = 11.318$; 1 : analysis for absolute minimum }

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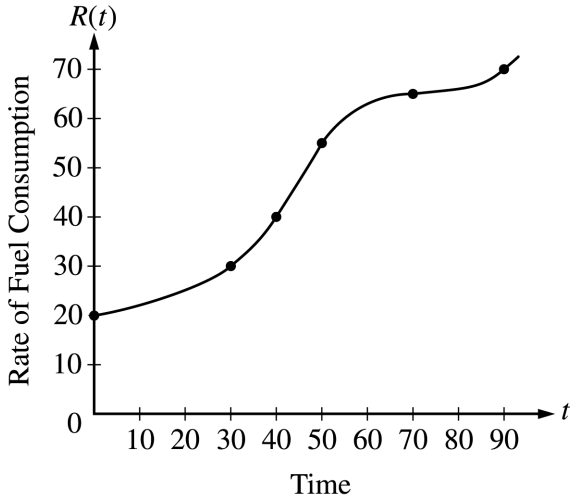

t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

   (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

   (c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

   (d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

END OF PART A OF SECTION II

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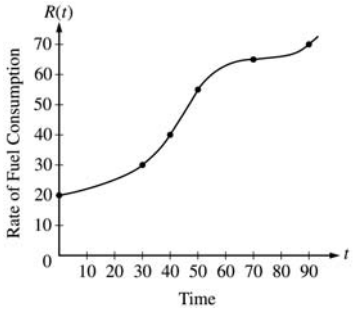
Question 3

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a) $R'(45) \approx \dfrac{R(50) - R(40)}{50 - 40} = \dfrac{55 - 40}{10}$
$= 1.5$ gal/min^2

$2 :$ { 1 : a difference quotient using numbers from table and interval that contains 45; 1 : 1.5 gal/min^2 }

(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.

$2 :$ { 1 : $R''(45) = 0$; 1 : reason }

(c) $\int_0^{90} R(t)\,dt \approx (30)(20) + (10)(30) + (10)(40)$
$+(20)(55) + (20)(65) = 3700$
Yes, this approximation is less because the graph of R is increasing on the interval.

$2 :$ { 1 : value of left Riemann sum; 1 : "less" with reason }

(d) $\int_0^b R(t)\,dt$ is the total amount of fuel in gallons consumed for the first b minutes.
$\frac{1}{b}\int_0^b R(t)\,dt$ is the average value of the rate of fuel consumption in gallons/min during the first b minutes.

$3 :$ { 2 : meanings { 1 : meaning of $\int_0^b R(t)\,dt$; 1 : meaning of $\frac{1}{b}\int_0^b R(t)\,dt$; $< -1 >$ if no reference to time b }; 1 : units in both answers }

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Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

3. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

   (a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

   (b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

   (c) Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2} \right)^2 dx$ in terms of the blood vessel.

   (d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

END OF PART A OF SECTION II

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2003 SCORING GUIDELINES (Form B)

Question 3

A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

(a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

(b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

(c) Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

(d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

(a) $\frac{1}{360}\int_0^{360}\frac{B(x)}{2}\,dx$

$2 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{integrand} \end{cases}$

(b) $\frac{1}{360}\left[120\left(\frac{B(60)}{2}+\frac{B(180)}{2}+\frac{B(300)}{2}\right)\right] =$
$\frac{1}{360}[60\,(30+30+24)] = 14$

$2 : \begin{cases} 1 : B(60)+B(180)+B(300) \\ 1 : \text{answer} \end{cases}$

(c) $\frac{B(x)}{2}$ is the radius, so $\pi\left(\frac{B(x)}{2}\right)^2$ is the area of the cross section at x. The expression is the volume in mm^3 of the blood vessel between 125 mm and 275 mm from the end of the vessel.

$2 : \begin{cases} 1 : \text{volume in mm}^3 \\ 1 : \text{between } x = 125 \text{ and } x = 275 \end{cases}$

(d) By the MVT, $B'(c_1) = 0$ for some c_1 in (60, 180) and $B'(c_2) = 0$ for some c_2 in (240, 360). The MVT applied to $B'(x)$ shows that $B''(x) = 0$ for some x in the interval $\left(c_1,\, c_2\right)$.

$3 : \begin{cases} 2 : \text{explains why there are two values of } x \text{ where } B'(x) \text{ has the same value} \\ 1 : \text{explains why that means } B''(x) = 0 \text{ for } 0 < x < 360 \end{cases}$

Note: max 1/3 if only explains why $B'(x) = 0$ at some x in (0, 360).

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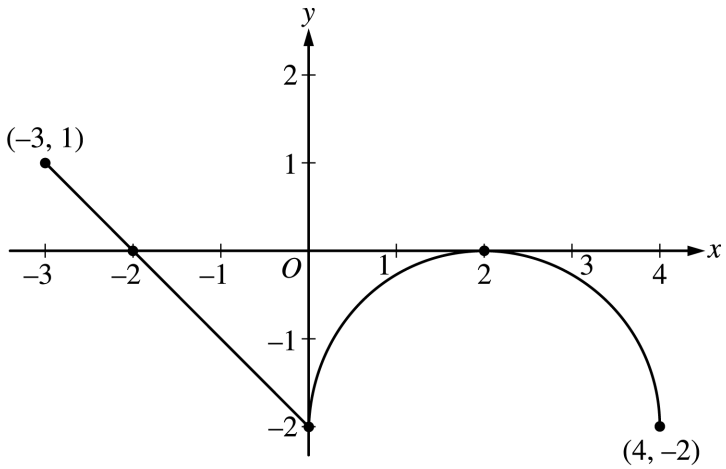
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

   (a) On what intervals, if any, is f increasing? Justify your answer.

   (b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

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Question 4

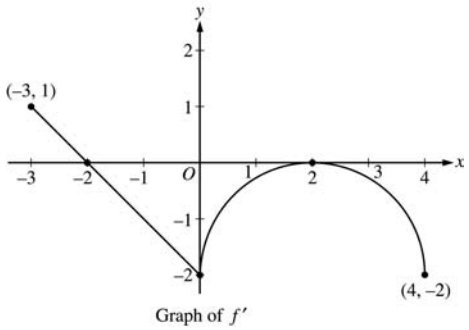
Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $\left(0, 3\right)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.



(a) The function f is increasing on $[-3, -2]$ since $f' > 0$ for $-3 \le x < -2$.

2 : { 1 : interval; 1 : reason }

(b) $x = 0$ and $x = 2$

f' changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$

2 : { 1 : $x = 0$ and $x = 2$ only; 1 : justification }

(c) $f'(0) = -2$

Tangent line is $y = -2x + 3$.

1 : equation

(d) $f(0) - f(-3) = \int_{-3}^{0} f'(t)\,dt$

$= \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2}$

$f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$

$f(4) - f(0) = \int_{0}^{4} f'(t)\,dt$

$= -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi$

$f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$

4 : { 1 : $\pm\left(\frac{1}{2} - 2\right)$ (difference of areas of triangles); 1 : answer for $f(-3)$ using FTC; 1 : $\pm\left(8 - \frac{1}{2}(2)^2\pi\right)$ (area of rectangle − area of semicircle); 1 : answer for $f(4)$ using FTC }

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

   (a) Find the acceleration of the particle at time $t = 3$.

   (b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

   (c) Find all values of t at which the particle changes direction. Justify your answer.

   (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

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2003 SCORING GUIDELINES (Form B)

Question 4

A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(a) $a(t) = v'(t) = -e^{1-t}$
$a(3) = -e^{-2}$

$2 : \begin{cases} 1 : v'(t) \\ 1 : a(3) \end{cases}$

(b) $a(3) < 0$
$v(3) = -1 + e^{-2} < 0$
Speed is increasing since $v(3) < 0$ and $a(3) < 0$.

1 : answer with reason

(c) $v(t) = 0$ when $1 = e^{1-t}$, so $t = 1$.

$v(t) > 0$ for $t < 1$ and $v(t) < 0$ for $t > 1$.
Therefore, the particle changes direction at $t = 1$.

$2 : \begin{cases} 1 : \text{solves } v(t) = 0 \text{ to get } t = 1 \\ 1 : \text{justifies change in direction at } t = 1 \end{cases}$

(d) Distance $= \int_0^3 |v(t)|\,dt$
$= \int_0^1 \left(-1 + e^{1-t}\right) dt + \int_1^3 \left(1 - e^{1-t}\right) dt$
$= \left(-t - e^{1-t}\Big|_0^1\right) + \left(t + e^{1-t}\Big|_1^3\right)$
$= (-1 - 1 + e) + \left(3 + e^{-2} - 1 - 1\right)$
$= e + e^{-2} - 1$

$4 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{antidifferentiation} \\ 1 : \text{evaluation} \end{cases}$

OR

OR

$x(t) = -t - e^{1-t}$
$x(0) = -e$
$x(1) = -2$
$x(3) = -e^{-2} - 3$
Distance $= (x(1) - x(0)) + (x(1) - x(3))$
$= (-2 + e) + \left(1 + e^{-2}\right)$
$= e + e^{-2} - 1$

$4 : \begin{cases} 1 : \text{any antiderivative} \\ 1 : \text{evaluates } x(t) \text{ when } t = 0,\ 1,\ 3 \\ 1 : \text{evaluates distance between points} \\ 1 : \text{evaluates total distance} \end{cases}$

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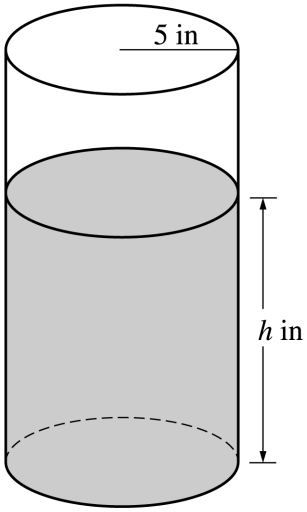

5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

   (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

   (c) At what time t is the coffeepot empty?

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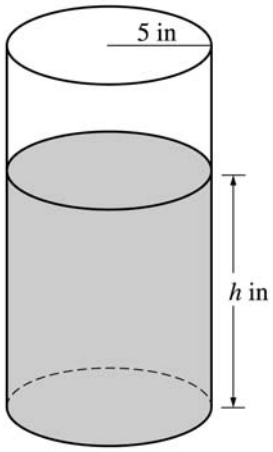
Question 5

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



(a) Show that $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$ for h as a function of t.

(c) At what time t is the coffeepot empty?

(a) $V = 25\pi h$

$\dfrac{dV}{dt} = 25\pi \dfrac{dh}{dt} = -5\pi\sqrt{h}$

$\dfrac{dh}{dt} = \dfrac{-5\pi\sqrt{h}}{25\pi} = -\dfrac{\sqrt{h}}{5}$

$3 : \begin{cases} 1 : \dfrac{dV}{dt} = -5\pi\sqrt{h} \\ 1 : \text{computes } \dfrac{dV}{dt} \\ 1 : \text{shows result} \end{cases}$

(b) $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$

$\dfrac{1}{\sqrt{h}}\,dh = -\dfrac{1}{5}\,dt$

$2\sqrt{h} = -\dfrac{1}{5}t + C$

$2\sqrt{17} = 0 + C$

$h = \left(-\dfrac{1}{10}t + \sqrt{17}\right)^2$

$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } h = 17 \text{ when } t = 0 \\ 1 : \text{solves for } h \end{cases}$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

(c) $\left(-\dfrac{1}{10}t + \sqrt{17}\right)^2 = 0$

$t = 10\sqrt{17}$

1 : answer

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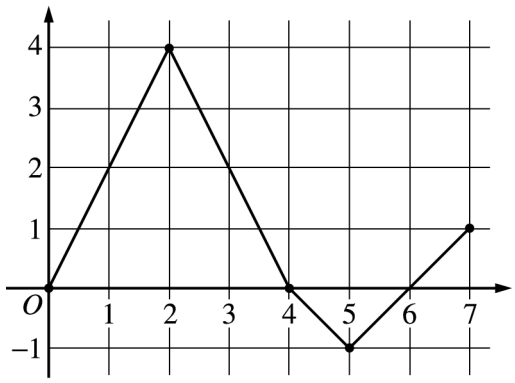
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

5. Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

   (a) Find $g(3)$, $g'(3)$, and $g''(3)$.

   (b) Find the average rate of change of g on the interval $0 \le x \le 3$.

   (c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b) ? Explain your reasoning.

   (d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

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Question 5

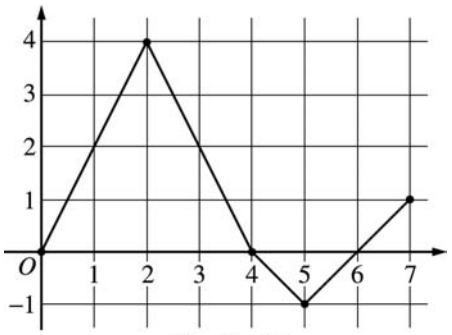
Let f be a function defined on the closed interval $[0,7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.



Graph of f

(a) $g(3) = \int_2^3 f(t)\,dt = \frac{1}{2}(4+2) = 3$

$g'(3) = f(3) = 2$

$g''(3) = f'(3) = \frac{0-4}{4-2} = -2$

$3 : \begin{cases} 1 : g(3) \\ 1 : g'(3) \\ 1 : g''(3) \end{cases}$

(b) $\frac{g(3)-g(0)}{3} = \frac{1}{3}\int_0^3 f(t)\,dt$

$= \frac{1}{3}\left(\frac{1}{2}(2)(4) + \frac{1}{2}(4+2)\right) = \frac{7}{3}$

$2 : \begin{cases} 1 : g(3) - g(0) = \int_0^3 f(t)\,dt \\ 1 : \text{answer} \end{cases}$

(c) There are two values of c.

We need $\frac{7}{3} = g'(c) = f(c)$

The graph of f intersects the line $y = \frac{7}{3}$ at two places between 0 and 3.

$2 : \begin{cases} 1 : \text{answer of 2} \\ 1 : \text{reason} \end{cases}$

Note: 1/2 if answer is 1 by MVT

(d) $x = 2$ and $x = 5$

because $g' = f$ changes from increasing to decreasing at $x = 2$, and from decreasing to increasing at $x = 5$.

$2 : \begin{cases} 1 : x = 2 \text{ and } x = 5 \text{ only} \\ 1 : \text{justification} \\ \quad (\text{ignore discussion at } x = 4) \end{cases}$

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6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

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Question 6

Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

(a) f is continuous at $x = 3$ because
$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 2$.
Therefore, $\lim_{x \to 3} f(x) = 2 = f(3)$.

$2 :$ { 1 : answers "yes" and equates the values of the left- and right-hand limits; 1 : explanation involving limits }

(b) $$\int_0^5 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx$$
$$= \frac{2}{3}(x+1)^{3/2}\Big|_0^3 + \left(5x - \frac{1}{2}x^2\right)\Big|_3^5$$
$$= \left(\frac{16}{3} - \frac{2}{3}\right) + \left(\frac{25}{2} - \frac{21}{2}\right) = \frac{20}{3}$$

Average value: $\frac{1}{5}\int_0^5 f(x)\,dx = \frac{4}{3}$

$4 :$ { 1 : $k\int_0^3 f(x)\,dx + k\int_3^5 f(x)\,dx$ (where $k \neq 0$); 1 : antiderivative of $\sqrt{x+1}$; 1 : antiderivative of $5 - x$; 1 : evaluation and answer }

(c) Since g is continuous at $x = 3$, $2k = 3m + 2$.

$$g'(x) = \begin{cases} \dfrac{k}{2\sqrt{x+1}} & \text{for } 0 < x < 3 \\ m & \text{for } 3 < x < 5 \end{cases}$$

$\lim_{x \to 3^-} g'(x) = \frac{k}{4}$ and $\lim_{x \to 3^+} g'(x) = m$
Since these two limits exist and g is differentiable at $x = 3$, the two limits are equal. Thus $\frac{k}{4} = m$.

$3 :$ { 1 : $2k = 3m + 2$; 1 : $\frac{k}{4} = m$; 1 : values for k and m }

$8m = 3m + 2$; $m = \frac{2}{5}$ and $k = \frac{8}{5}$

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6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

   (a) Find $f''(3)$.

   (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

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Question 6

Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

(a) Find $f''(3)$.

(b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

(a) $f''(x) = \sqrt{f(x)} + x \cdot \dfrac{f'(x)}{2\sqrt{f(x)}} = \sqrt{f(x)} + \dfrac{x^2}{2}$

$f''(3) = \sqrt{25} + \dfrac{9}{2} = \dfrac{19}{2}$

$3 : \begin{cases} 2 : f''(x) \\ \quad < -2 > \text{ product or} \\ \quad \text{chain rule error} \\ 1 : \text{value at } x = 3 \end{cases}$

(b) $\dfrac{1}{\sqrt{y}}\,dy = x\,dx$

$2\sqrt{y} = \dfrac{1}{2}x^2 + C$

$2\sqrt{25} = \dfrac{1}{2}(3)^2 + C\,;\ C = \dfrac{11}{2}$

$\sqrt{y} = \dfrac{1}{4}x^2 + \dfrac{11}{4}$

$y = \left(\dfrac{1}{4}x^2 + \dfrac{11}{4}\right)^2 = \dfrac{1}{16}\left(x^2 + 11\right)^2$

$6 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } f(3) = 25 \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

---

1. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

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AP® CALCULUS AB
2004 SCORING GUIDELINES

Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(a) $\int_0^{30} F(t)\,dt = 2474$ cars	3 : 1 : limits; 1 : integrand; 1 : answer
(b) $F'(7) = -1.872$ or -1.873 Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.	1 : answer with reason
(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min	3 : 1 : limits; 1 : integrand; 1 : answer
(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2	1 : answer
Units of cars/min in (c) and cars/min^2 in (d)	1 : units in (c) and (d)

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

---

1. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.
   (c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

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GO ON TO THE NEXT PAGE.

AP® CALCULUS AB
2004 SCORING GUIDELINES (Form B)

Question 1

Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.

(c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

(a) Area $= \int_1^{10} \sqrt{x-1}\, dx = 18$

3 : { 1 : limits; 1 : integrand; 1 : answer }

(b) Volume $= \pi \int_1^{10} \left(9 - \left(3 - \sqrt{x-1}\right)^2\right) dx$

$= 212.057$ or 212.058

3 : { 1 : limits and constant; 1 : integrand; 1 : answer }

(c) Volume $= \pi \int_0^{3} \left(10 - \left(y^2 + 1\right)\right)^2 dy$

$= 407.150$

3 : { 1 : limits and constant; 1 : integrand; 1 : answer }

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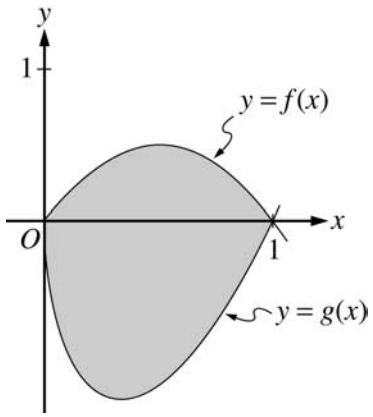

2. Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

   (a) Find the area of the shaded region enclosed by the graphs of f and g.

   (b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

   (c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

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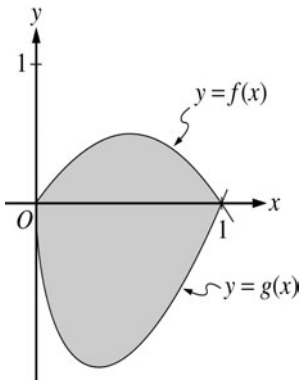
Question 2

Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.



(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

(a) Area $= \int_0^1 (f(x) - g(x))\,dx$

$= \int_0^1 \left(2x(1-x) - 3(x-1)\sqrt{x}\right)\,dx = 1.133$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi\int_0^1 \left((2-g(x))^2 - (2-f(x))^2\right)\,dx$

$= \pi\int_0^1 \left(\left(2-3(x-1)\sqrt{x}\right)^2 - (2-2x(1-x))^2\right)\,dx$

$= 16.179$

$4 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if integral not of form } c\int_a^b \left(R^2(x) - r^2(x)\right)\,dx \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \int_0^1 (h(x) - g(x))^2\,dx$

$\int_0^1 \left(kx(1-x) - 3(x-1)\sqrt{x}\right)^2\,dx = 15$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

2. For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

   (a) Show that the number of mosquitoes is increasing at time $t = 6$.

   (b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

   (c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

   (d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

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2004 SCORING GUIDELINES (Form B)

Question 2

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.

1 : shows that $R(6) > 0$

(b) $R'(6) = -1.913$
Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.

2 : $\begin{cases} 1 : \text{considers } R'(6) \\ 1 : \text{answer with reason} \end{cases}$

(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$
To the nearest whole number, there are 964 mosquitoes.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$
$R(t) > 0$ on $0 < t < 2.5\pi$
$R(t) < 0$ on $2.5\pi < t < 7.5\pi$
$R(t) > 0$ on $7.5\pi < t < 31$
The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$.

$1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357,$

There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.

4 : $\begin{cases} 2 : \text{absolute maximum value} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{answer} \\ 2 : \text{analysis} \\ \quad 1 : \text{computes interior critical points} \\ \quad 1 : \text{completes analysis} \end{cases}$

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3. A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

   (a) Find the acceleration of the particle at time $t = 2$.

   (b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

   (c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

   (d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

END OF PART A OF SECTION II

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Question 3

A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$.

At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

(a) $a(2) = v'(2) = -0.132$ or -0.133

1 : answer

(b) $v(2) = -0.436$
Speed is increasing since $a(2) < 0$ and $v(2) < 0$.

1 : answer with reason

(c) $v(t) = 0$ when $\tan^{-1}\left(e^t\right) = 1$
$t = \ln(\tan(1)) = 0.443$ is the only critical value for y.

$v(t) > 0$ for $0 < t < \ln(\tan(1))$
$v(t) < 0$ for $t > \ln(\tan(1))$

$y(t)$ has an absolute maximum at $t = 0.443$.

3 : { 1 : sets $v(t) = 0$; 1 : identifies $t = 0.443$ as a candidate; 1 : justifies absolute maximum }

(d) $y(2) = -1 + \int_0^2 v(t)\,dt = -1.360$ or -1.361
The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.

4 : { 1 : $\int_0^2 v(t)\,dt$; 1 : handles initial condition; 1 : value of $y(2)$; 1 : answer with reason }

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t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

   (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

   (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

   (c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.

   (d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

END OF PART A OF SECTION II

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Question 3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

(a) Midpoint Riemann sum is
$10 \cdot [v(5) + v(15) + v(25) + v(35)]$
$= 10 \cdot [9.2 + 7.0 + 2.4 + 4.3] = 229$
The integral gives the total distance in miles that the plane flies during the 40 minutes.

3 : { 1 : $v(5) + v(15) + v(25) + v(35)$; 1 : answer; 1 : meaning with units }

(b) By the Mean Value Theorem, $v'(t) = 0$ somewhere in the interval $(0, 15)$ and somewhere in the interval $(25, 30)$. Therefore the acceleration will equal 0 for at least two values of t.

2 : { 1 : two instances; 1 : justification }

(c) $f'(23) = -0.407$ or -0.408 miles per minute^2

1 : answer with units

(d) Average velocity $= \frac{1}{40}\int_0^{40} f(t)\,dt$
$= 5.916$ miles per minute

3 : { 1 : limits; 1 : integrand; 1 : answer }

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

   (a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

   (b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

   (c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P ? Justify your answer.

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Question 4

Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

(a) $2x + 8yy' = 3y + 3xy'$

$(8y - 3x)y' = 3y - 2x$

$y' = \frac{3y - 2x}{8y - 3x}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{solves for } y' \end{cases}$

(b) $\frac{3y - 2x}{8y - 3x} = 0;\ 3y - 2x = 0$

When $x = 3,\ 3y = 6$

$y = 2$

$3^2 + 4 \cdot 2^2 = 25$ and $7 + 3 \cdot 3 \cdot 2 = 25$

Therefore, $P = (3, 2)$ is on the curve and the slope is 0 at this point.

$3 : \begin{cases} 1 : \frac{dy}{dx} = 0 \\ 1 : \text{shows slope is 0 at } (3, 2) \\ 1 : \text{shows } (3, 2) \text{ lies on curve} \end{cases}$

(c) $\frac{d^2y}{dx^2} = \frac{(8y - 3x)(3y' - 2) - (3y - 2x)(8y' - 3)}{(8y - 3x)^2}$

At $P = (3, 2),\ \frac{d^2y}{dx^2} = \frac{(16 - 9)(-2)}{(16 - 9)^2} = -\frac{2}{7}$.

Since $y' = 0$ and $y'' < 0$ at P, the curve has a local maximum at P.

$4 : \begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{value of } \frac{d^2y}{dx^2} \text{ at } (3, 2) \\ 1 : \text{conclusion with justification} \end{cases}$

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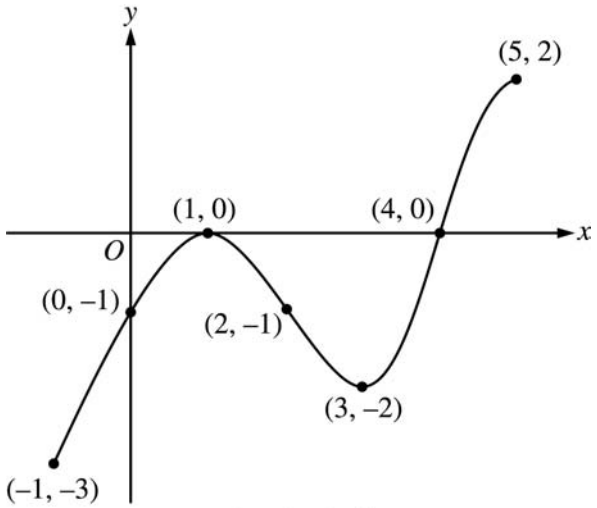
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

4. The figure above shows the graph of f', the derivative of the function f, on the closed interval $-1 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.

   (a) Find the x-coordinate of each of the points of inflection of the graph of f. Give a reason for your answer.

   (b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \le x \le 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \le x \le 5$? Show the analysis that leads to your answers.

   (c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

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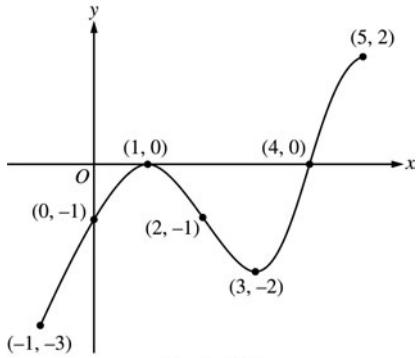
2004 SCORING GUIDELINES (Form B)

Question 4

The figure above shows the graph of f', the derivative of the function f, on the closed interval $-1 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.



Graph of f'

(a) Find the x-coordinate of each of the points of inflection of the graph of f. Give a reason for your answer.

(b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \le x \le 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \le x \le 5$? Show the analysis that leads to your answers.

(c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

(a) $x = 1$ and $x = 3$ because the graph of f' changes from increasing to decreasing at $x = 1$, and changes from decreasing to increasing at $x = 3$.

$2 : \begin{cases} 1 : x = 1, x = 3 \\ 1 : \text{reason} \end{cases}$

(b) The function f decreases from $x = -1$ to $x = 4$, then increases from $x = 4$ to $x = 5$.
Therefore, the absolute minimum value for f is at $x = 4$.
The absolute maximum value must occur at $x = -1$ or at $x = 5$.

$$f(5) - f(-1) = \int_{-1}^{5} f'(t)\,dt < 0$$

Since $f(5) < f(-1)$, the absolute maximum value occurs at $x = -1$.

$4 : \begin{cases} 1 : \text{indicates } f \text{ decreases then increases} \\ 1 : \text{eliminates } x = 5 \text{ for maximum} \\ 1 : \text{absolute minimum at } x = 4 \\ 1 : \text{absolute maximum at } x = -1 \end{cases}$

(c) $g'(x) = f(x) + xf'(x)$
$g'(2) = f(2) + 2f'(2) = 6 + 2(-1) = 4$
$g(2) = 2f(2) = 12$

Tangent line is $y = 4(x - 2) + 12$

$3 : \begin{cases} 2 : g'(x) \\ 1 : \text{tangent line} \end{cases}$

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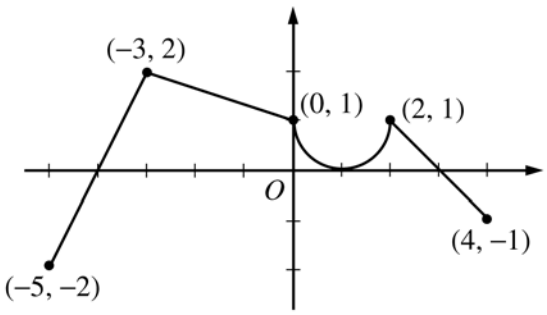
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

   (a) Find $g(0)$ and $g'(0)$.

   (b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

   (c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

   (d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

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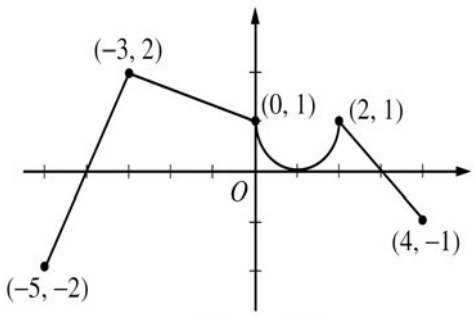
Question 5

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

(a) $g(0) = \int_{-3}^{0} f(t)\,dt = \frac{1}{2}(3)(2+1) = \frac{9}{2}$

$g'(0) = f(0) = 1$

$2 : \begin{cases} 1 : g(0) \\ 1 : g'(0) \end{cases}$

(b) g has a relative maximum at $x = 3$.
This is the only x-value where $g' = f$ changes from positive to negative.

$2 : \begin{cases} 1 : x = 3 \\ 1 : \text{justification} \end{cases}$

(c) The only x-value where f changes from negative to positive is $x = -4$. The other candidates for the location of the absolute minimum value are the endpoints.

$g(-5) = 0$

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -1$

$g(4) = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right) = \frac{13 - \pi}{2}$

So the absolute minimum value of g is -1.

$3 : \begin{cases} 1 : \text{identifies } x = -4 \text{ as a candidate} \\ 1 : g(-4) = -1 \\ 1 : \text{justification and answer} \end{cases}$

(d) $x = -3, 1, 2$

$2 : \begin{cases} \text{correct values} \\ \langle -1 \rangle \text{ each missing or extra value} \end{cases}$

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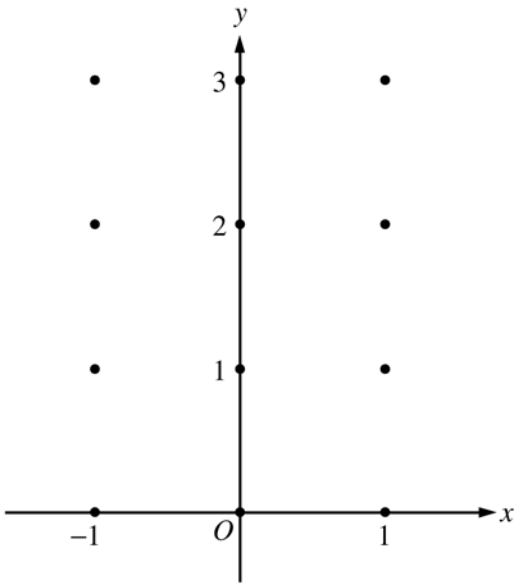
5. Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

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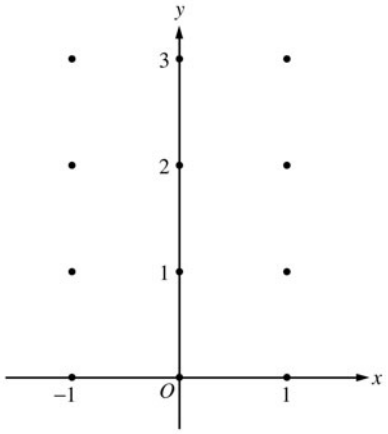
Question 5

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

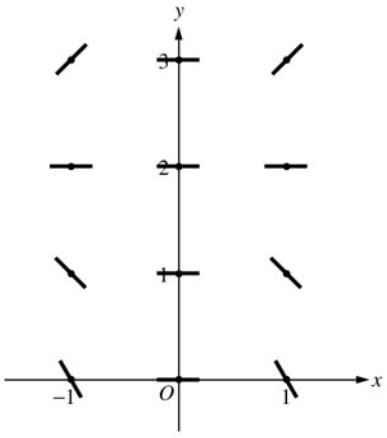
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.



(a)



2 : { 1 : zero slope at each point (x, y) where $x = 0$ or $y = 2$; 1 : { positive slope at each point (x, y) where $x \neq 0$ and $y > 2$; negative slope at each point (x, y) where $x \neq 0$ and $y < 2$ }

(b) Slopes are negative at points (x, y) where $x \neq 0$ and $y < 2$.

1 : description

(c) $\frac{1}{y-2}dy = x^4 dx$

$\ln|y-2| = \frac{1}{5}x^5 + C$

$|y-2| = e^C e^{\frac{1}{5}x^5}$

$y - 2 = Ke^{\frac{1}{5}x^5}$, $K = \pm e^C$

$-2 = Ke^0 = K$

$y = 2 - 2e^{\frac{1}{5}x^5}$

6 : { 1 : separates variables ; 2 : antiderivatives ; 1 : constant of integration ; 1 : uses initial condition ; 1 : solves for y ; $0/1$ if y is not exponential }

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

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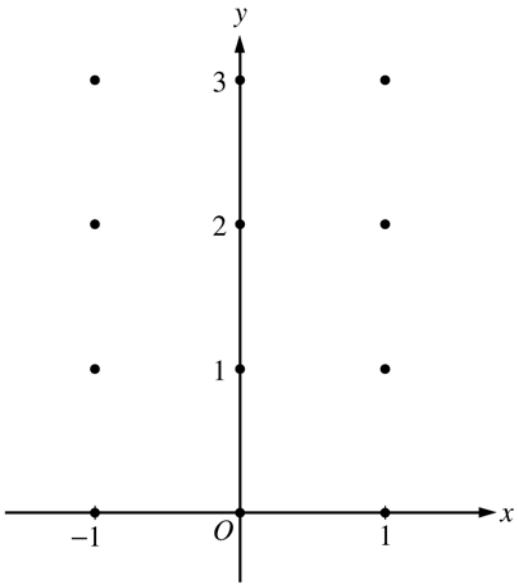
6. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

END OF EXAMINATION

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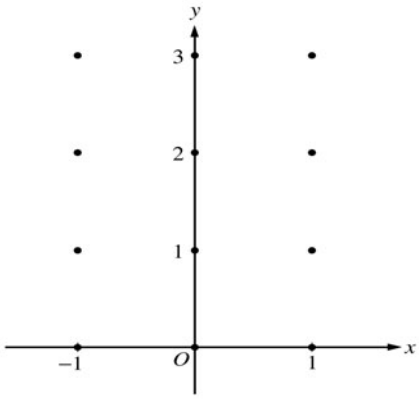
Question 6

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

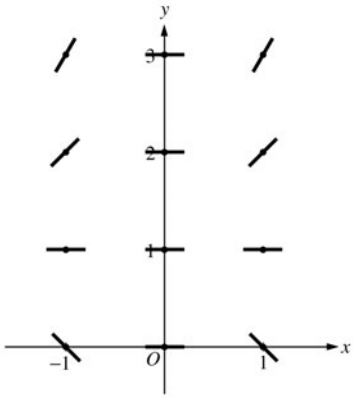


(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

(a)



$2 : \begin{cases} 1 : \text{zero slope at each point } (x, y) \text{ where } x = 0 \text{ or } y = 1 \\ 1 : \begin{cases} \text{positive slope at each point } (x, y) \text{ where } x \neq 0 \text{ and } y > 1 \\ \text{negative slope at each point } (x, y) \text{ where } x \neq 0 \text{ and } y < 1 \end{cases} \end{cases}$

(b) Slopes are positive at points (x, y) where $x \neq 0$ and $y > 1$.

1 : description

(c) $\frac{1}{y-1}dy = x^2dx$

$\ln|y-1| = \frac{1}{3}x^3 + C$

$|y-1| = e^C e^{\frac{1}{3}x^3}$

$y - 1 = Ke^{\frac{1}{3}x^3}$, $K = \pm e^C$

$2 = Ke^0 = K$

$y = 1 + 2e^{\frac{1}{3}x^3}$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \\ \quad 0/1 \text{ if } y \text{ is not exponential} \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

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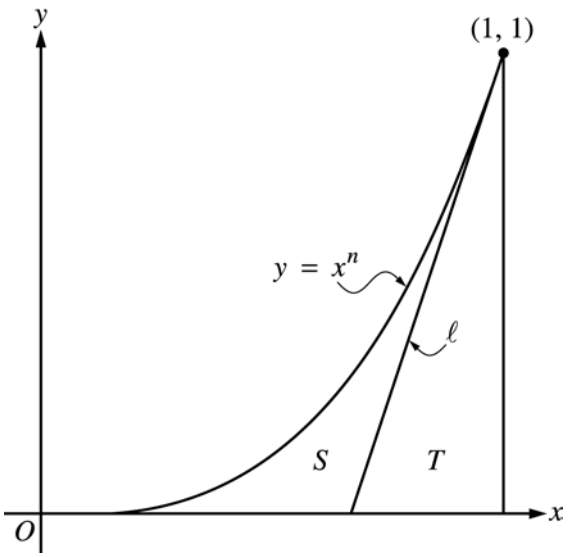

6. Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

   (a) Find $\int_0^1 x^n \, dx$ in terms of n.

   (b) Let T be the triangular region bounded by ℓ, the x-axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.

   (c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.

END OF EXAMINATION

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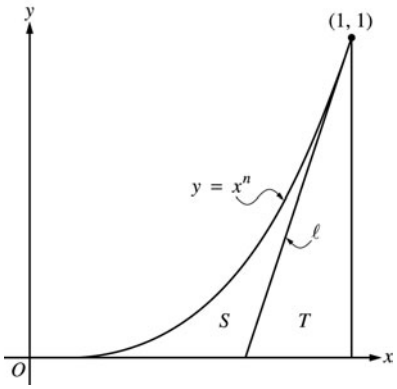
Question 6

Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

(a) Find $\int_0^1 x^n \, dx$ in terms of n.

(b) Let T be the triangular region bounded by ℓ, the x-axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.

(c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.



(a) $\int_0^1 x^n \, dx = \left. \frac{x^{n+1}}{n+1} \right|_0^1 = \frac{1}{n+1}$

$2 : \begin{cases} 1 : \text{antiderivative of } x^n \\ 1 : \text{answer} \end{cases}$

(b) Let b be the length of the base of triangle T.

$\frac{1}{b}$ is the slope of line ℓ, which is n

$\text{Area}(T) = \frac{1}{2} b(1) = \frac{1}{2n}$

$3 : \begin{cases} 1 : \text{slope of line } \ell \text{ is } n \\ 1 : \text{base of } T \text{ is } \frac{1}{n} \\ 1 : \text{shows area is } \frac{1}{2n} \end{cases}$

(c) $\text{Area}(S) = \int_0^1 x^n \, dx - \text{Area}(T)$

$= \frac{1}{n+1} - \frac{1}{2n}$

$\frac{d}{dn} \text{Area}(S) = -\frac{1}{(n+1)^2} + \frac{1}{2n^2} = 0$

$2n^2 = (n+1)^2$

$\sqrt{2}\, n = (n+1)$

$n = \frac{1}{\sqrt{2} - 1} = 1 + \sqrt{2}$

$4 : \begin{cases} 1 : \text{area of } S \text{ in terms of } n \\ 1 : \text{derivative} \\ 1 : \text{sets derivative equal to 0} \\ 1 : \text{solves for } n \end{cases}$

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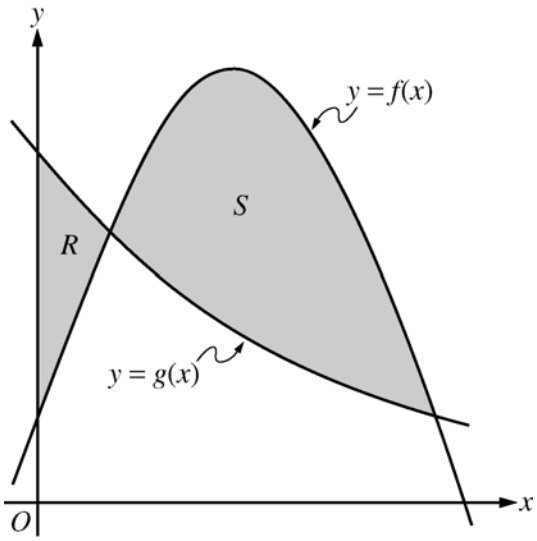
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the area of S.
   (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

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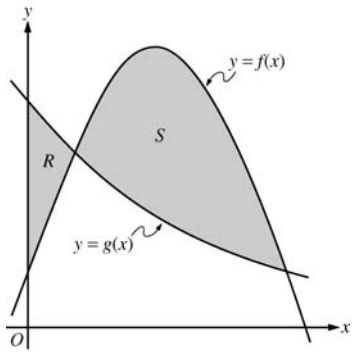
2005 SCORING GUIDELINES

Question 1

Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.



(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

$f(x) = g(x)$ when $\frac{1}{4} + \sin(\pi x) = 4^{-x}$.

f and g intersect when $x = 0.178218$ and when $x = 1$.
Let $a = 0.178218$.

(a) $\int_0^a (g(x) - f(x))\,dx = 0.064$ or 0.065

3 : { 1 : limits; 1 : integrand; 1 : answer }

(b) $\int_a^1 (f(x) - g(x))\,dx = 0.410$

3 : { 1 : limits; 1 : integrand; 1 : answer }

(c) $\pi\int_a^1 \left((f(x)+1)^2 - (g(x)+1)^2\right)dx = 4.558$ or 4.559

3 : { 2 : integrand; 1 : limits, constant, and answer }

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2. The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1+3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

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Question 2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

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(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

(a) $\int_0^6 R(t)\,dt = 31.815$ or 31.816 yd^3

2 : { 1 : integral; 1 : answer with units }

(b) $Y(t) = 2500 + \int_0^t (S(x) - R(x))\,dx$

3 : { 1 : integrand; 1 : limits; 1 : answer }

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$ or -1.909 yd^3/hr

1 : answer

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.

The only value in $[0, 6]$ to satisfy $S(t) = R(t)$ is $a = 5.117865$.

t	$Y(t)$
0	2500
a	2492.3694
6	2493.2766

The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.

3 : { 1 : sets $Y'(t) = 0$; 1 : critical t-value; 1 : answer with justification }

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Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

   (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

   (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

   (c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

   (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

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END OF PART A OF SECTION II

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Question 3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

(a) $\dfrac{T(8) - T(6)}{8 - 6} = \dfrac{55 - 62}{2} = -\dfrac{7}{2}$ °C/cm

1 : answer

(b) $\dfrac{1}{8}\int_0^8 T(x)\,dx$

Trapezoidal approximation for $\int_0^8 T(x)\,dx$:

$A = \dfrac{100 + 93}{2}\cdot 1 + \dfrac{93 + 70}{2}\cdot 4 + \dfrac{70 + 62}{2}\cdot 1 + \dfrac{62 + 55}{2}\cdot 2$

Average temperature $\approx \dfrac{1}{8}A = 75.6875$°C

3 : 1 : $\dfrac{1}{8}\int_0^8 T(x)\,dx$; 1 : trapezoidal sum; 1 : answer

(c) $\int_0^8 T'(x)\,dx = T(8) - T(0) = 55 - 100 = -45$°C

The temperature drops 45°C from the heated end of the wire to the other end of the wire.

2 : 1 : value; 1 : meaning

(d) Average rate of change of temperature on $[1, 5]$ is $\dfrac{70 - 93}{5 - 1} = -5.75$.

Average rate of change of temperature on $[5, 6]$ is $\dfrac{62 - 70}{6 - 5} = -8$.

No. By the MVT, $T'(c_1) = -5.75$ for some c_1 in the interval $(1, 5)$ and $T'(c_2) = -8$ for some c_2 in the interval $(5, 6)$. It follows that T' must decrease somewhere in the interval (c_1, c_2). Therefore T'' is not positive for every x in $[0, 8]$.

2 : 1 : two slopes of secant lines; 1 : answer with explanation

Units of °C/cm in (a), and °C in (b) and (c)

1 : units in (a), (b), and (c)

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

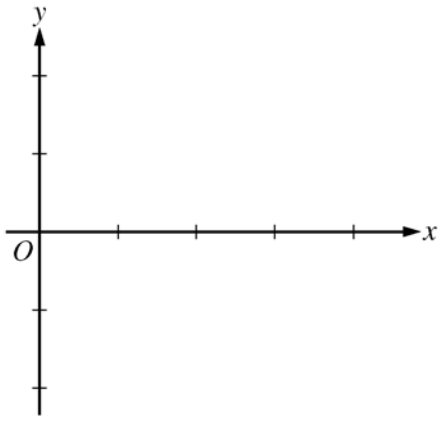
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

   (a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

   (b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

   (Note: Use the axes provided in the pink test booklet.)



   (c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

   (d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

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GO ON TO THE NEXT PAGE.

AP® CALCULUS AB
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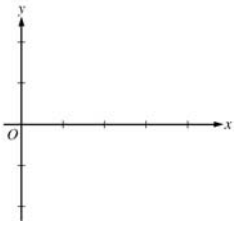
Question 4

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.
(Note: Use the axes provided in the pink test booklet.)



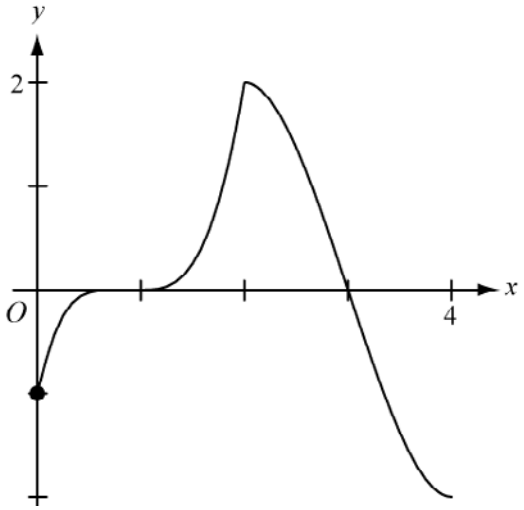
(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

(a) f has a relative maximum at $x = 2$ because f' changes from positive to negative at $x = 2$.

$2 : \begin{cases} 1 : \text{relative extremum at } x = 2 \\ 1 : \text{relative maximum with justification} \end{cases}$

(b)



$2 : \begin{cases} 1 : \text{points at } x = 0,\ 1,\ 2,\ 3 \text{ and behavior at } (2, 2) \\ 1 : \text{appropriate increasing/decreasing and concavity behavior} \end{cases}$

(c) $g'(x) = f(x) = 0$ at $x = 1, 3$.
g' changes from negative to positive at $x = 1$ so g has a relative minimum at $x = 1$. g' changes from positive to negative at $x = 3$ so g has a relative maximum at $x = 3$.

$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{critical points} \\ 1 : \text{answer with justification} \end{cases}$

(d) The graph of g has a point of inflection at $x = 2$ because $g'' = f'$ changes sign at $x = 2$.

$2 : \begin{cases} 1 : x = 2 \\ 1 : \text{answer with justification} \end{cases}$

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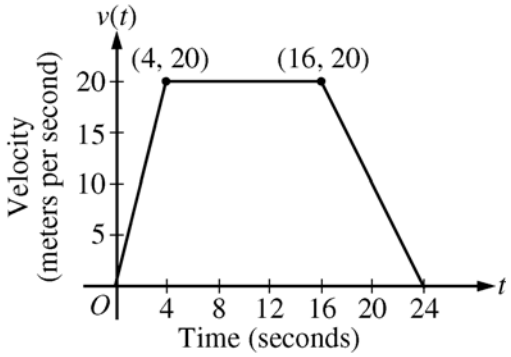

5. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

   (a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

   (b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

   (c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

   (d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

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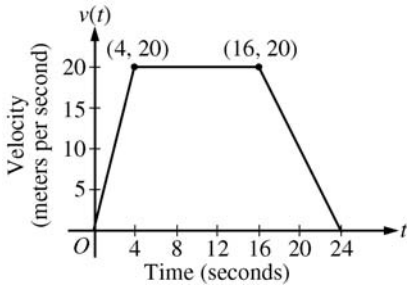
Question 5

A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.



(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

(a) $\int_0^{24} v(t)\,dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$

The car travels 360 meters in these 24 seconds.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning with units} \end{cases}$

(b) $v'(4)$ does not exist because

$\lim_{t \to 4^-}\left(\frac{v(t) - v(4)}{t - 4}\right) = 5 \ne 0 = \lim_{t \to 4^+}\left(\frac{v(t) - v(4)}{t - 4}\right)$.

$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2}\ \text{m/sec}^2$

$3 : \begin{cases} 1 : v'(4) \text{ does not exist, with explanation} \\ 1 : v'(20) \\ 1 : \text{units} \end{cases}$

(c) $a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$

$a(t)$ does not exist at $t = 4$ and $t = 16$.

$2 : \begin{cases} 1 : \text{finds the values } 5,\ 0,\ -\frac{5}{2} \\ 1 : \text{identifies constants with correct intervals} \end{cases}$

(d) The average rate of change of v on $[8, 20]$ is $\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6}\ \text{m/sec}^2$.

No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.

$2 : \begin{cases} 1 : \text{average rate of change of } v \text{ on } [8, 20] \\ 1 : \text{answer with explanation} \end{cases}$

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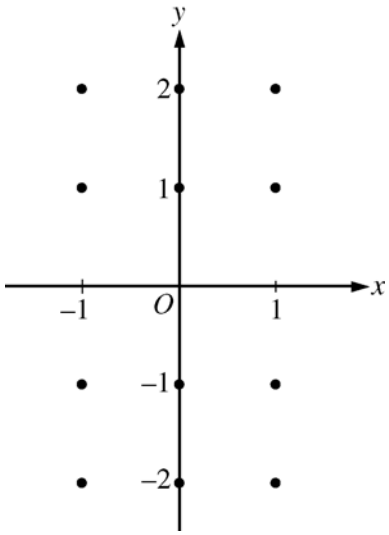
6. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

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END OF EXAM

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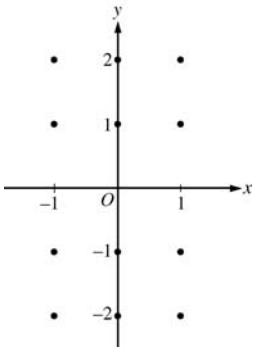
Question 6

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

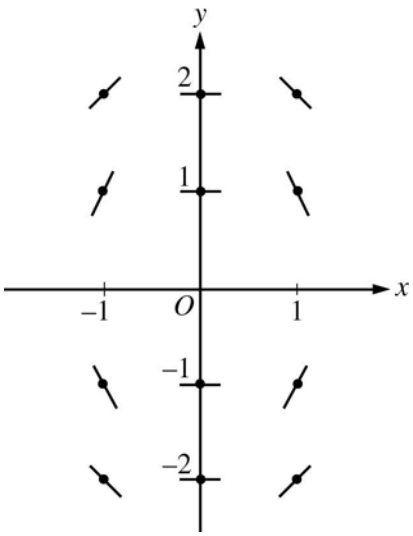
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)

(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.



(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) The line tangent to f at $(1, -1)$ is $y + 1 = 2(x - 1)$.
Thus, $f(1.1)$ is approximately -0.8.

$2 : \begin{cases} 1 : \text{equation of the tangent line} \\ 1 : \text{approximation for } f(1.1) \end{cases}$

(c) $\frac{dy}{dx} = -\frac{2x}{y}$

$y\,dy = -2x\,dx$

$\frac{y^2}{2} = -x^2 + C$

$\frac{1}{2} = -1 + C;\ C = \frac{3}{2}$

$y^2 = -2x^2 + 3$

Since the particular solution goes through $(1, -1)$, y must be negative.

Thus the particular solution is $y = -\sqrt{3 - 2x^2}$.

$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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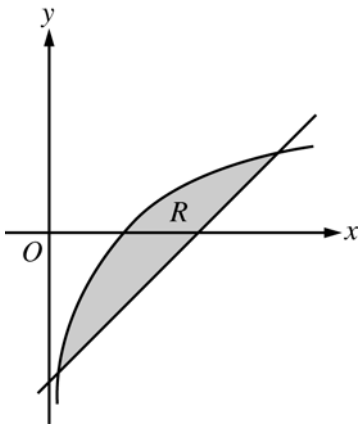
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

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2006 SCORING GUIDELINES

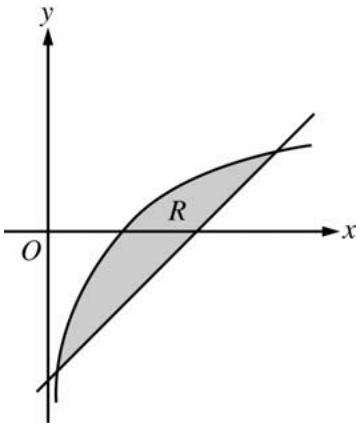
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x-2))\,dx = 1.949$

3 : { 1 : integrand; 1 : limits; 1 : answer }

(b) Volume $= \pi \int_S^T \left((\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx$
$= 34.198$ or 34.199

3 : { 2 : integrand; 1 : limits, constant, and answer }

(c) Volume $= \pi \int_{S-2}^{T-2} \left((y+2)^2 - \left(e^y\right)^2 \right) dy$

3 : { 2 : integrand; 1 : limits and constant }

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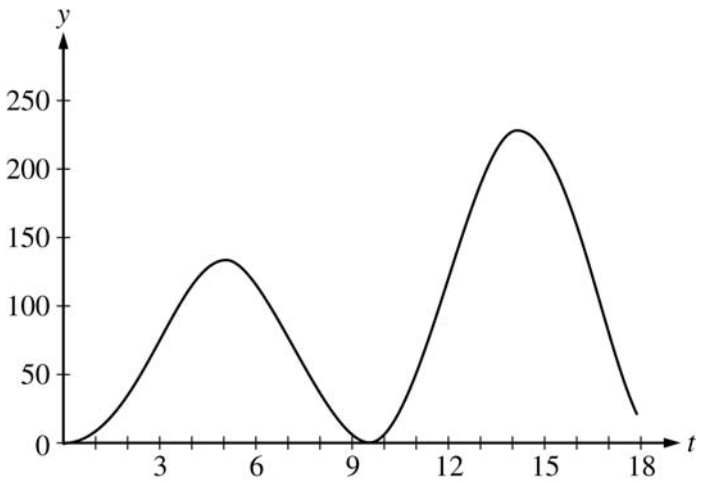

2. At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

   (a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

   (b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

   (c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

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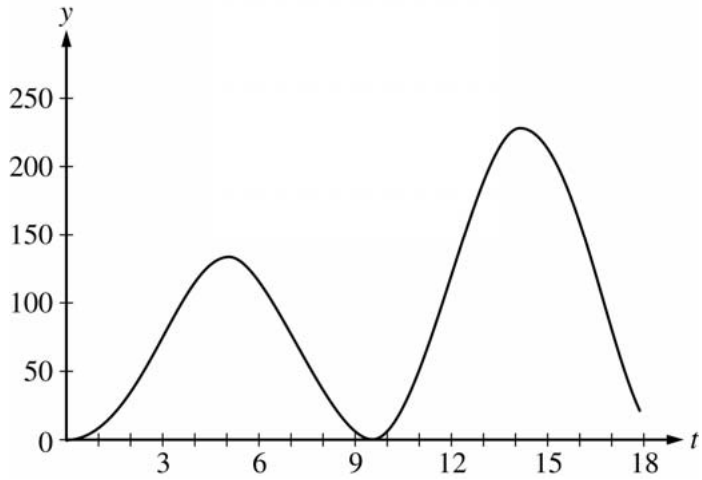
AP® CALCULUS AB
2006 SCORING GUIDELINES

Question 2

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.



(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

(a) $\int_0^{18} L(t)\,dt \approx 1658$ cars

2 : { 1 : setup; 1 : answer }

(b) $L(t) = 150$ when $t = 12.42831,\ 16.12166$
Let $R = 12.42831$ and $S = 16.12166$
$L(t) \ge 150$ for t in the interval $[R, S]$
$\frac{1}{S-R}\int_R^S L(t)\,dt = 199.426$ cars per hour

3 : { 1 : t-interval when $L(t) \ge 150$; 1 : average value integral; 1 : answer with units }

(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$\int_{13}^{15} L(t)\,dt = 431.931 > 400$

4 : { 1 : considers 400 cars; 1 : valid interval $[h, h+2]$; 1 : value of $\int_h^{h+2} L(t)\,dt$; 1 : answer and explanation }

OR

The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \ge 200$ on that interval.
$L(t) \ge 200$ on any two-hour subinterval of $[13.25304, 15.32386]$.

4 : { 1 : considers 200 cars per hour; 1 : solves $L(t) \ge 200$; 1 : discusses 2 hour interval; 1 : answer and explanation }

Yes, a traffic signal is required.

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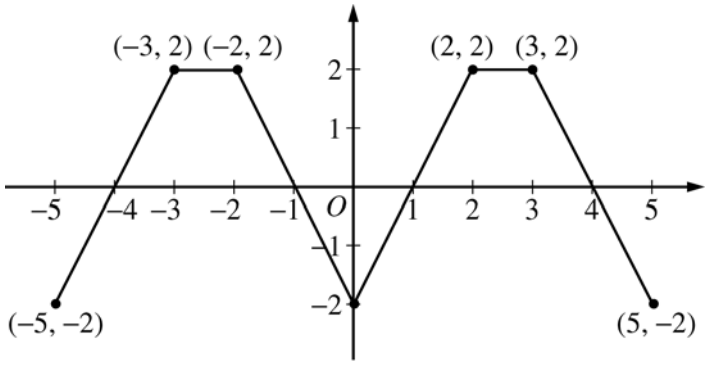

Graph of f

3. The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(4)$, $g'(4)$, and $g''(4)$.

   (b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.

   (c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

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END OF PART A OF SECTION II

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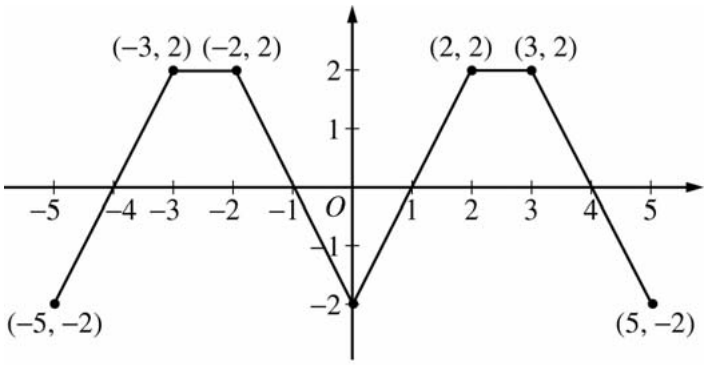
Question 3

The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

(b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.



Graph of f

(c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

(a) $g(4) = \int_0^4 f(t)\,dt = 3$

$g'(4) = f(4) = 0$

$g''(4) = f'(4) = -2$

$3 : \begin{cases} 1 : g(4) \\ 1 : g'(4) \\ 1 : g''(4) \end{cases}$

(b) g has a relative minimum at $x = 1$ because $g' = f$ changes from negative to positive at $x = 1$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c) $g(0) = 0$ and the function values of g increase by 2 for every increase of 5 in x.

$g(10) = 2g(5) = 4$

$g(108) = \int_0^{105} f(t)\,dt + \int_{105}^{108} f(t)\,dt$
$= 21g(5) + g(3) = 44$

$g'(108) = f(108) = f(3) = 2$

An equation for the line tangent to the graph of g at $x = 108$ is $y - 44 = 2(x - 108)$.

$4 : \begin{cases} 1 : g(10) \\ 3 : \begin{cases} 1 : g(108) \\ 1 : g'(108) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

---

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

   (a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

   (c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

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AP® CALCULUS AB
2006 SCORING GUIDELINES

Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

(a) Average acceleration of rocket A is

$$\frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft/sec}^2$$

1 : answer

(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds.

A midpoint Riemann sum is
$20[v(20) + v(40) + v(60)]$
$= 20[22 + 35 + 44] = 2020$ ft

3 : { 1 : explanation; 1 : uses $v(20)$, $v(40)$, $v(60)$; 1 : value }

(c) Let $v_B(t)$ be the velocity of rocket B at time t.

$v_B(t) = \int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$
$2 = v_B(0) = 6 + C$
$v_B(t) = 6\sqrt{t+1} - 4$
$v_B(80) = 50 > 49 = v(80)$

Rocket B is traveling faster at time $t = 80$ seconds.

4 : { 1 : $6\sqrt{t+1}$; 1 : constant of integration; 1 : uses initial condition; 1 : finds $v_B(80)$, compares to $v(80)$, and draws a conclusion }

Units of ft/sec^2 in (a) and ft in (b)

1 : units in (a) and (b)

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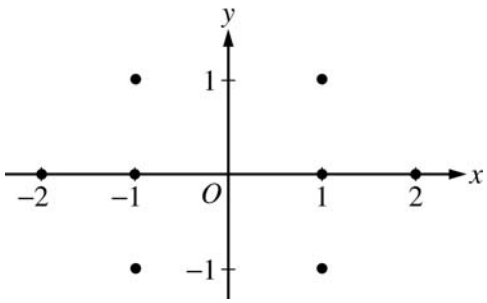
5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

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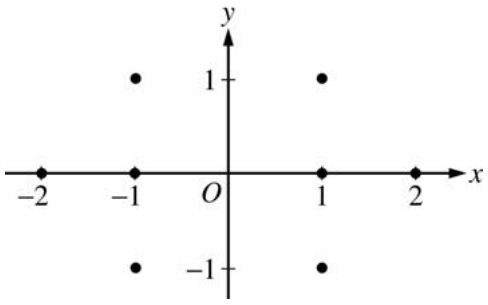
Question 5

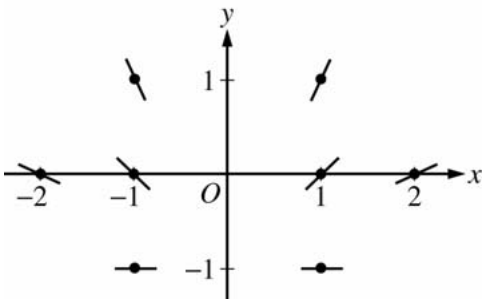
Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.
(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

(a)



2 : sign of slope at each point and relative steepness of slope lines in rows and columns

(b) $\frac{1}{1+y}\,dy = \frac{1}{x}\,dx$

$\ln|1+y| = \ln|x| + K$

$|1+y| = e^{\ln|x|+K}$
$1+y = C|x|$
$2 = C$
$1+y = 2|x|$
$y = 2|x| - 1$ and $x < 0$
or
$y = -2x - 1$ and $x < 0$

7 :
- 6 :
  - 1 : separates variables
  - 2 : antiderivatives
  - 1 : constant of integration
  - 1 : uses initial condition
  - 1 : solves for y

  Note: max $3/6$ [1-2-0-0-0] if no constant of integration

  Note: $0/6$ if no separation of variables
- 1 : domain

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6. The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \text{ and } f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

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END OF EXAM

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Question 6

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:
$f(0) = 2$, $f'(0) = -4$, and $f''(0) = 3$.

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

(a) $g'(x) = ae^{ax} + f'(x)$
$g'(0) = a - 4$

$g''(x) = a^2 e^{ax} + f''(x)$
$g''(0) = a^2 + 3$

$4 : \begin{cases} 1 : g'(x) \\ 1 : g'(0) \\ 1 : g''(x) \\ 1 : g''(0) \end{cases}$

(b) $h'(x) = f'(x)\cos(kx) - k\sin(kx) f(x)$
$h'(0) = f'(0)\cos(0) - k\sin(0) f(0) = f'(0) = -4$
$h(0) = \cos(0) f(0) = 2$
The equation of the tangent line is $y = -4x + 2$.

$5 : \begin{cases} 2 : h'(x) \\ 3 : \begin{cases} 1 : h'(0) \\ 1 : h(0) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

---

1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

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AP® CALCULUS AB
2007 SCORING GUIDELINES

Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

$\dfrac{20}{1+x^2} = 2$ when $x = \pm 3$

1 : correct limits in an integral in (a), (b), or (c)

(a) Area $= \displaystyle\int_{-3}^{3}\left(\frac{20}{1+x^2} - 2\right)dx = 37.961$ or 37.962

2 : { 1 : integrand; 1 : answer }

(b) Volume $= \pi\displaystyle\int_{-3}^{3}\left(\left(\frac{20}{1+x^2}\right)^2 - 2^2\right)dx = 1871.190$

3 : { 2 : integrand; 1 : answer }

(c) Volume $= \dfrac{\pi}{2}\displaystyle\int_{-3}^{3}\left(\frac{1}{2}\left(\frac{20}{1+x^2} - 2\right)\right)^2 dx$

$= \dfrac{\pi}{8}\displaystyle\int_{-3}^{3}\left(\frac{20}{1+x^2} - 2\right)^2 dx = 174.268$

3 : { 2 : integrand; 1 : answer }

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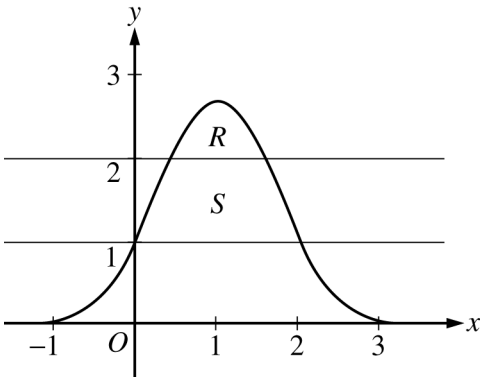
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.
   (a) Find the area of R.
   (b) Find the area of S.
   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

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AP® CALCULUS AB
2007 SCORING GUIDELINES (Form B)

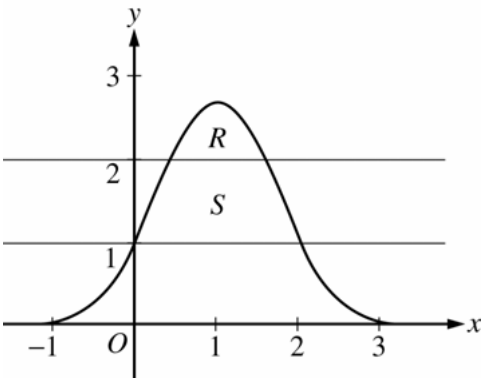
Question 1

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$

Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \displaystyle\int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

3 : { 1 : integrand; 1 : limits; 1 : answer }

(b) $e^{2x-x^2} = 1$ when $x = 0,\ 2$

Area of $S = \displaystyle\int_0^2 \left(e^{2x-x^2} - 1\right) dx$ − Area of R

$= 2.06016 -$ Area of $R = 1.546$

OR

$\displaystyle\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q - P) \cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$

$= 0.219064 + 1.107886 + 0.219064 = 1.546$

3 : { 1 : integrand; 1 : limits; 1 : answer }

(c) Volume $= \pi \displaystyle\int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2-1)^2\right) dx$

3 : { 2 : integrand; 1 : constant and limits }

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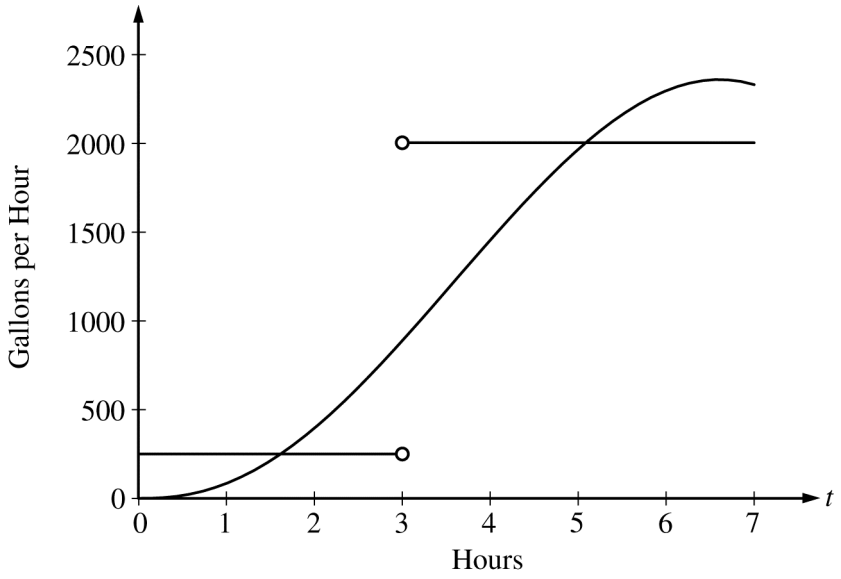

2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

   (i) The rate at which water enters the tank is $f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \le t \le 7$.

   (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$

   The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

   (a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

   (b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

   (c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

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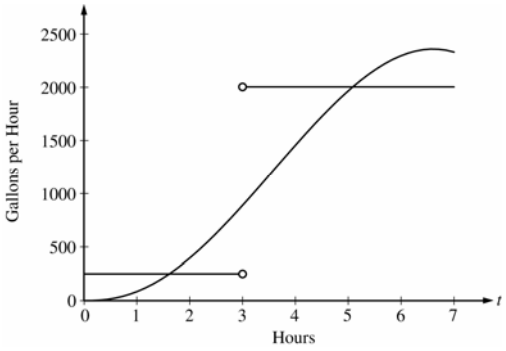
2007 SCORING GUIDELINES

Question 2

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is $g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases}$ gallons per hour.



The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a) $\int_0^7 f(t)\,dt \approx 8264$ gallons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) The amount of water in the tank is decreasing on the intervals $0 \le t \le 1.617$ and $3 \le t \le 5.076$ because $f(t) < g(t)$ for $0 \le t < 1.617$ and $3 < t < 5.076$.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0$, 3, and 7.

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t)\,dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t)\,dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

$5 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$

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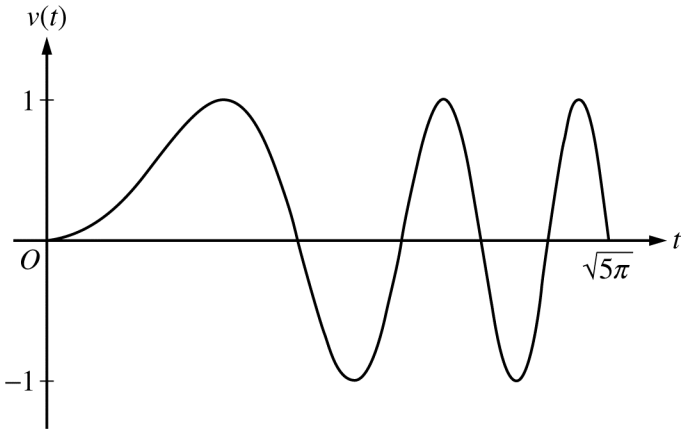

2. A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin\left(t^2\right)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

   (a) Find the acceleration of the particle at time $t = 3$.

   (b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

   (c) Find the position of the particle at time $t = 3$.

   (d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

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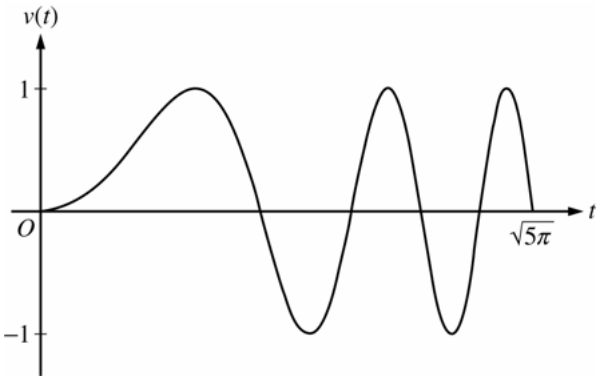
AP® CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 2

A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin\left(t^2\right)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.



(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

(a) $a(3) = v'(3) = 6\cos 9 = -5.466$ or -5.467

$1 : a(3)$

(b) Distance $= \int_0^3 |v(t)|\, dt = 1.702$

OR

For $0 < t < 3$, $v(t) = 0$ when $t = \sqrt{\pi} = 1.77245$ and $t = \sqrt{2\pi} = 2.50663$

$x(0) = 5$

$x\left(\sqrt{\pi}\right) = 5 + \int_0^{\sqrt{\pi}} v(t)\, dt = 5.89483$

$x\left(\sqrt{2\pi}\right) = 5 + \int_0^{\sqrt{2\pi}} v(t)\, dt = 5.43041$

$x(3) = 5 + \int_0^3 v(t)\, dt = 5.77356$

$\left|x\left(\sqrt{\pi}\right) - x(0)\right| + \left|x\left(\sqrt{2\pi}\right) - x\left(\sqrt{\pi}\right)\right| + \left|x(3) - x\left(\sqrt{2\pi}\right)\right| = 1.702$

$2 : \begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$

(c) $x(3) = 5 + \int_0^3 v(t)\, dt = 5.773$ or 5.774

$3 : \begin{cases} 2 \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = 5 \end{cases} \\ 1 : \text{answer} \end{cases}$

(d) The particle's rightmost position occurs at time $t = \sqrt{\pi} = 1.772$.

The particle changes from moving right to moving left at those times t for which $v(t) = 0$ with $v(t)$ changing from positive to negative, namely at $t = \sqrt{\pi}, \sqrt{3\pi}, \sqrt{5\pi}$ $(t = 1.772, 3.070, 3.963)$.

Using $x(T) = 5 + \int_0^T v(t)\, dt$, the particle's positions at the times it changes from rightward to leftward movement are:

T:	0	$\sqrt{\pi}$	$\sqrt{3\pi}$	$\sqrt{5\pi}$
$x(T)$:	5	5.895	5.788	5.752

The particle is farthest to the right when $T = \sqrt{\pi}$.

$3 : \begin{cases} 1 : \text{sets } v(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

   (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

   (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

   (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

   (d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

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END OF PART A OF SECTION II

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AP® CALCULUS AB
2007 SCORING GUIDELINES

Question 3

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	– 4	4	2
4	–1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$
$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$
Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r, $1 < r < 3$, such that $h(r) = -5$.

$2 : \begin{cases} 1 : h(1) \text{ and } h(3) \\ 1 : \text{conclusion, using IVT} \end{cases}$

(b) $\dfrac{h(3) - h(1)}{3 - 1} = \dfrac{-7 - 3}{3 - 1} = -5$
Since h is continuous and differentiable, by the Mean Value Theorem, there exists a value c, $1 < c < 3$, such that $h'(c) = -5$.

$2 : \begin{cases} 1 : \dfrac{h(3) - h(1)}{3 - 1} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c) $w'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -2$

$2 : \begin{cases} 1 : \text{apply chain rule} \\ 1 : \text{answer} \end{cases}$

(d) $g(1) = 2$, so $g^{-1}(2) = 1$.
$\left(g^{-1}\right)'(2) = \dfrac{1}{g'\left(g^{-1}(2)\right)} = \dfrac{1}{g'(1)} = \dfrac{1}{5}$
An equation of the tangent line is $y - 1 = \dfrac{1}{5}(x - 2)$.

$3 : \begin{cases} 1 : g^{-1}(2) \\ 1 : \left(g^{-1}\right)'(2) \\ 1 : \text{tangent line equation} \end{cases}$

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3. The wind chill is the temperature, in degrees Fahrenheit $(°F)$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32°F$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

   (a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

   (b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

   (c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant $32°F$. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

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END OF PART A OF SECTION II

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2007 SCORING GUIDELINES (Form B)

Question 3

The wind chill is the temperature, in degrees Fahrenheit (°F), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is 32°F, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant 32°F. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286

When $v = 20$ mph, the wind chill is decreasing at 0.286 °F/mph.

2 : { 1 : value; 1 : explanation }

(b) The average rate of change of W over the interval $5 \le v \le 60$ is $\dfrac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254.

$W'(v) = \dfrac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.

3 : { 1 : average rate of change; 1 : $W'(v)$ = average rate of change; 1 : value of v }

(c) $\left.\dfrac{dW}{dt}\right|_{t=3} = \left(\dfrac{dW}{dv} \cdot \dfrac{dv}{dt}\right)\Big|_{t=3} = W'(35) \cdot 5 = -0.892$ °F/hr

OR

$W = 55.6 - 22.1(20 + 5t)^{0.16}$

$\left.\dfrac{dW}{dt}\right|_{t=3} = -0.892$ °F/hr

3 : { 1 : $\dfrac{dv}{dt} = 5$; 1 : uses $v(3) = 35$, or uses $v(t) = 20 + 5t$; 1 : answer }

Units of °F/mph in (a) and °F/hr in (c)

1 : units in (a) and (c)

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

   (a) Find the time t at which the particle is farthest to the left. Justify your answer.

   (b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

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2007 SCORING GUIDELINES

Question 4

A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

(a) Find the time t at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

(a) $x'(t) = -e^{-t}\sin t + e^{-t}\cos t = e^{-t}(\cos t - \sin t)$

$x'(t) = 0$ when $\cos t = \sin t$. Therefore, $x'(t) = 0$ on $0 \le t \le 2\pi$ for $t = \frac{\pi}{4}$ and $t = \frac{5\pi}{4}$.

The candidates for the absolute minimum are at $t = 0, \frac{\pi}{4}, \frac{5\pi}{4}$, and 2π.

t	$x(t)$
0	$e^{0}\sin(0) = 0$
$\frac{\pi}{4}$	$e^{-\frac{\pi}{4}}\sin\left(\frac{\pi}{4}\right) > 0$
$\frac{5\pi}{4}$	$e^{-\frac{5\pi}{4}}\sin\left(\frac{5\pi}{4}\right) < 0$
2π	$e^{-2\pi}\sin(2\pi) = 0$

The particle is farthest to the left when $t = \frac{5\pi}{4}$.

$5:\begin{cases} 2: x'(t) \\ 1: \text{sets } x'(t) = 0 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$

(b) $x''(t) = -e^{-t}(\cos t - \sin t) + e^{-t}(-\sin t - \cos t)$

$= -2e^{-t}\cos t$

$Ax''(t) + x'(t) + x(t)$

$= A\left(-2e^{-t}\cos t\right) + e^{-t}(\cos t - \sin t) + e^{-t}\sin t$

$= (-2A + 1)e^{-t}\cos t$

$= 0$

Therefore, $A = \frac{1}{2}$.

$4:\begin{cases} 2: x''(t) \\ 1: \text{substitutes } x''(t), x'(t), \text{ and } x(t) \text{ into } Ax''(t) + x'(t) + x(t) \\ 1: \text{answer} \end{cases}$

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

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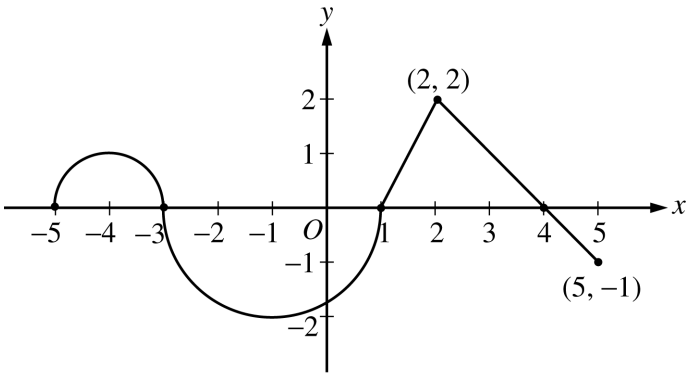

Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.
   (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
   (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
   (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
   (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

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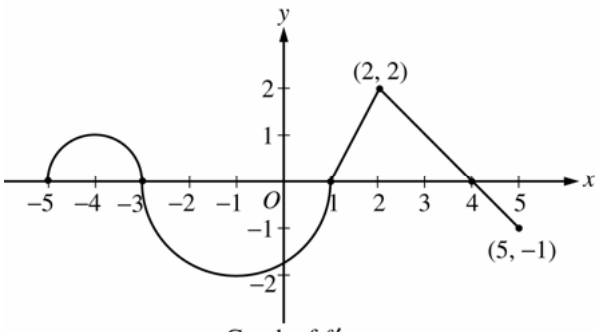
2007 SCORING GUIDELINES (Form B)

Question 4

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3,\ 1,\ 4$
f' changes from positive to negative at -3 and 4.
Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1,$ and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1,$ and 2.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{explanation} \end{cases}$

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$

$f(1) = 3$

$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

$3 : \begin{cases} 1 : \text{identifies } x = 1 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{value and explanation} \end{cases}$

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$.

   (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

   (a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

   (b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

   (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

   (d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

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Question 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft
Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{conclusion with reason} \end{cases}$

(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$
$\left.\frac{dV}{dt}\right|_{t=5} = 4\pi(30)^2\, 2 = 7200\pi \text{ ft}^3/\text{min}$

$3 : \begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{12} r'(t)\,dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$
$= 19.3$ ft
$\int_0^{12} r'(t)\,dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$

(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t)\,dt$.

1 : conclusion with reason

Units of ft^3/min in part (b) and ft in part (c)

1 : units in (b) and (c)

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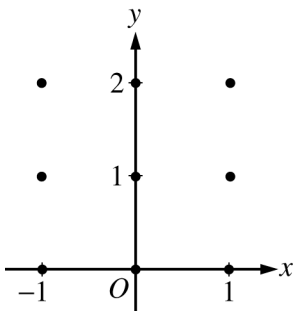
5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

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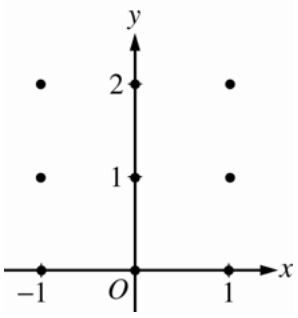
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.



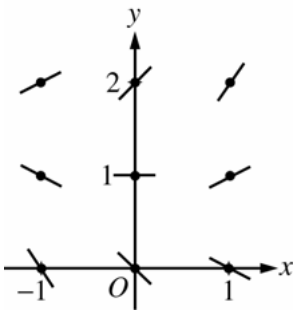
(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

(a)



2 : Sign of slope at each point and relative steepness of slope lines in rows and columns.

(b) $\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx} = \frac{1}{2}x + y - \frac{1}{2}$

Solution curves will be concave up on the half-plane above the line $y = -\frac{1}{2}x + \frac{1}{2}$.

3 : $\begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{description} \end{cases}$

(c) $\left.\frac{dy}{dx}\right|_{(0,\,1)} = 0 + 1 - 1 = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{(0,\,1)} = 0 + 1 - \frac{1}{2} > 0$

Thus, f has a relative minimum at $(0, 1)$.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) Substituting $y = mx + b$ into the differential equation:

$m = \frac{1}{2}x + (mx + b) - 1 = \left(m + \frac{1}{2}\right)x + (b - 1)$

Then $0 = m + \frac{1}{2}$ and $m = b - 1$: $m = -\frac{1}{2}$ and $b = \frac{1}{2}$.

2 : $\begin{cases} 1 : \text{value for } m \\ 1 : \text{value for } b \end{cases}$

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6. Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

   (a) Find $f'(x)$ and $f''(x)$.

   (b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

   (c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

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END OF EXAM

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Question 6

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

(a) Find $f'(x)$ and $f''(x)$.

(b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

(c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

(a) $f'(x) = \dfrac{k}{2\sqrt{x}} - \dfrac{1}{x}$

$f''(x) = -\dfrac{1}{4}kx^{-3/2} + x^{-2}$

$2 : \begin{cases} 1 : f'(x) \\ 1 : f''(x) \end{cases}$

(b) $f'(1) = \dfrac{1}{2}k - 1 = 0 \Rightarrow k = 2$

When $k = 2$, $f'(1) = 0$ and $f''(1) = -\dfrac{1}{2} + 1 > 0$.

f has a relative minimum value at $x = 1$ by the Second Derivative Test.

$4 : \begin{cases} 1 : \text{sets } f'(1) = 0 \text{ or } f'(x) = 0 \\ 1 : \text{solves for } k \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) At this inflection point, $f''(x) = 0$ and $f(x) = 0$.

$f''(x) = 0 \Rightarrow \dfrac{-k}{4x^{3/2}} + \dfrac{1}{x^2} = 0 \Rightarrow k = \dfrac{4}{\sqrt{x}}$

$f(x) = 0 \Rightarrow k\sqrt{x} - \ln x = 0 \Rightarrow k = \dfrac{\ln x}{\sqrt{x}}$

Therefore, $\dfrac{4}{\sqrt{x}} = \dfrac{\ln x}{\sqrt{x}}$

$\Rightarrow 4 = \ln x$

$\Rightarrow x = e^4$

$\Rightarrow k = \dfrac{4}{e^2}$

$3 : \begin{cases} 1 : f''(x) = 0 \text{ or } f(x) = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{answer} \end{cases}$

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6. Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

   (a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

   (b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

   (c) Show that if $f''(x) = 0$ for all x, then the graph of g does not have a point of inflection.

   (d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

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END OF EXAM

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Question 6

Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

(b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

(c) Show that if $f''(x) = 0$ for all x, then the graph of g does not have a point of inflection.

(d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

(a) The Mean Value Theorem guarantees that there is a value c, with $2 < c < 5$, so that
$f'(c) = \dfrac{f(5) - f(2)}{5 - 2} = \dfrac{2 - 5}{5 - 2} = -1.$

$2 : \begin{cases} 1 : \dfrac{f(5) - f(2)}{5 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(b) $g'(x) = f'(f(x)) \cdot f'(x)$
$g'(2) = f'(f(2)) \cdot f'(2) = f'(5) \cdot f'(2)$
$g'(5) = f'(f(5)) \cdot f'(5) = f'(2) \cdot f'(5)$
Thus, $g'(2) = g'(5)$.
Since f is twice-differentiable, g' is differentiable everywhere, so the Mean Value Theorem applied to g' on $[2, 5]$ guarantees there is a value k, with $2 < k < 5$, such that $g''(k) = \dfrac{g'(5) - g'(2)}{5 - 2} = 0.$

$3 : \begin{cases} 1 : g'(x) \\ 1 : g'(2) = f'(5) \cdot f'(2) = g'(5) \\ 1 : \text{uses MVT with } g' \end{cases}$

(c) $g''(x) = f''(f(x)) \cdot f'(x) \cdot f'(x) + f'(f(x)) \cdot f''(x)$
If $f''(x) = 0$ for all x, then
$g''(x) = 0 \cdot f'(x) \cdot f'(x) + f'(f(x)) \cdot 0 = 0$ for all x.
Thus, there is no x-value at which $g''(x)$ changes sign, so the graph of g has no inflection points.

$2 : \begin{cases} 1 : \text{considers } g'' \\ 1 : g''(x) = 0 \text{ for all } x \end{cases}$

OR

If $f''(x) = 0$ for all x, then f is linear, so $g = f \circ f$ is linear and the graph of g has no inflection points.

$2 : \begin{cases} 1 : f \text{ is linear} \\ 1 : g \text{ is linear} \end{cases}$

(d) Let $h(x) = f(x) - x$.
$h(2) = f(2) - 2 = 5 - 2 = 3$
$h(5) = f(5) - 5 = 2 - 5 = -3$
Since $h(2) > 0 > h(5)$, the Intermediate Value Theorem guarantees that there is a value r, with $2 < r < 5$, such that $h(r) = 0$.

$2 : \begin{cases} 1 : h(2) \text{ and } h(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$

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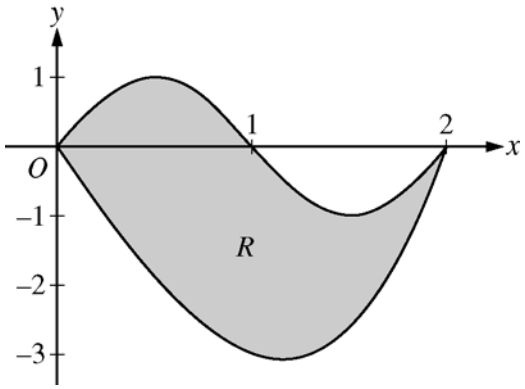
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

   (a) Find the area of R.

   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

   (d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

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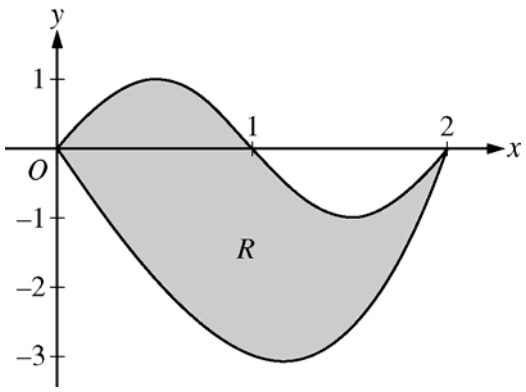
2008 SCORING GUIDELINES

Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$

Area $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 4$

3 : { 1 : limits; 1 : integrand; 1 : answer }

(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$

The area of the stated region is $\int_r^s \left(-2 - \left(x^3 - 4x\right)\right) dx$

2 : { 1 : limits; 1 : integrand }

(c) Volume $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right)^2 dx = 9.978$

2 : { 1 : integrand; 1 : answer }

(d) Volume $= \int_0^2 (3 - x)\left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 8.369$ or 8.370

2 : { 1 : integrand; 1 : answer }

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

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2008 SCORING GUIDELINES (Form B)

Question 1

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

The graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$ intersect at the points $(0, 0)$ and $(9, 3)$.

(a) $\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5$

OR

$\int_0^3 \left(3y - y^2\right) dy = 4.5$

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\pi \int_0^3 \left((3y+1)^2 - \left(y^2+1\right)^2\right) dy$

$= \frac{207\pi}{5} = 130.061$ or 130.062

$4 : \begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^3 \left(3y - y^2\right)^2 dy = 8.1$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

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t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

   (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

   (c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

   (d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

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Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

(a) $L'(5.5) \approx \dfrac{L(7) - L(4)}{7 - 4} = \dfrac{150 - 126}{3} = 8$ people per hour

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4}\left(\frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3)\right)$$

$= 155.25$ people

$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$

(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.

$3 : \begin{cases} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

OR

OR

The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.

$3 : \begin{cases} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

[*Note: There is a function L that satisfies the given conditions with $L'(t) = 0$ for exactly three values of t.*]

(d) $\displaystyle\int_0^3 r(t)\,dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

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2. For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

   (a) How many kilometers does the car travel during the first 2 hours?

   (b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

   (c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

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2008 SCORING GUIDELINES (Form B)

Question 2

For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

(a) $\int_0^2 r(t)\,dt = 206.370$ kilometers

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{dg}{dt} = \dfrac{dg}{dx} \cdot \dfrac{dx}{dt}; \quad \dfrac{dx}{dt} = r(t)$

$\left.\dfrac{dg}{dt}\right|_{t=2} = \left.\dfrac{dg}{dx}\right|_{x=206.370} \cdot r(2)$

$= (0.050)(120) = 6$ liters/hour

$3 : \begin{cases} 2 : \text{uses chain rule} \\ 1 : \text{answer with units} \end{cases}$

(c) Let T be the time at which the car's speed reaches 80 kilometers per hour.

Then, $r(T) = 80$ or $T = 0.331453$ hours.

At time T, the car has gone

$x(T) = \int_0^T r(t)\,dt = 10.794097$ kilometers

and has consumed $g(x(T)) = 0.537$ liters of gasoline.

$4 : \begin{cases} 1 : \text{equation } r(t) = 80 \\ 2 : \text{distance integral} \\ 1 : \text{answer} \end{cases}$

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3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

   (a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

   (b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

   (c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

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END OF PART A OF SECTION II

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AP® CALCULUS AB
2008 SCORING GUIDELINES

Question 3

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

(c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

(a) When $r = 100$ cm and $h = 0.5$ cm, $\frac{dV}{dt} = 2000 \text{ cm}^3/\text{min}$ and $\frac{dr}{dt} = 2.5 \text{ cm}/\text{min}$.

$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$

$2000 = 2\pi(100)(2.5)(0.5) + \pi(100)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = 0.038$ or $0.039 \text{ cm}/\text{min}$

$4 : \begin{cases} 1 : \frac{dV}{dt} = 2000 \text{ and } \frac{dr}{dt} = 2.5 \\ 2 : \text{expression for } \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

(b) $\frac{dV}{dt} = 2000 - R(t)$, so $\frac{dV}{dt} = 0$ when $R(t) = 2000$.
This occurs when $t = 25$ minutes.
Since $\frac{dV}{dt} > 0$ for $0 < t < 25$ and $\frac{dV}{dt} < 0$ for $t > 25$,
the oil slick reaches its maximum volume 25 minutes after the device begins working.

$3 : \begin{cases} 1 : R(t) = 2000 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) The volume of oil, in cm^3, in the slick at time $t = 25$ minutes is given by $60{,}000 + \int_0^{25} (2000 - R(t))\, dt$.

$2 : \begin{cases} 1 : \text{limits and initial condition} \\ 1 : \text{integrand} \end{cases}$

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Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

3. A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

   (a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

   (b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

   (c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

   (d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

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AP® CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

(a) $\frac{(0+7)}{2}\cdot 8 + \frac{(7+8)}{2}\cdot 6 + \frac{(8+2)}{2}\cdot 8 + \frac{(2+0)}{2}\cdot 2$
$= 115 \text{ ft}^2$

1 : trapezoidal approximation

(b) $\frac{1}{120}\int_0^{120} 115v(t)\,dt$
$= 1807.169$ or $1807.170 \text{ ft}^3/\text{min}$

3 : { 1 : limits and average value constant; 1 : integrand; 1 : answer }

(c) $\int_0^{24} 8\sin\left(\frac{\pi x}{24}\right)dx = 122.230$ or 122.231 ft^2

2 : { 1 : integra1; 1 : answer }

(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is $\frac{1}{20}\int_{40}^{60} C\cdot v(t)\,dt = 2181.912$ or $2181.913 \text{ ft}^3/\text{min}$.

Yes, water must be diverted since the average volumetric flow for this 20-minute period exceeds $2100 \text{ ft}^3/\text{min}$.

3 : { 1 : volumetric flow integral; 1 : average volumetric flow; 1 : answer with reason }

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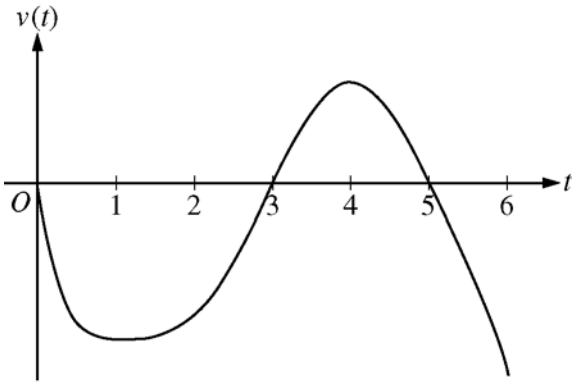
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

   (a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

   (b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

   (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

   (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

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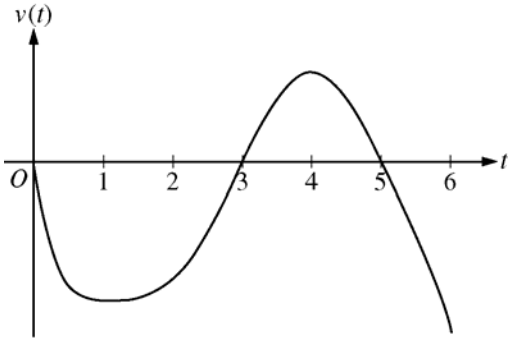
AP® CALCULUS AB
2008 SCORING GUIDELINES

Question 4



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

(a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$.

$$x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$$

$$x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$$

Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.

3 : { 1 : identifies $t = 3$ as a candidate; 1 : considers $\int_0^6 v(t)\,dt$; 1 : conclusion }

(b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$.

By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.

3 : { 1 : positions at $t = 3$, $t = 5$, and $t = 6$; 1 : description of motion; 1 : conclusion }

(c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.

1 : answer with reason

(d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.

2 : { 1 : answer; 1 : justification }

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4 + t^2}\, dt$ and $g(x) = f(\sin x)$.

   (a) Find $f'(x)$ and $g'(x)$.

   (b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.

   (c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \le x \le \pi$. Justify your answer.

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2008 SCORING GUIDELINES (Form B)

Question 4

The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4+t^2}\, dt$ and $g(x) = f(\sin x)$.

(a) Find $f'(x)$ and $g'(x)$.

(b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.

(c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \le x \le \pi$. Justify your answer.

(a) $f'(x) = 3\sqrt{4+(3x)^2}$

$g'(x) = f'(\sin x) \cdot \cos x$

$= 3\sqrt{4+(3\sin x)^2} \cdot \cos x$

$4 : \begin{cases} 2 : f'(x) \\ 2 : g'(x) \end{cases}$

(b) $g(\pi) = 0,\ g'(\pi) = -6$

Tangent line: $y = -6(x-\pi)$

$2 : \begin{cases} 1 : g(\pi) \text{ or } g'(\pi) \\ 1 : \text{tangent line equation} \end{cases}$

(c) For $0 < x < \pi$, $g'(x) = 0$ only at $x = \frac{\pi}{2}$.

$g(0) = g(\pi) = 0$

$g\left(\frac{\pi}{2}\right) = \int_0^3 \sqrt{4+t^2}\, dt > 0$

The maximum value of g on $[0, \pi]$ is $\int_0^3 \sqrt{4+t^2}\, dt$.

$3 : \begin{cases} 1 : \text{sets } g'(x) = 0 \\ 1 : \text{justifies maximum at } \frac{\pi}{2} \\ 1 : \text{integral expression for } g\left(\frac{\pi}{2}\right) \end{cases}$

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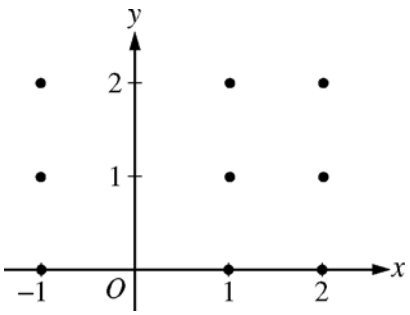
5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

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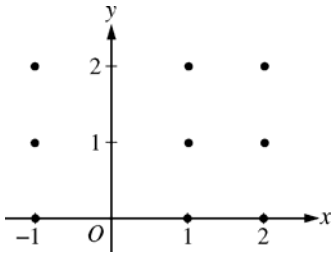
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

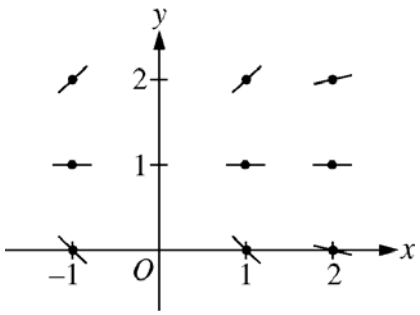
(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x\to\infty} f(x)$.



(a)



2 : { 1 : zero slopes; 1 : all other slopes }

(b) $\frac{1}{y-1}\,dy = \frac{1}{x^2}\,dx$

$\ln|y-1| = -\frac{1}{x} + C$

$|y-1| = e^{-\frac{1}{x}+C}$

$|y-1| = e^C e^{-\frac{1}{x}}$

$y - 1 = ke^{-\frac{1}{x}}$, where $k = \pm e^C$

$-1 = ke^{-\frac{1}{2}}$

$k = -e^{\frac{1}{2}}$

$f(x) = 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)}$, $x > 0$

6 : { 1 : separates variables; 2 : antidifferentiates; 1 : includes constant of integration; 1 : uses initial condition; 1 : solves for y }

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

(c) $\lim_{x\to\infty} 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)} = 1 - \sqrt{e}$

1 : limit

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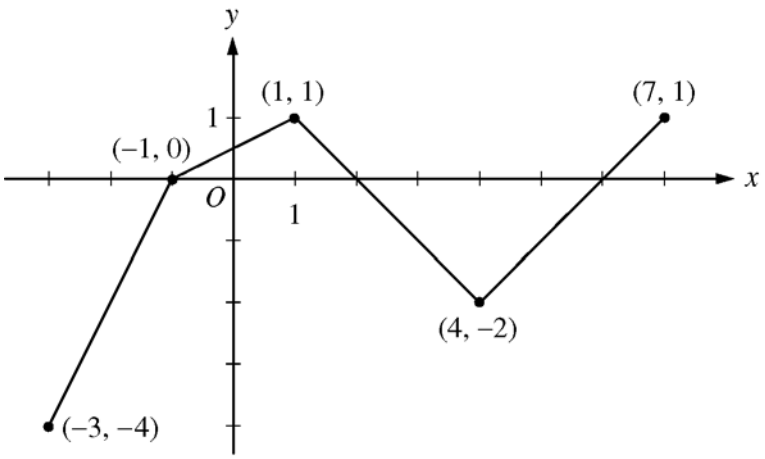

Graph of g'

5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

   (a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

   (b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

   (c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

   (d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

WRITE ALL WORK IN THE EXAM BOOKLET.

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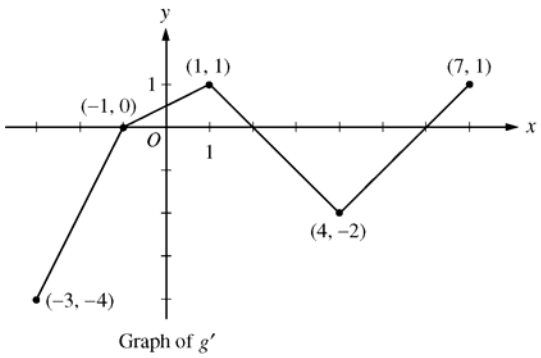
2008 SCORING GUIDELINES (Form B)

Question 5

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.



Graph of g'

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

(a) g' changes from increasing to decreasing at $x = 1$;
g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at $x = 1$ and $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) The only sign change of g' from positive to negative in the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x)\,dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$
$$g(2) = 5$$
$$g(7) = 5 + \int_2^{7} g'(x)\,dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \le x \le 7$ is $\frac{15}{2}$.

$3 : \begin{cases} 1 : \text{identifies } x = 2 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{maximum value and justification} \end{cases}$

(c) $\dfrac{g(7) - g(-3)}{7 - (-3)} = \dfrac{\frac{3}{2} - \frac{15}{2}}{10} = -\dfrac{3}{5}$

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

(d) $\dfrac{g'(7) - g'(-3)}{7 - (-3)} = \dfrac{1 - (-4)}{10} = \dfrac{1}{2}$

No, the MVT does *not* guarantee the existence of a value c with the stated properties because g' is not differentiable for at least one point in $-3 < x < 7$.

$2 : \begin{cases} 1 : \text{average value of } g'(x) \\ 1 : \text{answer "No" with reason} \end{cases}$

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6. Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

   (a) Write an equation for the line tangent to the graph of f at $x = e^2$.

   (b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

   (c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

   (d) Find $\lim\limits_{x \to 0^+} f(x)$.

END OF EXAM

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Question 6

Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

(d) Find $\lim\limits_{x \to 0^+} f(x)$.

(a) $f\left(e^2\right) = \dfrac{\ln e^2}{e^2} = \dfrac{2}{e^2}$, $f'\left(e^2\right) = \dfrac{1 - \ln e^2}{\left(e^2\right)^2} = -\dfrac{1}{e^4}$

An equation for the tangent line is $y = \dfrac{2}{e^2} - \dfrac{1}{e^4}\left(x - e^2\right)$.

$2 : \begin{cases} 1 : f\left(e^2\right) \text{ and } f'\left(e^2\right) \\ 1 : \text{answer} \end{cases}$

(b) $f'(x) = 0$ when $x = e$. The function f has a relative maximum at $x = e$ because $f'(x)$ changes from positive to negative at $x = e$.

$3 : \begin{cases} 1 : x = e \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$

(c) $f''(x) = \dfrac{-\dfrac{1}{x}x^2 - (1 - \ln x)2x}{x^4} = \dfrac{-3 + 2\ln x}{x^3}$ for all $x > 0$

$f''(x) = 0$ when $-3 + 2\ln x = 0$

$x = e^{3/2}$

The graph of f has a point of inflection at $x = e^{3/2}$ because $f''(x)$ changes sign at $x = e^{3/2}$.

$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer} \end{cases}$

(d) $\lim\limits_{x \to 0^+} \dfrac{\ln x}{x} = -\infty$ or Does Not Exist

$1 : \text{answer}$

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6. Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

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END OF EXAM

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2008 SCORING GUIDELINES (Form B)

Question 6

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

(a) $2x + 2 + 4y^3\dfrac{dy}{dx} + 4\dfrac{dy}{dx} = 0$

$\left(4y^3 + 4\right)\dfrac{dy}{dx} = -2x - 2$

$\dfrac{dy}{dx} = \dfrac{-2(x+1)}{4\left(y^3+1\right)} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$

(b) $\left.\dfrac{dy}{dx}\right|_{(-2,\,1)} = \dfrac{-(-2+1)}{2(1+1)} = \dfrac{1}{4}$

Tangent line: $y = 1 + \dfrac{1}{4}(x+2)$

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \end{cases}$

(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$.

On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$.

Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.

$3 : \begin{cases} 1 : y = -1 \\ 1 : \text{substitutes } y = -1 \text{ into the equation of the curve} \\ 1 : \text{answer} \end{cases}$

(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$.

The curve crosses the x-axis where $y = 0$.

$(-1)^2 + 2(-1) + 0^4 + 4 \cdot 0 \neq 5$

No, the curve cannot have a horizontal tangent where it crosses the x-axis.

$2 : \begin{cases} 1 : \text{works with } x = -1 \text{ or } y = 0 \\ 1 : \text{answer with reason} \end{cases}$

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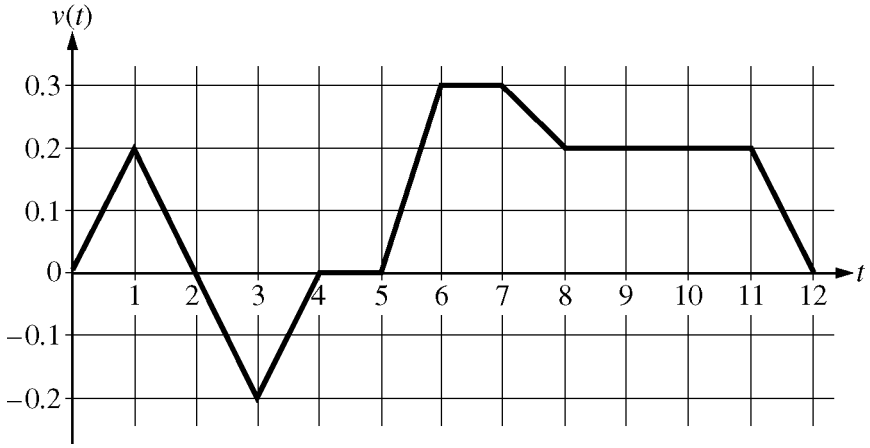
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

   (a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

   (c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

   (d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

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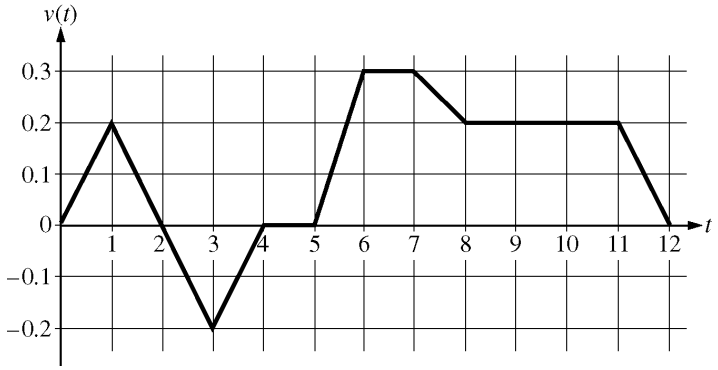
AP® CALCULUS AB
2009 SCORING GUIDELINES

Question 1



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

(a) $a(7.5) = v'(7.5) = \dfrac{v(8) - v(7)}{8 - 7} = -0.1$ miles/minute^2

2 : { 1 : answer; 1 : units }

(b) $\int_0^{12} |v(t)|\, dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t = 0$ to $t = 12$.

$$\int_0^{12} |v(t)|\, dt = \int_0^{2} v(t)\, dt - \int_2^{4} v(t)\, dt + \int_4^{12} v(t)\, dt$$
$$= 0.2 + 0.2 + 1.4 = 1.8 \text{ miles}$$

2 : { 1 : meaning of integral; 1 : value of integral }

(c) Caren turns around to go back home at time $t = 2$ minutes. This is the time at which her velocity changes from positive to negative.

2 : { 1 : answer; 1 : reason }

(d) $\int_0^{12} w(t)\, dt = 1.6$; Larry lives 1.6 miles from school.

$\int_0^{12} v(t)\, dt = 1.4$; Caren lives 1.4 miles from school.

Therefore, Caren lives closer to school.

3 : { 2 : Larry's distance from school (1 : integral; 1 : value); 1 : Caren's distance from school and conclusion }

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

   (a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

   (b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

   (c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?

GO ON TO THE NEXT PAGE.

AP® Calculus AB
Free-Response Scoring Guidelines

Question 1

The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

(a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

(c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?

(a) $\int_0^4 E(t)\,dt \approx 3981$ gallons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Let $S(t)$ be the amount of sewage in the treatment tank at time t. Then $S'(t) = E(t) - 645$ and $S'(t) = 0$ when $E(t) = 645$. On the interval $0 \le t \le 4$, $E(t) = 645$ when $t = 2.309$ and $t = 3.559$.

t (hours)	amount of sewage in treatment tank
0	0
2.309	$\int_0^{2.309} E(t)\,dt - 645(2.309) = 1637.178$
3.559	$\int_0^{3.559} E(t)\,dt - 645(3.559) = 1228.520$
4	$3981.022 - 645(4) = 1401.022$

The amount of sewage in the treatment tank is greatest at $t = 2.309$ hours. At that time, the amount of sewage in the tank, rounded to the nearest gallon, is 1637 gallons.

$4 : \begin{cases} 1 : \text{sets } E(t) = 645 \\ 1 : \text{identifies } t = 2.309 \text{ as a candidate} \\ 1 : \text{amount of sewage at } t = 2.309 \\ 1 : \text{conclusion} \end{cases}$

(c) Total cost $= \int_0^4 (0.15 - 0.02t)E(t)\,dt = 474.320$

The total cost of treating the sewage that enters the tank during the time interval $0 \le t \le 4$, to the nearest dollar, is \$474.

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

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AP® CALCULUS AB
2009 SCORING GUIDELINES (Form B)

Question 1

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

(a) $R(t) = 6 + \int_0^t \frac{1}{16}\left(3 + \sin\left(x^2\right)\right) dx$

$R(3) = 6.610$ or 6.611

3 : { 1 : integral; 1 : expression for $R(t)$; 1 : $R(3)$ }

(b) $A(t) = \pi(R(t))^2$

$A'(t) = 2\pi R(t) R'(t)$

$A'(3) = 8.858 \text{ cm}^2/\text{year}$

3 : { 1 : expression for $A(t)$; 1 : expression for $A'(t)$; 1 : answer with units }

(c) $\int_0^3 A'(t)\,dt = A(3) - A(0) = 24.200$ or 24.201

From time $t = 0$ to $t = 3$ years, the cross-sectional area grows by 24.201 square centimeters.

3 : { 1 : uses Fundamental Theorem of Calculus; 1 : value of $\int_0^3 A'(t)\,dt$; 1 : meaning of $\int_0^3 A'(t)\,dt$ }

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2. The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

   (a) How many people are in the auditorium when the concert begins?

   (b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

   (c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

   (d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

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AP® CALCULUS AB
2009 SCORING GUIDELINES

Question 2

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

(a) How many people are in the auditorium when the concert begins?

(b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

(c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

(d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

(a) $\int_0^2 R(t)\,dt = 980$ people

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $R'(t) = 0$ when $t = 0$ and $t = 1.36296$
The maximum rate may occur at 0, $a = 1.36296$, or 2.

$R(0) = 0$
$R(a) = 854.527$
$R(2) = 120$

The maximum rate occurs when $t = 1.362$ or 1.363.

$3 : \begin{cases} 1 : \text{considers } R'(t) = 0 \\ 1 : \text{interior critical point} \\ 1 : \text{answer and justification} \end{cases}$

(c) $w(2) - w(1) = \int_1^2 w'(t)\,dt = \int_1^2 (2 - t)R(t)\,dt = 387.5$
The total wait time for those who enter the auditorium after time $t = 1$ is 387.5 hours.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $\frac{1}{980}w(2) = \frac{1}{980}\int_0^2 (2 - t)R(t)\,dt = 0.77551$
On average, a person waits 0.775 or 0.776 hour.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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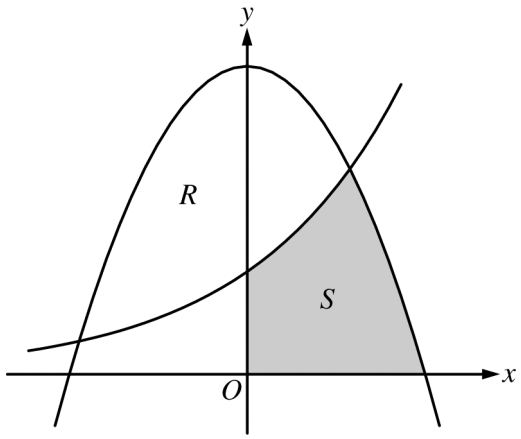
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



2. Let R and S in the figure above be defined as follows: R is the region in the first and second quadrants bounded by the graphs of $y = 3 - x^2$ and $y = 2^x$. S is the shaded region in the first quadrant bounded by the two graphs, the x-axis, and the y-axis.

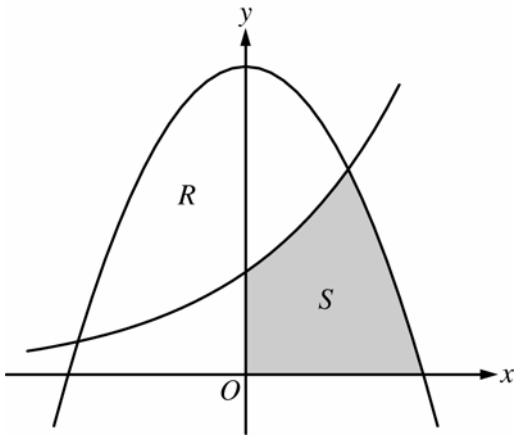
   (a) Find the area of S.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with one leg across the base of the solid. Write, but do not evaluate, an integral expression that gives the volume of the solid.

AP® Calculus AB
Free-Response Scoring Guidelines

Question 2



Let R and S in the figure above be defined as follows: R is the region in the first and second quadrants bounded by the graphs of $y = 3 - x^2$ and $y = 2^x$. S is the shaded region in the first quadrant bounded by the two graphs, the x-axis, and the y-axis.

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(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with one leg across the base of the solid. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$3 - x^2 = 2^x$ when $x = -1.63658$ and $x = 1$
Let $a = -1.63658$

(a) Area of $S = \int_0^1 2^x\,dx + \int_1^{\sqrt{3}} \left(3 - x^2\right) dx$
$= 2.240$

3 : { 1 : integrands; 1 : limits; 1 : answer }

(b) Volume $= \pi \int_a^1 \left(\left(3 - x^2 + 1\right)^2 - \left(2^x + 1\right)^2\right) dx$
$= 63.106$ or 63.107

4 : { 2 : integrand; 1 : limits and constant; 1 : answer }

(c) Volume $= \frac{1}{2}\int_a^1 \left(3 - x^2 - 2^x\right)^2\,dx$

2 : { 1 : integrand; 1 : limits and constant }

2. A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.

   (a) What was the distance between the road and the edge of the water at the end of the storm?

   (b) Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.

   (c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.

   (d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

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AP® CALCULUS AB
2009 SCORING GUIDELINES (Form B)

Question 2

A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.

(a) What was the distance between the road and the edge of the water at the end of the storm?

(b) Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.

(c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.

(d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

(a) $35 + \int_0^5 f(t)\,dt = 26.494$ or 26.495 meters

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Four hours after the storm began, the rate of change of the distance between the road and the edge of the water is increasing at a rate of 1.007 meters/hours^2.

$2 : \begin{cases} 1 : \text{interpretation of } f'(4) \\ 1 : \text{units} \end{cases}$

(c) $f'(t) = 0$ when $t = 0.66187$ and $t = 2.84038$
The minimum of f for $0 \le t \le 5$ may occur at 0, 0.66187, 2.84038, or 5.

$f(0) = -2$
$f(0.66187) = -1.39760$
$f(2.84038) = -2.26963$
$f(5) = -0.48027$

The distance between the road and the edge of the water was decreasing most rapidly at time $t = 2.840$ hours after the storm began.

$3 : \begin{cases} 1 : \text{considers } f'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $-\int_0^5 f(t)\,dt = \int_0^x g(p)\,dp$

$2 : \begin{cases} 1 : \text{integral of } g \\ 1 : \text{answer} \end{cases}$

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3. Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

   (a) Find Mighty's profit on the sale of a 25-meter cable.

   (b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\,dx$ in the context of this problem.

   (c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

   (d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

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END OF PART A OF SECTION II

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2009 SCORING GUIDELINES

Question 3

Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

(a) Find Mighty's profit on the sale of a 25-meter cable.

(b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\,dx$ in the context of this problem.

(c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

(d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

(a) Profit $= 120 \cdot 25 - \int_0^{25} 6\sqrt{x}\,dx = 2500$ dollars

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\int_{25}^{30} 6\sqrt{x}\,dx$ is the difference in cost, in dollars, of producing a cable of length 30 meters and a cable of length 25 meters.

1 : answer with units

(c) Profit $= 120k - \int_0^{k} 6\sqrt{x}\,dx$ dollars

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{expression} \end{cases}$

(d) Let $P(k)$ be the profit for a cable of length k.
$P'(k) = 120 - 6\sqrt{k} = 0$ when $k = 400$.
This is the only critical point for P, and P' changes from positive to negative at $k = 400$.
Therefore, the maximum profit is $P(400) = 16{,}000$ dollars.

$4 : \begin{cases} 1 : P'(k) = 0 \\ 1 : k = 400 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

---

t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

3. The temperature, in degrees Celsius (°C), of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

   (a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

   (c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

   (d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

AP® Calculus AB
Free-Response Scoring Guidelines

Question 3

t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

The temperature, in degrees Celsius $(°C)$, of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

(a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.

(b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

(c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

(d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

(a) $H'(10) \approx \dfrac{H(12) - H(8)}{12 - 8} = \dfrac{80 - 73}{4} = \dfrac{7}{4}°\text{C/min}$

2 : { 1 : difference quotient; 1 : answer with units }

(b) Average temperature is $\dfrac{1}{16}\displaystyle\int_0^{16} H(t)\,dt$

$\displaystyle\int_0^{16} H(t)\,dt \approx 4 \cdot (65 + 68 + 73 + 80)$

Average temperature $\approx \dfrac{4 \cdot 286}{16} = 71.5°\text{C}$

3 : { 1 : $\dfrac{1}{16}\displaystyle\int_0^{16} H(t)\,dt$; 1 : left Riemann sum; 1 : answer }

(c) The left Riemann sum approximation is an underestimate of the integral because the graph of H is increasing. Dividing by 16 will not change the inequality, so $71.5°\text{C}$ is an underestimate of the average temperature.

1 : answer with reason

(d) If a continuous function is increasing at an increasing rate, then the slopes of the secant lines of the graph of the function are increasing. The slopes of the secant lines for the four intervals in the table are $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$, and $\frac{10}{4}$, respectively.

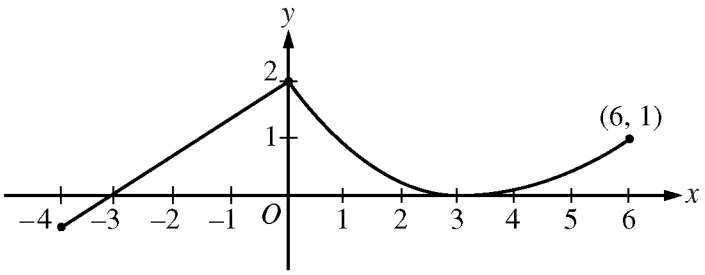
Since the slopes are increasing, the data are consistent with the claim.

OR

By the Mean Value Theorem, the slopes are also the values of $H'(c_k)$ for some times $c_1 < c_2 < c_3 < c_4$, respectively.

Since these derivative values are positive and increasing, the data are consistent with the claim.

3 : { 1 : considers slopes of four secant lines; 1 : explanation; 1 : conclusion consistent with explanation }



Graph of f

3. A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

   (a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

   (b) For how many values of a, $-4 \le a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0? Give a reason for your answer.

   (c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

   (d) The function g is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

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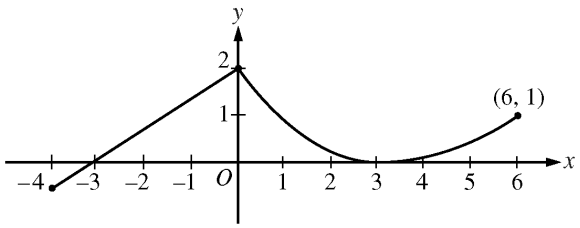
AP® CALCULUS AB
2009 SCORING GUIDELINES (Form B)

Question 3



Graph of f

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

(a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

(b) For how many values of a, $-4 \le a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0? Give a reason for your answer.

(c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

(d) The function g is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

(a) $\lim\limits_{h \to 0^-} \dfrac{f(h) - f(0)}{h} = \dfrac{2}{3}$

$\lim\limits_{h \to 0^+} \dfrac{f(h) - f(0)}{h} < 0$

Since the one-sided limits do not agree, f is not differentiable at $x = 0$.

$2 : \begin{cases} 1 : \text{sets up difference quotient at } x = 0 \\ 1 : \text{answer with justification} \end{cases}$

(b) $\dfrac{f(6) - f(a)}{6 - a} = 0$ when $f(a) = f(6)$. There are two values of a for which this is true.

$2 : \begin{cases} 1 : \text{expression for average rate of change} \\ 1 : \text{answer with reason} \end{cases}$

(c) Yes, $a = 3$. The function f is differentiable on the interval $3 < x < 6$ and continuous on $3 \le x \le 6$.

Also, $\dfrac{f(6) - f(3)}{6 - 3} = \dfrac{1 - 0}{6 - 3} = \dfrac{1}{3}$.

By the Mean Value Theorem, there is a value c, $3 < c < 6$, such that $f'(c) = \dfrac{1}{3}$.

$2 : \begin{cases} 1 : \text{answers "yes" and identifies } a = 3 \\ 1 : \text{justification} \end{cases}$

(d) $g'(x) = f(x)$, $g''(x) = f'(x)$

$g''(x) > 0$ when $f'(x) > 0$

This is true for $-4 < x < 0$ and $3 < x < 6$.

$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{considers } g''(x) > 0 \\ 1 : \text{answer} \end{cases}$

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

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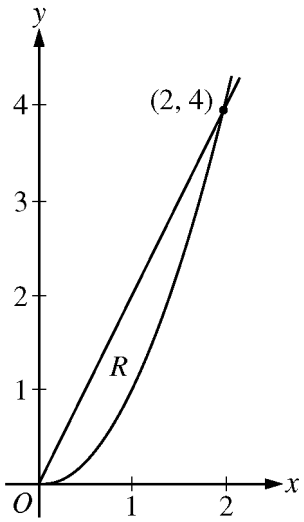

4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

   (c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

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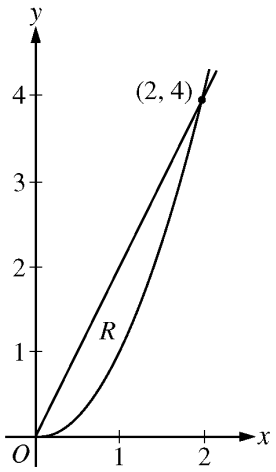
AP® CALCULUS AB
2009 SCORING GUIDELINES

Question 4

Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

(a) Area $= \int_0^2 \left(2x - x^2\right) dx$

$= x^2 - \frac{1}{3}x^3 \Big|_{x=0}^{x=2}$

$= \frac{4}{3}$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(b) Volume $= \int_0^2 \sin\left(\frac{\pi}{2}x\right) dx$

$= -\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Big|_{x=0}^{x=2}$

$= \frac{4}{\pi}$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(c) Volume $= \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$

3 : { 2 : integrand; 1 : limits }

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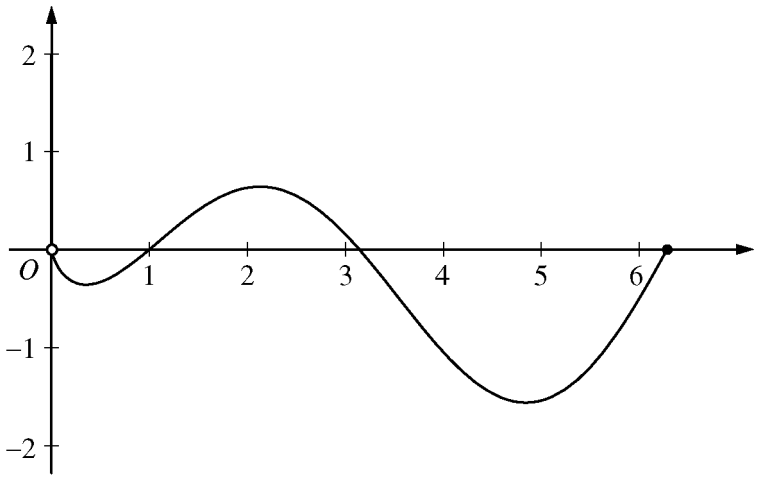
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$.

   The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

   (a) Find $g(1)$ and $g'(1)$.

   (b) On what intervals, if any, is g increasing? Justify your answer.

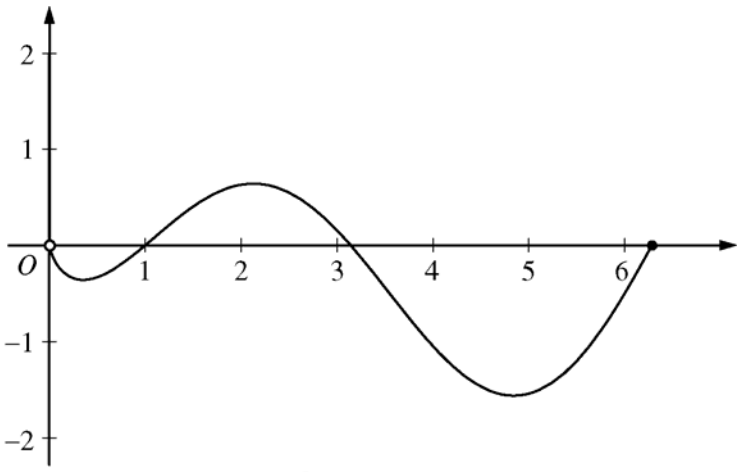
   (c) For $0 < x \le 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.

   (d) For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x-axis? Explain why or why not.

GO ON TO THE NEXT PAGE.

AP® Calculus AB
Free-Response Scoring Guidelines

Question 4



Graph of f

Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$. The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

(a) Find $g(1)$ and $g'(1)$.

(b) On what intervals, if any, is g increasing? Justify your answer.

(c) For $0 < x \le 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.

(d) For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x-axis? Explain why or why not.

(a) $g(1) = \int_1^1 f(t)\,dt = 0$ and $g'(1) = f(1) = 0$

$2 : \begin{cases} 1 : g(1) \\ 1 : g'(1) \end{cases}$

(b) Since $g'(x) = f(x)$, g is increasing on the interval $1 \le x \le \pi$ because $f(x) > 0$ for $1 < x < \pi$.

$2 : \begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$

(c) For $0 < x < 2\pi$, $g'(x) = f(x) = 0$ when $x = 1,\ \pi$. $g' = f$ changes from negative to positive only at $x = 1$. The absolute minimum must occur at $x = 1$ or at the right endpoint. Since $g(1) = 0$ and $g(2\pi) = \int_1^{2\pi} f(t)\,dt = \int_1^{\pi} f(t)\,dt + \int_{\pi}^{2\pi} f(t)\,dt < 0$ by comparison of the two areas, the absolute minimum occurs at $x = 2\pi$.

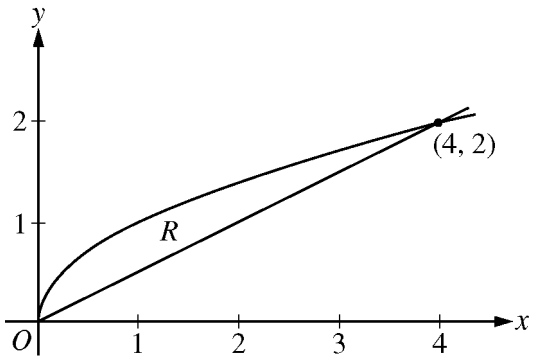
$3 : \begin{cases} 1 : \text{identifies 1 and } 2\pi \text{ as candidates} \\ \quad\quad \text{- or -} \\ \quad \text{indicates that the graph of } g \\ \quad \text{decreases, increases, then decreases} \\ 1 : \text{justifies } g(2\pi) < g(1) \\ 1 : \text{answer} \end{cases}$

(d) Yes, the graph of g is tangent to the x-axis at $x = 1$ since $g(1) = 0$ and $g'(1) = 0$.

$2 : \begin{cases} 1 : \text{answer of ``yes'' with } x = 1 \\ 1 : \text{explanation} \end{cases}$

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

   (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

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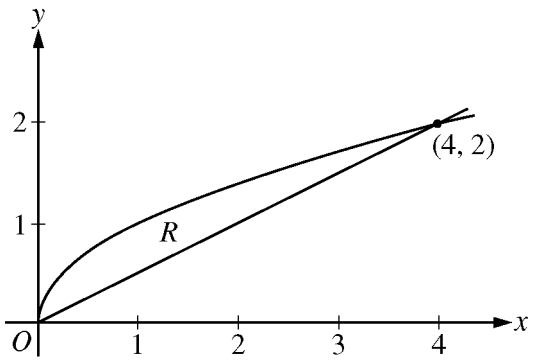
2009 SCORING GUIDELINES (Form B)

Question 4

Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

(a) Area $= \displaystyle\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx = \frac{2}{3}x^{3/2} - \frac{x^2}{4}\Big|_{x=0}^{x=4} = \frac{4}{3}$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(b) Volume $= \displaystyle\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)^2 dx = \int_0^4 \left(x - x^{3/2} + \frac{x^2}{4}\right) dx$

$= \displaystyle\frac{x^2}{2} - \frac{2x^{5/2}}{5} + \frac{x^3}{12}\Big|_{x=0}^{x=4} = \frac{8}{15}$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(c) Volume $= \pi \displaystyle\int_0^4 \left(\left(2 - \frac{x}{2}\right)^2 - \left(2 - \sqrt{x}\right)^2\right) dx$

3 : { 1 : limits and constant; 2 : integrand }

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x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

   (a) Estimate $f'(4)$. Show the work that leads to your answer.

   (b) Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

   (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

   (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

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Question 5

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

(a) Estimate $f'(4)$. Show the work that leads to your answer.

(b) Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

(a) $f'(4) \approx \dfrac{f(5) - f(3)}{5 - 3} = -3$

1 : answer

(b) $\displaystyle\int_2^{13} (3 - 5f'(x))\,dx = \int_2^{13} 3\,dx - 5\int_2^{13} f'(x)\,dx$

$= 3(13 - 2) - 5(f(13) - f(2)) = 8$

2 : { 1 : uses Fundamental Theorem of Calculus; 1 : answer }

(c) $\displaystyle\int_2^{13} f(x)\,dx \approx f(2)(3 - 2) + f(3)(5 - 3) + f(5)(8 - 5) + f(8)(13 - 8) = 18$

2 : { 1 : left Riemann sum; 1 : answer }

(d) An equation for the tangent line is $y = -2 + 3(x - 5)$.
Since $f''(x) < 0$ for all x in the interval $5 \le x \le 8$, the line tangent to the graph of $y = f(x)$ at $x = 5$ lies above the graph for all x in the interval $5 < x \le 8$.
Therefore, $f(7) \le -2 + 3 \cdot 2 = 4$.

An equation for the secant line is $y = -2 + \frac{5}{3}(x - 5)$.
Since $f''(x) < 0$ for all x in the interval $5 \le x \le 8$, the secant line connecting $(5, f(5))$ and $(8, f(8))$ lies below the graph of $y = f(x)$ for all x in the interval $5 < x < 8$.
Therefore, $f(7) \ge -2 + \frac{5}{3} \cdot 2 = \frac{4}{3}$.

4 : { 1 : tangent line; 1 : shows $f(7) \le 4$; 1 : secant line; 1 : shows $f(7) \ge \frac{4}{3}$ }

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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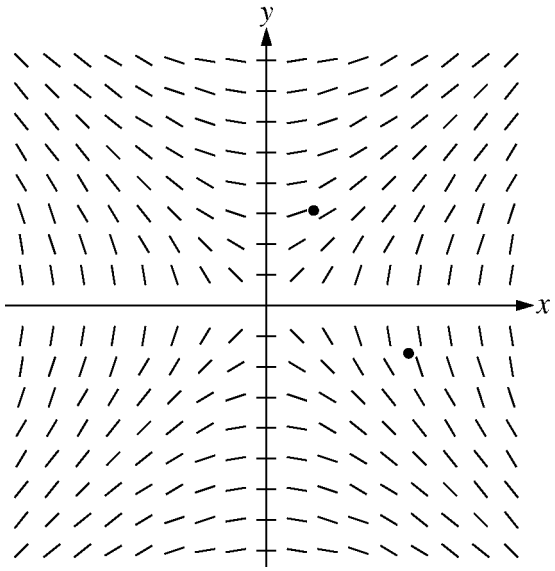
5. Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$, where $y \neq 0$.

   (a) The slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(3, -1)$, and sketch the solution curve that passes through the point $(1, 2)$.

   (Note: The points $(3, -1)$ and $(1, 2)$ are indicated in the figure.)



   (b) Write an equation for the line tangent to the solution curve that passes through the point $(1, 2)$.

   (c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(3) = -1$, and state its domain.

GO ON TO THE NEXT PAGE.

AP® Calculus AB
Free-Response Scoring Guidelines

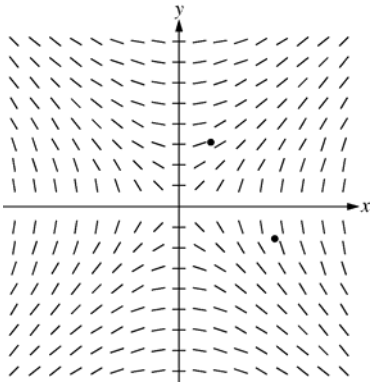
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$, where $y \neq 0$.

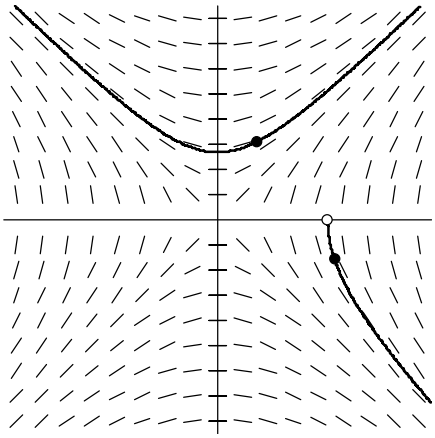
(a) The slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(3, -1)$, and sketch the solution curve that passes through the point $(1, 2)$. (Note: The points $(3, -1)$ and $(1, 2)$ are indicated in the figure.)

(b) Write an equation for the line tangent to the solution curve that passes through the point $(1, 2)$.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(3) = -1$, and state its domain.



(a)



$2 : \begin{cases} 1 : \text{solution curve through } (3, -1) \\ 1 : \text{solution curve through } (1, 2) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field or the x-axis.

(b) $\left.\frac{dy}{dx}\right|_{(1, 2)} = \frac{1}{2}$

An equation for the line tangent to the solution curve is $y - 2 = \frac{1}{2}(x - 1)$.

1 : equation of tangent line

(c) $y\,dy = x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + A$

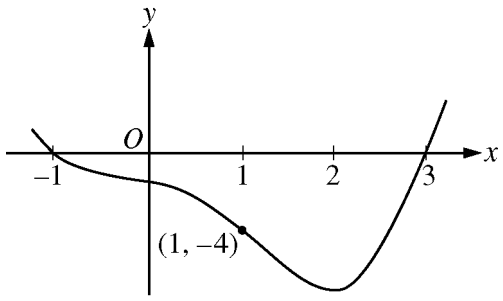
$y^2 = x^2 + C$

$C = -8$

Since the particular solution goes through $(3, -1)$, y must be negative.

$y = -\sqrt{x^2 - 8}$ for $x > \sqrt{8}$

$6 : \begin{cases} 5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases} \\ \text{Note: max } 2/5 \text{ [1-1-0-0-0] if no constant of integration} \\ 1 : \text{domain} \end{cases}$



Graph of f'

5. Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

   (a) Write an equation for the line tangent to the graph of g at $x = 1$.

   (b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

   (c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

   (d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

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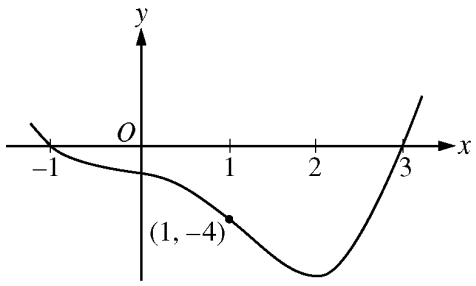
Question 5

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.



Graph of f'

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

(a) $g(1) = e^{f(1)} = e^2$

$g'(x) = e^{f(x)}f'(x)$, $g'(1) = e^{f(1)}f'(1) = -4e^2$

The tangent line is given by $y = e^2 - 4e^2(x - 1)$.

$3 : \begin{cases} 1 : g'(x) \\ 1 : g(1) \text{ and } g'(1) \\ 1 : \text{tangent line equation} \end{cases}$

(b) $g'(x) = e^{f(x)}f'(x)$

$e^{f(x)} > 0$ for all x

So, g' changes from positive to negative only when f' changes from positive to negative. This occurs at $x = -1$ only. Thus, g has a local maximum at $x = -1$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) $g''(-1) = e^{f(-1)}\left[(f'(-1))^2 + f''(-1)\right]$

$e^{f(-1)} > 0$ and $f'(-1) = 0$

Since f' is decreasing on a neighborhood of -1, $f''(-1) < 0$. Therefore, $g''(-1) < 0$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $\dfrac{g'(3) - g'(1)}{3 - 1} = \dfrac{e^{f(3)}f'(3) - e^{f(1)}f'(1)}{2} = 2e^2$

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

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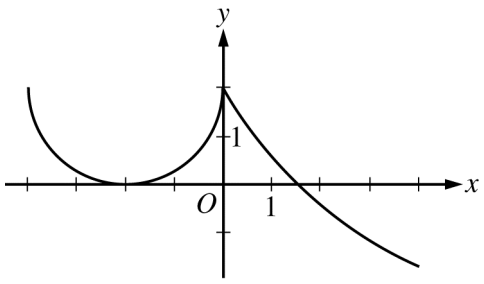

Graph of f'

6. The derivative of a function f is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \le 4 \end{cases}$.

The graph of the continuous function f', shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.

(a) For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

(b) Find $f(-4)$ and $f(4)$.

(c) For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

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END OF EXAM

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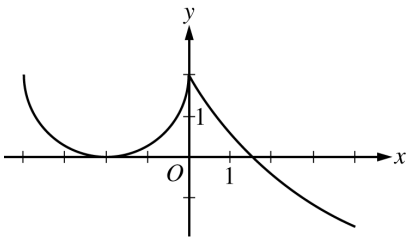
2009 SCORING GUIDELINES

Question 6

The derivative of a function f is defined by

$$f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \le 4 \end{cases}.$$

The graph of the continuous function f', shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.



Graph of f'

(a) For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

(b) Find $f(-4)$ and $f(4)$.

(c) For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

(a) f' changes from decreasing to increasing at $x = -2$ and from increasing to decreasing at $x = 0$. Therefore, the graph of f has points of inflection at $x = -2$ and $x = 0$.

$2 : \begin{cases} 1 : \text{identifies } x = -2 \text{ or } x = 0 \\ 1 : \text{answer with justification} \end{cases}$

(b) $f(-4) = 5 + \int_0^{-4} g(x)\,dx$

$= 5 - (8 - 2\pi) = 2\pi - 3$

$f(4) = 5 + \int_0^4 \left(5e^{-x/3} - 3\right) dx$

$= 5 + \left(-15e^{-x/3} - 3x\right)\Big|_{x=0}^{x=4}$

$= 8 - 15e^{-4/3}$

$5 : \begin{cases} 2 : f(-4) \\ \quad 1 : \text{integral} \\ \quad 1 : \text{value} \\ 3 : f(4) \\ \quad 1 : \text{integral} \\ \quad 1 : \text{antiderivative} \\ \quad 1 : \text{value} \end{cases}$

(c) Since $f'(x) > 0$ on the intervals $-4 < x < -2$ and $-2 < x < 3\ln\left(\frac{5}{3}\right)$, f is increasing on the interval $-4 \le x \le 3\ln\left(\frac{5}{3}\right)$.

Since $f'(x) < 0$ on the interval $3\ln\left(\frac{5}{3}\right) < x < 4$, f is decreasing on the interval $3\ln\left(\frac{5}{3}\right) \le x \le 4$.

Therefore, f has an absolute maximum at $x = 3\ln\left(\frac{5}{3}\right)$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

---

6. Let $g(x) = xe^{-x} + be^{-x}$, where b is a positive constant.

   (a) Find $\lim_{x \to \infty} g(x)$.

   (b) For what positive value of b does g have an absolute maximum at $x = \frac{2}{3}$? Justify your answer.

   (c) Find all values of b, if any, for which the graph of g has a point of inflection on the interval $0 < x < \infty$. Justify your answer.

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Free-Response Scoring Guidelines

Question 6

Let $g(x) = xe^{-x} + be^{-x}$, where b is a positive constant.

(a) Find $\lim_{x\to\infty} g(x)$.

(b) For what positive value of b does g have an absolute maximum at $x = \frac{2}{3}$? Justify your answer.

(c) Find all values of b, if any, for which the graph of g has a point of inflection on the interval $0 < x < \infty$. Justify your answer.

(a) $\lim_{x\to\infty} g(x) = 0$

1 : answer

(b) $g'(x) = e^{-x} - xe^{-x} - be^{-x} = (1 - x - b)e^{-x}$

$g'\left(\frac{2}{3}\right) = \left(\frac{1}{3} - b\right)e^{-2/3} = 0 \Rightarrow b = \frac{1}{3}$

When $b = \frac{1}{3}$, $g'(x) = \left(\frac{2}{3} - x\right)e^{-x}$.

For $x < \frac{2}{3}$, $g'(x) > 0$ and for $x > \frac{2}{3}$, $g'(x) < 0$.

Therefore, when $b = \frac{1}{3}$, g has an absolute maximum at $x = \frac{2}{3}$.

4 : { 2 : $g'(x)$; 1 : solves $g'\left(\frac{2}{3}\right) = 0$ for b; 1 : justification }

(c) $g''(x) = -e^{-x} - (1 - x - b)e^{-x} = (x - 2 + b)e^{-x}$

If $0 < b < 2$, then $g''(x)$ will change sign at $x = 2 - b > 0$. Therefore, the graph of g will have a point of inflection on the interval $0 < x < \infty$ when $0 < b < 2$.

4 : { 2 : $g''(x)$; 1 : interval for b; 1 : justification }

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	–10	–8	–4	7

6. The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

   (a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

   (c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

   (d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

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END OF EXAM

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2009 SCORING GUIDELINES (Form B)

Question 6

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	–10	–8	–4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

---

1 : units in (a) and (b)

(a) $a(36) = v'(36) \approx \dfrac{v(40) - v(32)}{40 - 32} = \dfrac{11}{8}$ meters/sec^2

1 : answer

(b) $\int_{20}^{40} v(t)\,dt$ is the particle's change in position in meters from time $t = 20$ seconds to time $t = 40$ seconds.

$$\int_{20}^{40} v(t)\,dt \approx \frac{v(20) + v(25)}{2} \cdot 5 + \frac{v(25) + v(32)}{2} \cdot 7 + \frac{v(32) + v(40)}{2} \cdot 8$$
$$= -75 \text{ meters}$$

3 : { 1 : meaning of $\int_{20}^{40} v(t)\,dt$; 2 : trapezoidal approximation }

(c) $v(8) > 0$ and $v(20) < 0$
$v(32) < 0$ and $v(40) > 0$
Therefore, the particle changes direction in the intervals $8 < t < 20$ and $32 < t < 40$.

2 : { 1 : answer ; 1 : explanation }

(d) Since $v'(t) = a(t) > 0$ for $0 < t < 8$, $v(t) \ge 3$ on this interval.
Therefore, $x(8) = x(0) + \int_0^8 v(t)\,dt \ge 7 + 8 \cdot 3 > 30$.

2 : { 1 : $v'(t) = a(t)$; 1 : explanation of $x(8) > 30$ }

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9 \end{cases}.$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

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Question 1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

(a) $\int_0^6 f(t)\,dt = 142.274$ or 142.275 cubic feet

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.

1 : answer

(c) $h(0) = 0$

For $0 < t \le 6$, $h(t) = h(0) + \int_0^t g(s)\,ds = 0 + \int_0^t 0\,ds = 0$.

For $6 < t \le 7$, $h(t) = h(6) + \int_6^t g(s)\,ds = 0 + \int_6^t 125\,ds = 125(t-6)$.

For $7 < t \le 9$, $h(t) = h(7) + \int_7^t g(s)\,ds = 125 + \int_7^t 108\,ds = 125 + 108(t-7)$.

Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \le t \le 6 \\ 125(t-6) & \text{for } 6 < t \le 7 \\ 125 + 108(t-7) & \text{for } 7 < t \le 9 \end{cases}$

3 : $\begin{cases} 1 : h(t) \text{ for } 0 \le t \le 6 \\ 1 : h(t) \text{ for } 6 < t \le 7 \\ 1 : h(t) \text{ for } 7 < t \le 9 \end{cases}$

(d) Amount of snow is $\int_0^9 f(t)\,dt - h(9) = 26.334$ or 26.335 cubic feet.

3 : $\begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$

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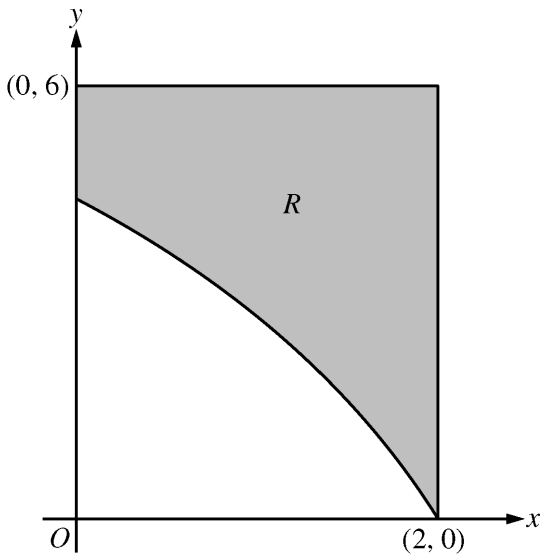
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

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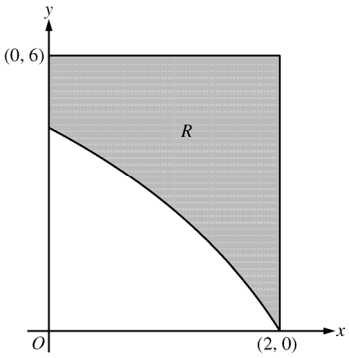
Question 1

In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.



	1 : Correct limits in an integral in (a), (b), or (c)
(a) $\int_0^2 (6 - 4\ln(3 - x))\,dx = 6.816$ or 6.817	2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $\pi\int_0^2 \left((8 - 4\ln(3 - x))^2 - (8 - 6)^2\right) dx$ $= 168.179$ or 168.180	3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\int_0^2 (6 - 4\ln(3 - x))^2\,dx = 26.266$ or 26.267	3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

   (a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

   (b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$.

   Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

   (c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

   (d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

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Question 2

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$.

Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

(a) $E'(6) \approx \dfrac{E(7) - E(5)}{7 - 5} = 4$ hundred entries per hour

1 : answer

(b) $\frac{1}{8}\int_0^8 E(t)\,dt \approx$

$\frac{1}{8}\left(2 \cdot \dfrac{E(0) + E(2)}{2} + 3 \cdot \dfrac{E(2) + E(5)}{2} + 2 \cdot \dfrac{E(5) + E(7)}{2} + 1 \cdot \dfrac{E(7) + E(8)}{2}\right)$

$= 10.687$ or 10.688

$\frac{1}{8}\int_0^8 E(t)\,dt$ is the average number of hundreds of entries in the box between noon and 8 P.M.

3 : { 1 : trapezoidal sum; 1 : approximation; 1 : meaning }

(c) $23 - \int_8^{12} P(t)\,dt = 23 - 16 = 7$ hundred entries

2 : { 1 : integral; 1 : answer }

(d) $P'(t) = 0$ when $t = 9.183503$ and $t = 10.816497$.

t	$P(t)$
8	0
9.183503	5.088662
10.816497	2.911338
12	8

Entries are being processed most quickly at time $t = 12$.

3 : { 1 : considers $P'(t) = 0$; 1 : identifies candidates; 1 : answer with justification }

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2. The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

   (a) Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.

   (b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.

   (c) Write an equation for the line tangent to the graph of g at $x = 0.3$.

   (d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

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Question 2

The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

(a) Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.

(b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 0.3$.

(d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

(a) The graph of g has a horizontal tangent line when $g'(x) = 0$.
This occurs at $x = 0.163$ and $x = 0.359$.

$2 : \begin{cases} 1 : \text{sets } g'(x) = 0 \\ 1 : \text{answer} \end{cases}$

(b) $g''(x) = 0$ at $x = 0.129458$ and $x = 0.222734$

The graph of g is concave down on $(0.1295, 0.2227)$ because $g''(x) < 0$ on this interval.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) $g'(0.3) = -0.472161$

$g(0.3) = 2 + \int_1^{0.3} g'(x)\,dx = 1.546007$

An equation for the line tangent to the graph of g is $y = 1.546 - 0.472(x - 0.3)$.

$4 : \begin{cases} 1 : g'(0.3) \\ 1 : \text{integral expression} \\ 1 : g(0.3) \\ 1 : \text{equation} \end{cases}$

(d) $g''(x) > 0$ for $0.3 < x < 1$

Therefore the line tangent to the graph of g at $x = 0.3$ lies below the graph of g for $0.3 < x < 1$.

$1 : \text{answer with reason}$

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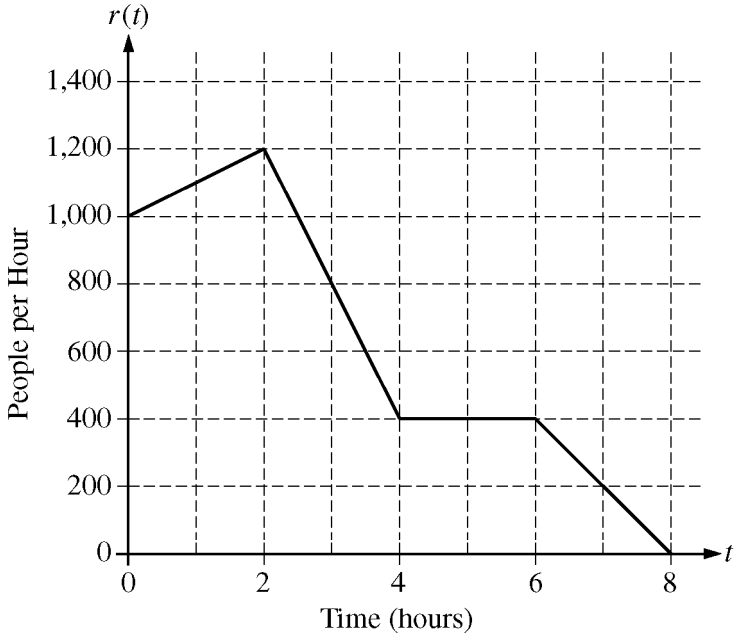

3. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

   (a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

   (b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

   (c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

   (d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

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END OF PART A OF SECTION II

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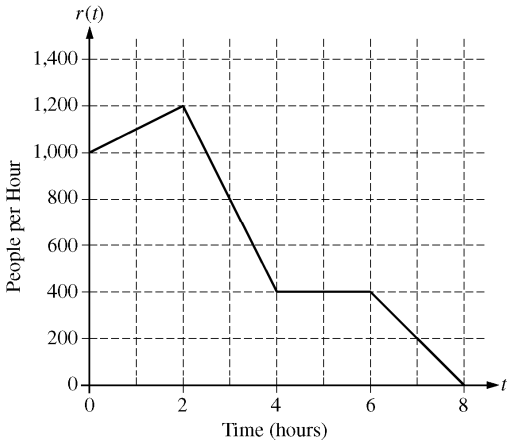
Question 3

There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.



(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

(c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

(d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

(a) $\int_0^3 r(t)\,dt = 2\cdot\frac{1000+1200}{2} + \frac{1200+800}{2} = 3200$ people

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) The number of people waiting in line is increasing because people move onto the ride at a rate of 800 people per hour and for $2 < t < 3$, $r(t) > 800$.

1 : answer with reason

(c) $r(t) = 800$ only at $t = 3$
For $0 \le t < 3$, $r(t) > 800$. For $3 < t \le 8$, $r(t) < 800$.
Therefore, the line is longest at time $t = 3$.
There are $700 + 3200 - 800\cdot 3 = 1500$ people waiting in line at time $t = 3$.

$3 : \begin{cases} 1 : \text{identifies } t = 3 \\ 1 : \text{number of people in line} \\ 1 : \text{justification} \end{cases}$

(d) $0 = 700 + \int_0^t r(s)\,ds - 800t$

$3 : \begin{cases} 1 : 800t \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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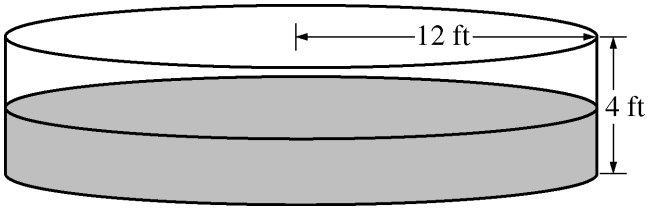
t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



3. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

   (a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

   (b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

   (c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

   (d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

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END OF PART A OF SECTION II

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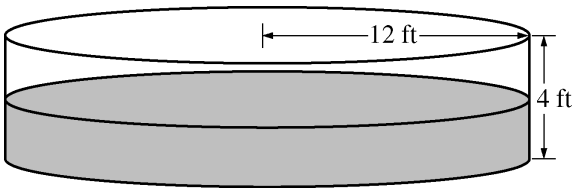
2010 SCORING GUIDELINES (Form B)

Question 3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

(a) $\displaystyle\int_0^{12} P(t)\,dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$

(b) $\displaystyle\int_0^{12} R(t)\,dt = 225.594 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $1000 + \displaystyle\int_0^{12} P(t)\,dt - \int_0^{12} R(t)\,dt = 1434.406$

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3.

1 : answer

(d) $V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241$ or $43.242 \text{ ft}^3/\text{hr}$

$V = \pi(12)^2 h$

$\dfrac{dV}{dt} = 144\pi \dfrac{dh}{dt}$

$\left.\dfrac{dh}{dt}\right|_{t=8} = \dfrac{1}{144\pi} \cdot \left.\dfrac{dV}{dt}\right|_{t=8} = 0.095$ or $0.096 \text{ ft}/\text{hr}$

$4 : \begin{cases} 1 : V'(8) \\ 1 : \text{equation relating } \dfrac{dV}{dt} \text{ and } \dfrac{dh}{dt} \\ 1 : \left.\dfrac{dh}{dt}\right|_{t=8} \\ 1 : \text{units of } \text{ft}^3/\text{hr} \text{ and } \text{ft}/\text{hr} \end{cases}$

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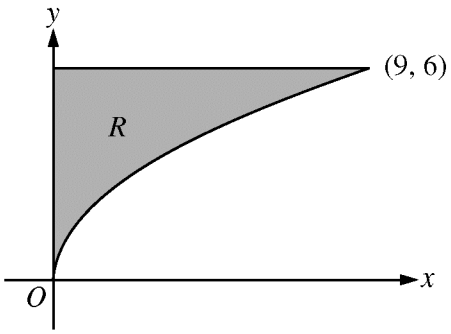
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

   (c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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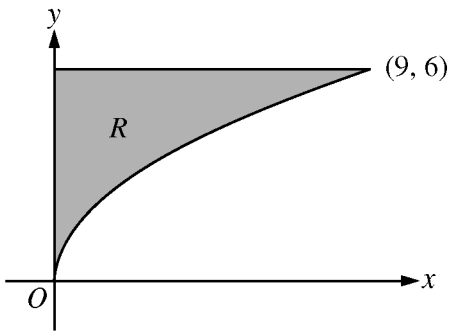
Question 4



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) Area $= \int_0^9 \left(6 - 2\sqrt{x}\right) dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big|_{x=0}^{x=9} = 18$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(b) Volume $= \pi\int_0^9 \left(\left(7 - 2\sqrt{x}\right)^2 - (7-6)^2\right) dx$

3 : { 2 : integrand; 1 : limits and constant }

(c) Solving $y = 2\sqrt{x}$ for x yields $x = \frac{y^2}{4}$.

Each rectangular cross section has area $\left(3\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$.

Volume $= \int_0^6 \frac{3}{16}y^4\, dy$

3 : { 2 : integrand; 1 : answer }

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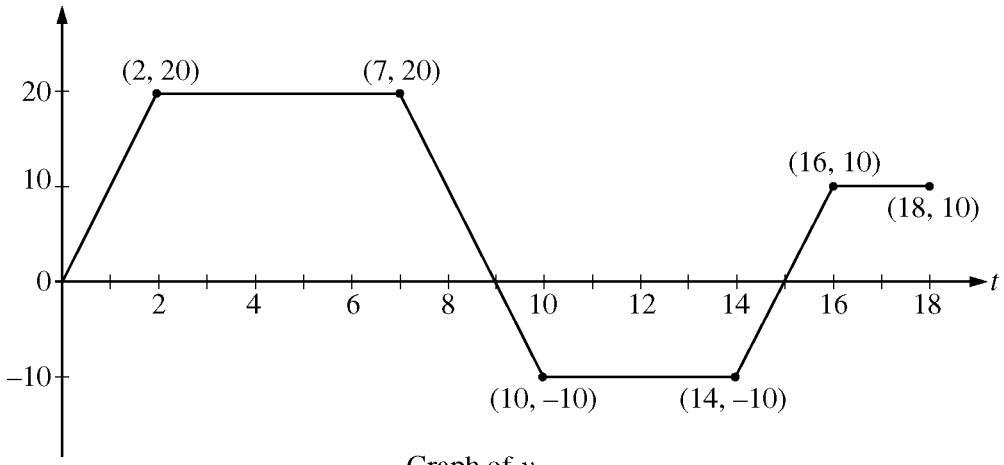
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

   (a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

   (b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A ? How far from building A is the squirrel at that time?

   (c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

   (d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

WRITE ALL WORK IN THE EXAM BOOKLET.

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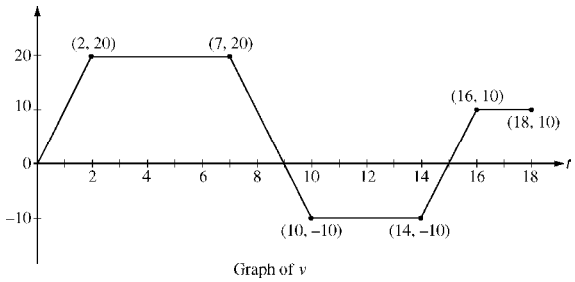
Question 4

A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.



Graph of v

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at this time?

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

(a) The squirrel changes direction whenever its velocity changes sign. This occurs at $t = 9$ and $t = 15$.

$2 : \begin{cases} 1 : t\text{-values} \\ 1 : \text{explanation} \end{cases}$

(b) Velocity is 0 at $t = 0$, $t = 9$, and $t = 15$.

t	position at time t
0	0
9	$\frac{9+5}{2} \cdot 20 = 140$
15	$140 - \frac{6+4}{2} \cdot 10 = 90$
18	$90 + \frac{3+2}{2} \cdot 10 = 115$

The squirrel is farthest from building A at time $t = 9$; its greatest distance from the building is 140.

$2 : \begin{cases} 1 : \text{identifies candidates} \\ 1 : \text{answers} \end{cases}$

(c) The total distance traveled is $\int_0^{18} |v(t)|\, dt = 140 + 50 + 25 = 215$.

1 : answer

(d) For $7 < t < 10$, $a(t) = \frac{20 - (-10)}{7 - 10} = -10$

$v(t) = 20 - 10(t - 7) = -10t + 90$

$x(7) = \frac{7+5}{2} \cdot 20 = 120$

$x(t) = x(7) + \int_7^t (-10u + 90)\, du$

$= 120 + \left(-5u^2 + 90u\right)\Big|_{u=7}^{u=t}$

$= -5t^2 + 90t - 265$

$4 : \begin{cases} 1 : a(t) \\ 1 : v(t) \\ 2 : x(t) \end{cases}$

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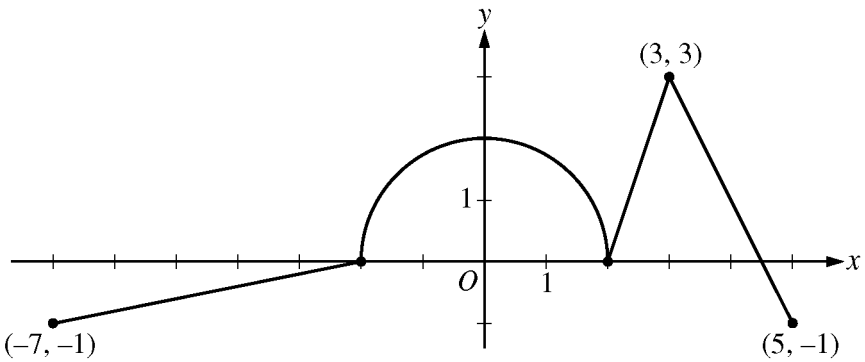

Graph of g'

5. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

   (a) Find $g(3)$ and $g(-2)$.

   (b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

   (c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

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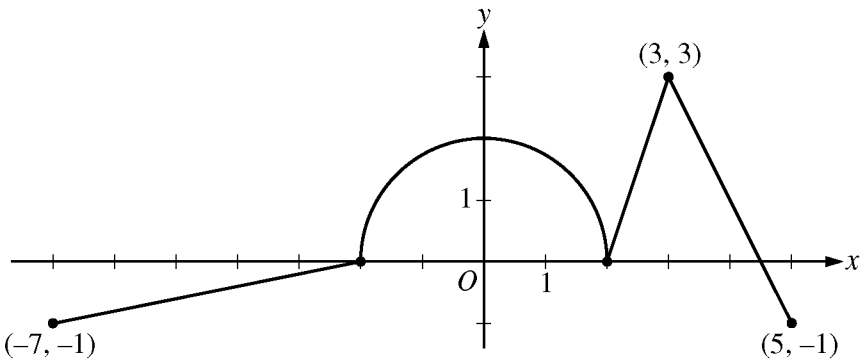
Question 5



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(a) $g(3) = 5 + \int_0^3 g'(x)\,dx = 5 + \frac{\pi \cdot 2^2}{4} + \frac{3}{2} = \frac{13}{2} + \pi$

$g(-2) = 5 + \int_0^{-2} g'(x)\,dx = 5 - \pi$

$3 : \begin{cases} 1 : \text{uses } g(0) = 5 \\ 1 : g(3) \\ 1 : g(-2) \end{cases}$

(b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.

$2 : \begin{cases} 1 : \text{identifies } x = 0, 2, 3 \\ 1 : \text{explanation} \end{cases}$

(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$

On the interval $-2 \le x \le 2$, $g'(x) = \sqrt{4 - x^2}$.

On this interval, $g'(x) = x$ when $x = \sqrt{2}$.

The only other solution to $g'(x) = x$ is $x = 3$.

$h'(x) = g'(x) - x > 0$ for $0 \le x < \sqrt{2}$

$h'(x) = g'(x) - x \le 0$ for $\sqrt{2} < x \le 5$

Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.

$4 : \begin{cases} 1 : h'(x) \\ 1 : \text{identifies } x = \sqrt{2}, 3 \\ 1 : \text{answer for } \sqrt{2} \text{ with analysis} \\ 1 : \text{answer for 3 with analysis} \end{cases}$

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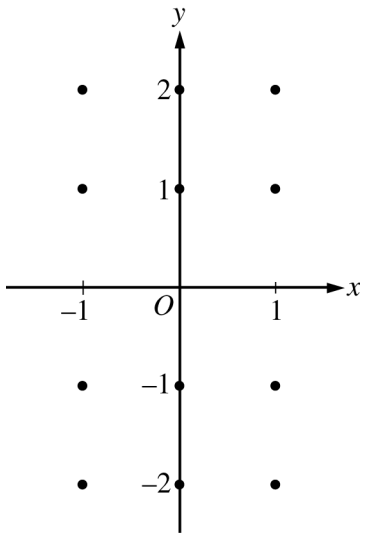
5. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the axes provided in the exam booklet.)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

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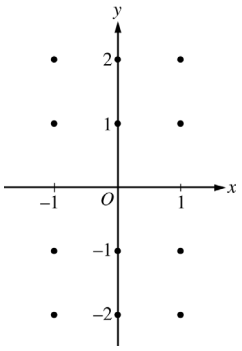
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

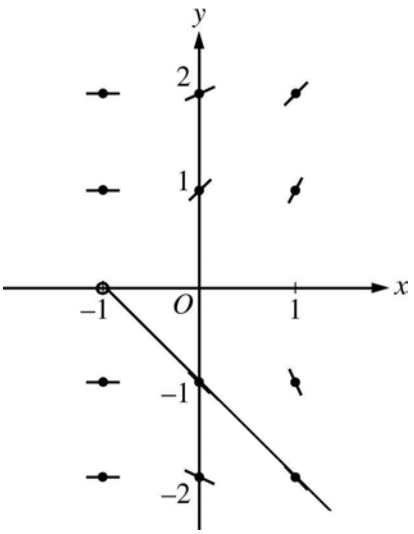
(**Note: Use the axes provided in the exam booklet.**)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the *xy*-plane for which $y \neq 0$. Describe all points in the *xy*-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

(a)



3 : { 1 : zero slopes; 1 : nonzero slopes; 1 : solution curve through $(0, -1)$ }

(b) $-1 = \frac{x+1}{y} \Rightarrow y = -x - 1$

$\frac{dy}{dx} = -1$ for all (x, y) with $y = -x - 1$ and $y \neq 0$

1 : description

(c) $\int y\, dy = \int (x+1)\, dx$

$\frac{y^2}{2} = \frac{x^2}{2} + x + C$

$\frac{(-2)^2}{2} = \frac{0^2}{2} + 0 + C \Rightarrow C = 2$

$y^2 = x^2 + 2x + 4$

Since the solution goes through $(0,-2)$, y must be negative. Therefore $y = -\sqrt{x^2 + 2x + 4}$.

5 : { 1 : separates variables; 1 : antiderivatives; 1 : constant of integration; 1 : uses initial condition; 1 : solves for y }

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

   (a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

   (b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

   (c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

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END OF EXAM

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Question 6

Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

(a) $f'(1) = \left.\frac{dy}{dx}\right|_{(1,\,2)} = 8$

An equation of the tangent line is $y = 2 + 8(x - 1)$.

$2 : \begin{cases} 1 : f'(1) \\ 1 : \text{answer} \end{cases}$

(b) $f(1.1) \approx 2.8$

Since $y = f(x) > 0$ on the interval $1 \le x < 1.1$,

$\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right) > 0$ on this interval.

Therefore on the interval $1 < x < 1.1$, the line tangent to the graph of $y = f(x)$ at $x = 1$ lies below the curve and the approximation 2.8 is less than $f(1.1)$.

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{conclusion with explanation} \end{cases}$

(c) $\frac{dy}{dx} = xy^3$

$\int \frac{1}{y^3}\,dy = \int x\,dx$

$-\frac{1}{2y^2} = \frac{x^2}{2} + C$

$-\frac{1}{2 \cdot 2^2} = \frac{1^2}{2} + C \Rightarrow C = -\frac{5}{8}$

$y^2 = \frac{1}{\frac{5}{4} - x^2}$

$f(x) = \frac{2}{\sqrt{5 - 4x^2}}, \quad \frac{-\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

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   (b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

   (c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

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Question 6

Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

(a) $f'(1) = \left.\frac{dy}{dx}\right|_{(1,\,2)} = 8$

An equation of the tangent line is $y = 2 + 8(x - 1)$.

$2 : \begin{cases} 1 : f'(1) \\ 1 : \text{answer} \end{cases}$

(b) $f(1.1) \approx 2.8$

Since $y = f(x) > 0$ on the interval $1 \le x < 1.1$,

$\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right) > 0$ on this interval.

Therefore on the interval $1 < x < 1.1$, the line tangent to the graph of $y = f(x)$ at $x = 1$ lies below the curve and the approximation 2.8 is less than $f(1.1)$.

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{conclusion with explanation} \end{cases}$

(c) $\frac{dy}{dx} = xy^3$

$\int \frac{1}{y^3}\,dy = \int x\,dx$

$-\frac{1}{2y^2} = \frac{x^2}{2} + C$

$-\frac{1}{2 \cdot 2^2} = \frac{1^2}{2} + C \Rightarrow C = -\frac{5}{8}$

$y^2 = \frac{1}{\frac{5}{4} - x^2}$

$f(x) = \frac{2}{\sqrt{5 - 4x^2}}, \quad \frac{-\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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6. Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

   (a) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

   (b) For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

   (c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

   (d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

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Question 6

Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(a) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

(b) For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

(c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

(a) $r'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$
$r'(t) = 0$ when $t = 1$ and $t = 3$
$r'(t) > 0$ for $0 < t < 1$ and $3 < t < 6$
$r'(t) < 0$ for $1 < t < 3$

Therefore R is moving to the right for $0 < t < 1$ and $3 < t < 6$.

$2 : \begin{cases} 1 : r'(t) \\ 1 : \text{answer} \end{cases}$

(b) $p'(t) = -2 \cdot \frac{\pi}{4}\sin\left(\frac{\pi}{4}t\right)$
$p'(t) = 0$ when $t = 0$ and $t = 4$

$p'(t) < 0$ for $0 < t < 4$
$p'(t) > 0$ for $4 < t < 6$

Therefore the particles travel in opposite directions for $0 < t < 1$ and $3 < t < 4$.

$3 : \begin{cases} 1 : p'(t) \\ 1 : \text{sign analysis for } p'(t) \\ 1 : \text{answer} \end{cases}$

(c) $p''(t) = -2 \cdot \frac{\pi}{4} \cdot \frac{\pi}{4}\cos\left(\frac{\pi}{4}t\right)$
$p''(3) = -2\left(\frac{\pi}{4}\right)^2 \cos\left(\frac{3\pi}{4}\right) = \frac{\pi^2}{8} \cdot \frac{\sqrt{2}}{2} > 0$
$p'(3) < 0$
Therefore particle P is slowing down at time $t = 3$.

$2 : \begin{cases} 1 : p''(3) \\ 1 : \text{answer with reason} \end{cases}$

(d) $\frac{1}{2}\int_1^3 \left| p(t) - r(t) \right| dt$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

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CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

   (a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

   (b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

   (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

   (d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

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AP® CALCULUS AB
2011 SCORING GUIDELINES

Question 1

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

(a) $v(5.5) = -0.45337$, $a(5.5) = -1.35851$

The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.

2 : conclusion with reason

(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$

2 : { 1 : integral; 1 : answer }

(c) Distance $= \int_0^6 |v(t)|\,dt = 12.573$

2 : { 1 : integral; 1 : answer }

(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$.
$v(t)$ changes sign from positive to negative at time $t = b$.
$x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135

3 : { 1 : considers $v(t) = 0$; 1 : integral; 1 : answer }

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CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

   (a) According to the model, what is the height of the water in the can at the end of the 60-day period?

   (b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

   (c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

   (d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

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AP® CALCULUS AB
2011 SCORING GUIDELINES (Form B)

Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

(c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

(a) $S(60) = \int_0^{60} S'(t)\,dt = 171.813$ mm

3 : { 1 : limits; 1 : integrand; 1 : answer }

(b) $\frac{S(60) - S(0)}{60} = 2.863$ or 2.864 mm/day

1 : answer

(c) $V(t) = 100\pi S(t)$
$V'(7) = 100\pi S'(7) = 602.218$

The volume of water in the can is increasing at a rate of 602.218 mm^3/day.

2 : { 1 : relationship between V and S; 1 : answer }

(d) $D(0) = -0.675 < 0$ and $D(60) = 69.37730 > 0$

Because D is continuous, the Intermediate Value Theorem implies that there is a time t, $0 < t < 60$, at which $D(t) = 0$. At this time, the heights of water in the two cans are changing at the same rate.

2 : { 1 : considers $D(0)$ and $D(60)$; 1 : justification }

1 : units in (b) or (c)

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t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

   (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

   (b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

   (c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

   (d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

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END OF PART A OF SECTION II

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2011 SCORING GUIDELINES

Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

(a) $H'(3.5) \approx \dfrac{H(5) - H(2)}{5 - 2}$

$= \dfrac{52 - 60}{3} = -2.666$ or -2.667 degrees Celsius per minute

1 : answer

(b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{1}{10}\left(2 \cdot \frac{66 + 60}{2} + 3 \cdot \frac{60 + 52}{2} + 4 \cdot \frac{52 + 44}{2} + 1 \cdot \frac{44 + 43}{2}\right)$

$= 52.95$

3 : { 1 : meaning of expression; 1 : trapezoidal sum; 1 : estimate }

(c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$

The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

2 : { 1 : value of integral; 1 : meaning of expression }

(d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275$; $\quad H(10) - B(10) = 8.817$

The biscuits are 8.817 degrees Celsius cooler than the tea.

3 : { 1 : integrand; 1 : uses $B(0) = 100$; 1 : answer }

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CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

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END OF PART A OF SECTION II

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AP® CALCULUS AB
2011 SCORING GUIDELINES (Form B)

Question 2

A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

(a) $\lim\limits_{t \to 5^-} r(t) = \lim\limits_{t \to 5^-} \left(\dfrac{600t}{t+3} \right) = 375 = r(5)$

$\lim\limits_{t \to 5^+} r(t) = \lim\limits_{t \to 5^+} \left(1000e^{-0.2t} \right) = 367.879$

Because the left-hand and right-hand limits are not equal, r is not continuous at $t = 5$.

2 : conclusion with analysis

(b) $\dfrac{1}{8}\displaystyle\int_0^8 r(t)\,dt = \dfrac{1}{8}\left(\int_0^5 \dfrac{600t}{t+3}\,dt + \int_5^8 1000e^{-0.2t}\,dt \right)$

$= 258.052$ or 258.053

3 : { 1 : integrand; 1 : limits and constant; 1 : answer }

(c) $r'(3) = 50$

The rate at which water is draining out of the tank at time $t = 3$ hours is increasing at 50 liters / hour^2.

2 : { 1 : $r'(3)$; 1 : meaning of $r'(3)$ }

(d) $12{,}000 - \displaystyle\int_0^A r(t)\,dt = 9000$

2 : { 1 : integral; 1 : equation }

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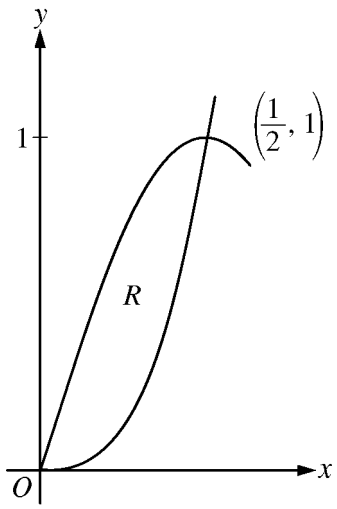
CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

   (a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

   (b) Find the area of R.

   (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

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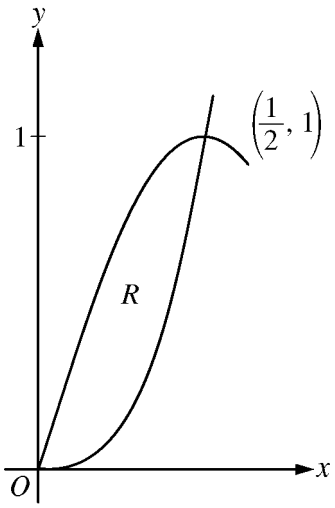
Question 3

Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



(a) $f\left(\frac{1}{2}\right) = 1$

$f'(x) = 24x^2$, so $f'\left(\frac{1}{2}\right) = 6$

An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.

2 : $\begin{cases} 1 : f'\left(\frac{1}{2}\right) \\ 1 : \text{answer} \end{cases}$

(b) $\text{Area} = \int_0^{1/2} (g(x) - f(x))\,dx$

$= \int_0^{1/2} \left(\sin(\pi x) - 8x^3\right) dx$

$= \left[-\frac{1}{\pi}\cos(\pi x) - 2x^4\right]_{x=0}^{x=1/2}$

$= -\frac{1}{8} + \frac{1}{\pi}$

4 : $\begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) $\pi\int_0^{1/2} \left((1 - f(x))^2 - (1 - g(x))^2\right) dx$

$= \pi \int_0^{1/2} \left(\left(1 - 8x^3\right)^2 - (1 - \sin(\pi x))^2\right) dx$

3 : $\begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$

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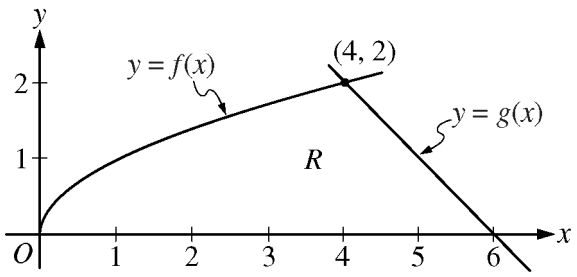
CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

   (c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

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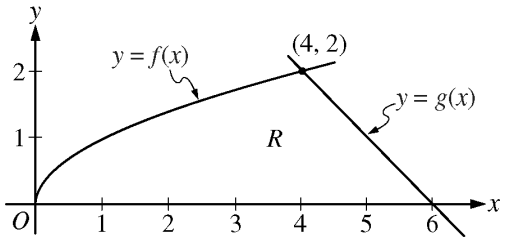
AP® CALCULUS AB
2011 SCORING GUIDELINES (Form B)

Question 3

The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

(a) Area $= \int_0^4 \sqrt{x}\,dx + \frac{1}{2} \cdot 2 \cdot 2 = \frac{2}{3}x^{3/2}\Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$

3 : { 1 : integral; 1 : antiderivative; 1 : answer }

(b) $y = \sqrt{x} \Rightarrow x = y^2$

$y = 6 - x \Rightarrow x = 6 - y$

Width $= (6 - y) - y^2$

Volume $= \int_0^2 2y\left(6 - y - y^2\right)dy$

3 : { 2 : integrand; 1 : answer }

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$

The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

3 : { 1 : $f'(x)$; 1 : equation; 1 : answer }

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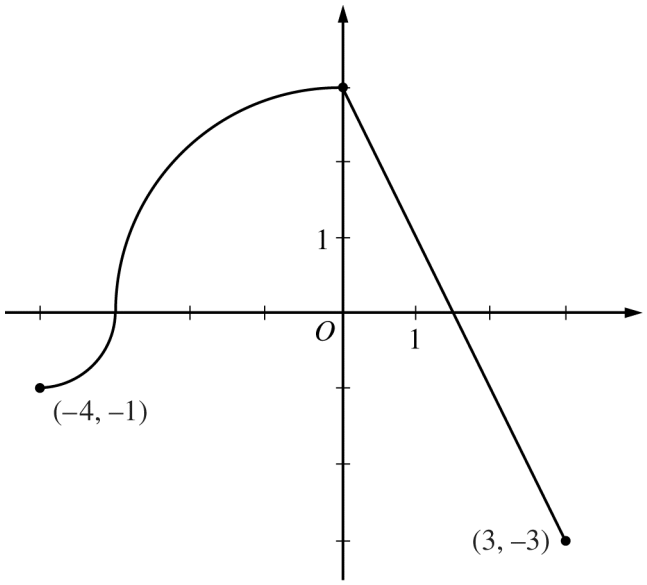

Graph of f

4. The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

   (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

   (b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

   (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

   (d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

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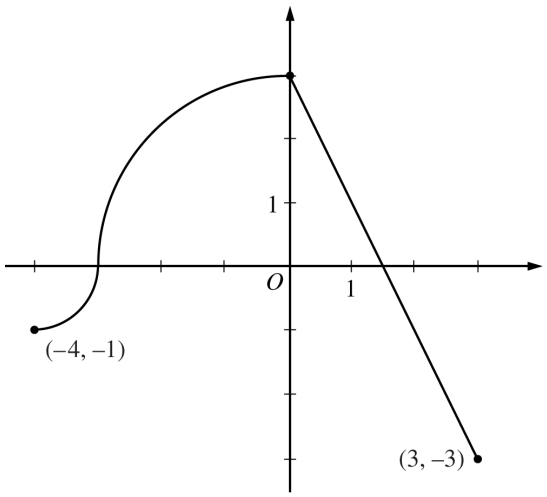
Question 4

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t)\,dt$.



Graph of f

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \frac{9\pi}{4}$

$g'(x) = 2 + f(x)$

$g'(-3) = 2 + f(-3) = 2$

3 : { 1 : $g(-3)$; 1 : $g'(x)$; 1 : $g'(-3)$ }

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.

$g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.

Therefore g has an absolute maximum at $x = \frac{5}{2}$.

3 : { 1 : considers $g'(x) = 0$; 1 : identifies interior candidate; 1 : answer with justification }

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \le x \le 3$ is $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

2 : { 1 : average rate of change; 1 : explanation }

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CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

4. Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

   (a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

   (b) Find all intervals on which the graph of f is concave down. Justify your answer.

   (c) Given that $f(1) = 2$, determine the function f.

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2011 SCORING GUIDELINES (Form B)

Question 4

Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

(a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

(b) Find all intervals on which the graph of f is concave down. Justify your answer.

(c) Given that $f(1) = 2$, determine the function f.

(a) $f'(x) = 0$ at $x = 4$
$f'(x) > 0$ for $0 < x < 4$
$f'(x) < 0$ for $x > 4$
Therefore f has a relative maximum at $x = 4$.

$3 : \begin{cases} 1 : x = 4 \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$

(b) $f''(x) = -x^{-3} + (4 - x)\left(-3x^{-4}\right)$
$= -x^{-3} - 12x^{-4} + 3x^{-3}$
$= 2x^{-4}(x - 6)$
$= \dfrac{2(x - 6)}{x^4}$
$f''(x) < 0$ for $0 < x < 6$

The graph of f is concave down on the interval $0 < x < 6$.

$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer with justification} \end{cases}$

(c) $f(x) = 2 + \int_1^x \left(4t^{-3} - t^{-2}\right) dt$
$= 2 + \left[-2t^{-2} + t^{-1}\right]_{t=1}^{t=x}$
$= 3 - 2x^{-2} + x^{-1}$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

   (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

   (b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

   (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

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Question 5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

(a) $\left.\frac{dW}{dt}\right|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$

The tangent line is $y = 1400 + 44t$.

$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411$ tons

2 : { 1 : $\frac{dW}{dt}$ at $t = 0$; 1 : answer }

(b) $\frac{d^2W}{dt^2} = \frac{1}{25}\frac{dW}{dt} = \frac{1}{625}(W - 300)$ and $W \geq 1400$

Therefore $\frac{d^2W}{dt^2} > 0$ on the interval $0 \leq t \leq \frac{1}{4}$.

The answer in part (a) is an underestimate.

2 : { 1 : $\frac{d^2W}{dt^2}$; 1 : answer with reason }

(c) $\frac{dW}{dt} = \frac{1}{25}(W - 300)$

$\int \frac{1}{W - 300}\, dW = \int \frac{1}{25}\, dt$

$\ln|W - 300| = \frac{1}{25}t + C$

$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$

$W - 300 = 1100e^{\frac{1}{25}t}$

$W(t) = 300 + 1100e^{\frac{1}{25}t}, \ 0 \leq t \leq 20$

5 : { 1 : separation of variables ; 1 : antiderivatives ; 1 : constant of integration ; 1 : uses initial condition ; 1 : solves for W }

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

5. Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

   (a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

   (b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\,dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\,dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

   (c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

   (d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

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2011 SCORING GUIDELINES (Form B)

Question 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

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(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

(a) $a(5) \approx \dfrac{v(10) - v(0)}{10 - 0} = \dfrac{0.3}{10} = 0.03$ meters/sec^2

1 : answer

(b) $\int_0^{60} |v(t)|\,dt$ is the total distance, in meters, that Ben rides over the 60-second interval $t = 0$ to $t = 60$.

$\int_0^{60} |v(t)|\,dt \approx 2.0 \cdot 10 + 2.3(40 - 10) + 2.5(60 - 40) = 139$ meters

2 : { 1 : meaning of integral; 1 : approximation }

(c) Because $\dfrac{B(60) - B(40)}{60 - 40} = \dfrac{49 - 9}{20} = 2$, the Mean Value Theorem implies there is a time t, $40 < t < 60$, such that $v(t) = 2$.

2 : { 1 : difference quotient; 1 : conclusion with justification }

(d) $2L(t)L'(t) = 2B(t)B'(t)$

$L'(40) = \dfrac{B(40)v(40)}{L(40)} = \dfrac{9 \cdot 2.5}{\sqrt{144 + 81}} = \dfrac{3}{2}$ meters/sec

3 : { 1 : derivatives; 1 : uses $B'(t) = v(t)$; 1 : answer }

1 : units in (a) or (b)

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6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

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END OF EXAM

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2011 SCORING GUIDELINES

Question 6

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \ne 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

(a) $\lim\limits_{x \to 0^-} (1 - 2\sin x) = 1$

$\lim\limits_{x \to 0^+} e^{-4x} = 1$

$f(0) = 1$

So, $\lim\limits_{x \to 0} f(x) = f(0)$.

Therefore f is continuous at $x = 0$.

2 : analysis

(b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$

$-2\cos x \ne -3$ for all values of $x < 0$.

$-4e^{-4x} = -3$ when $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0$.

Therefore $f'(x) = -3$ for $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$.

3 : $\begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$

(c) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

$= \int_{-1}^{0} (1 - 2\sin x)\,dx + \int_{0}^{1} e^{-4x}\,dx$

$= \left[x + 2\cos x\right]_{x=-1}^{x=0} + \left[-\frac{1}{4}e^{-4x}\right]_{x=0}^{x=1}$

$= (3 - 2\cos(-1)) + \left(-\frac{1}{4}e^{-4} + \frac{1}{4}\right)$

Average value $= \frac{1}{2}\int_{-1}^{1} f(x)\,dx$

$= \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4}$

4 : $\begin{cases} 1 : \int_{-1}^{0} (1 - 2\sin x)\,dx \text{ and } \int_{0}^{1} e^{-4x}\,dx \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

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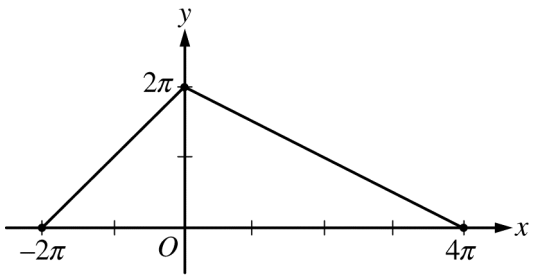

Graph of g

6. Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

   (a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

   (b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

   (c) Let $h(x) = \int_{0}^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

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END OF EXAM

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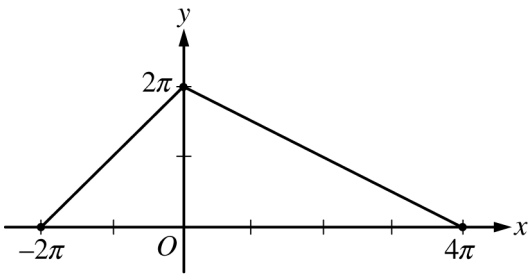
2011 SCORING GUIDELINES (Form B)

Question 6

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.



Graph of g

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

(b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

(c) Let $h(x) = \int_{0}^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

(a) $\int_{-2\pi}^{4\pi} f(x)\,dx = \int_{-2\pi}^{4\pi} \left(g(x) - \cos\left(\frac{x}{2}\right)\right) dx$

$= 6\pi^2 - \left[2\sin\left(\frac{x}{2}\right)\right]_{x=-2\pi}^{x=4\pi}$

$= 6\pi^2$

$2 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right) = \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi < x < 0 \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0 < x < 4\pi \end{cases}$

$f'(x)$ does not exist at $x = 0$.
For $-2\pi < x < 0$, $f'(x) \neq 0$.
For $0 < x < 4\pi$, $f'(x) = 0$ when $x = \pi$.

f has critical points at $x = 0$ and $x = \pi$.

$4 : \begin{cases} 1 : \frac{d}{dx}\left(\cos\left(\frac{x}{2}\right)\right) \\ 1 : g'(x) \\ 1 : x = 0 \\ 1 : x = \pi \end{cases}$

(c) $h'(x) = g(3x) \cdot 3$

$h'\left(-\frac{\pi}{3}\right) = 3g(-\pi) = 3\pi$

$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

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CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

   (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

   (b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

   (c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

   (d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

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Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is $55°$F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a) $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{67.9 - 61.8}{6}$
$= 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately $1.017\,°$F per minute at time $t = 12$ minutes.

2 : { 1 : estimate; 1 : interpretation with units }

(b) $\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by $16\,°$F over the interval from $t = 0$ to $t = 20$ minutes.

2 : { 1 : value; 1 : interpretation with units }

(c) $\frac{1}{20}\int_0^{20} W(t)\,dt \approx \frac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$
$= \frac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$
$= \frac{1}{20} \cdot 1215.8 = 60.79$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

3 : { 1 : left Riemann sum; 1 : approximation; 1 : underestimate with reason }

(d) $W(25) = 71.0 + \int_{20}^{25} W'(t)\,dt$
$= 71.0 + 2.043155 = 73.043$

2 : { 1 : integral; 1 : answer }

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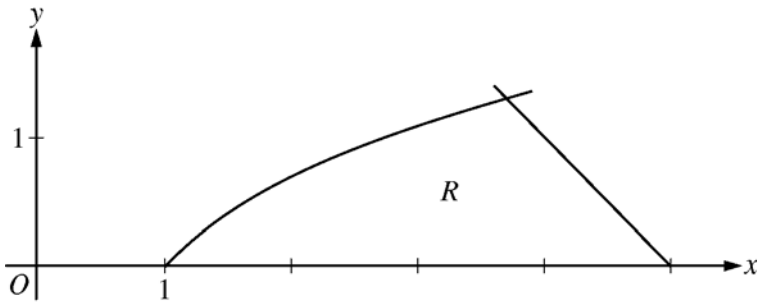

2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

   (a) Find the area of R.

   (b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

   (c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

END OF PART A OF SECTION II

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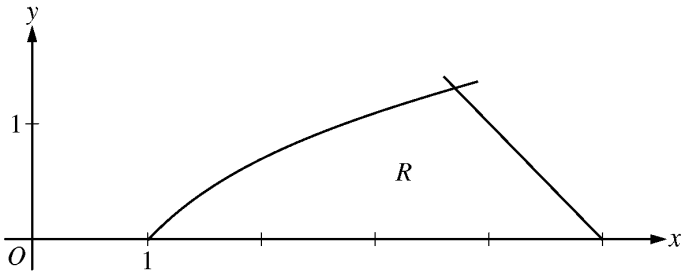
Question 2

Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$\ln x = 5 - x \implies x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) $\text{Area} = \int_0^B \left(5 - y - e^y\right) dy$
$= 2.986 \text{ (or } 2.985)$

OR

$\text{Area} = \int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx$
$= 2.986 \text{ (or } 2.985)$

3 : { 1 : integrand; 1 : limits; 1 : answer }

(b) $\text{Volume} = \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$

3 : { 2 : integrands; 1 : expression for total volume }

(c) $\int_0^k \left(5 - y - e^y\right) dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$

3 : { 1 : integrand; 1 : limits; 1 : equation }

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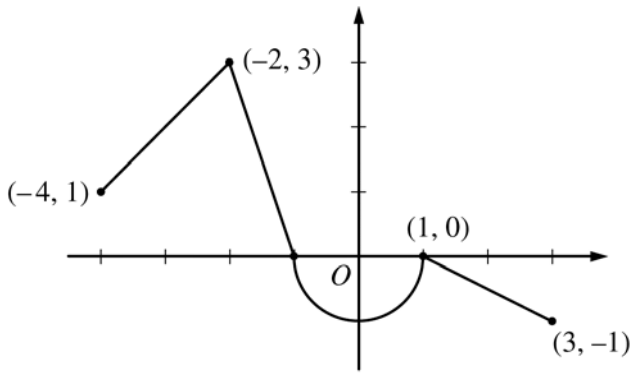
CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

   (a) Find the values of $g(2)$ and $g(-2)$.

   (b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

   (c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

   (d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

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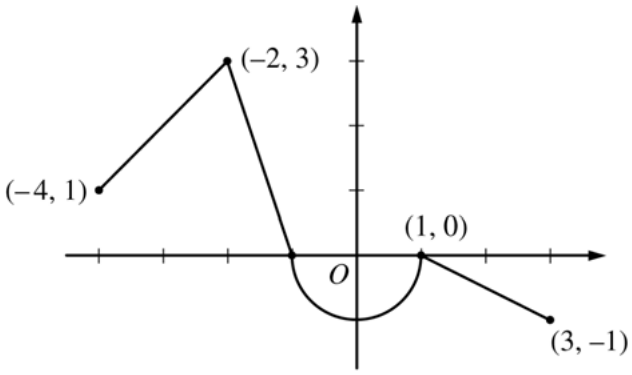
Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.



Graph of f

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^1 f(t)\,dt$

$= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$

$2 : \begin{cases} 1 : g(2) \\ 1 : g(-2) \end{cases}$

(b) $g'(x) = f(x) \Rightarrow g'(-3) = f(-3) = 2$

$g''(x) = f'(x) \Rightarrow g''(-3) = f'(-3) = 1$

$2 : \begin{cases} 1 : g'(-3) \\ 1 : g''(-3) \end{cases}$

(c) The graph of g has a horizontal tangent line where $g'(x) = f(x) = 0$. This occurs at $x = -1$ and $x = 1$.

$g'(x)$ changes sign from positive to negative at $x = -1$. Therefore, g has a relative maximum at $x = -1$.

$g'(x)$ does not change sign at $x = 1$. Therefore, g has neither a relative maximum nor a relative minimum at $x = 1$.

$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : x = -1 \text{ and } x = 1 \\ 1 : \text{answers with justifications} \end{cases}$

(d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ because $g''(x) = f'(x)$ changes sign at each of these values.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$

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4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

   (a) Find $f'(x)$.

   (b) Write an equation for the line tangent to the graph of f at $x = -3$.

   (c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

   Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

   (d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

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Question 4

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

(a) $f'(x) = \frac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$

$2 : f'(x)$

(b) $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$

$f(-3) = \sqrt{25 - 9} = 4$

An equation for the tangent line is $y = 4 + \frac{3}{4}(x + 3)$.

$2 : \begin{cases} 1 : f'(-3) \\ 1 : \text{answer} \end{cases}$

(c) $\lim_{x \to -3^-} g(x) = \lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \sqrt{25 - x^2} = 4$

$\lim_{x \to -3^+} g(x) = \lim_{x \to -3^+} (x + 7) = 4$

Therefore, $\lim_{x \to -3} g(x) = 4$.

$g(-3) = f(-3) = 4$

So, $\lim_{x \to -3} g(x) = g(-3)$.

Therefore, g is continuous at $x = -3$.

$2 : \begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with explanation} \end{cases}$

(d) Let $u = 25 - x^2 \Rightarrow du = -2x\, dx$

$$\int_0^5 x\sqrt{25 - x^2}\, dx = -\frac{1}{2}\int_{25}^0 \sqrt{u}\, du$$

$$= \left[-\frac{1}{2} \cdot \frac{2}{3}u^{3/2}\right]_{u=25}^{u=0}$$

$$= -\frac{1}{3}(0 - 125) = \frac{125}{3}$$

$3 : \begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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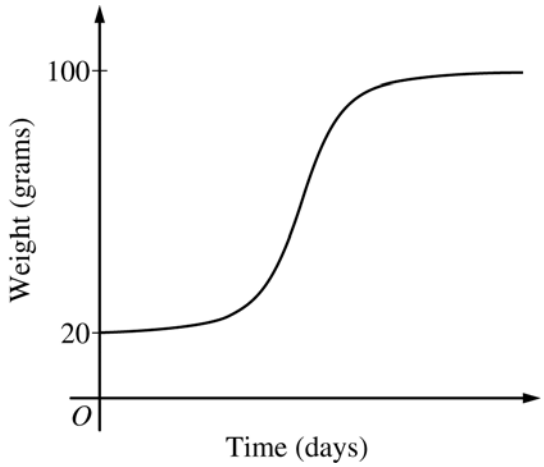
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

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Question 5

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

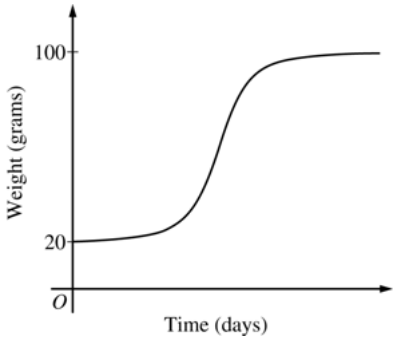
$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.



(a) $\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(60) = 12$

$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(30) = 6$

Because $\left.\frac{dB}{dt}\right|_{B=40} > \left.\frac{dB}{dt}\right|_{B=70}$, the bird is gaining weight faster when it weighs 40 grams.

2 : { 1 : uses $\frac{dB}{dt}$; 1 : answer with reason }

(b) $\frac{d^2B}{dt^2} = -\frac{1}{5}\frac{dB}{dt} = -\frac{1}{5}\cdot\frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$

Therefore, the graph of B is concave down for $20 \le B < 100$. A portion of the given graph is concave up.

2 : { 1 : $\frac{d^2B}{dt^2}$ in terms of B ; 1 : explanation }

(c) $\frac{dB}{dt} = \frac{1}{5}(100 - B)$

$\int \frac{1}{100 - B}\, dB = \int \frac{1}{5}\, dt$

$-\ln|100 - B| = \frac{1}{5}t + C$

Because $20 \le B < 100$, $|100 - B| = 100 - B$.

$-\ln(100 - 20) = \frac{1}{5}(0) + C \;\Rightarrow\; -\ln(80) = C$

$100 - B = 80e^{-t/5}$

$B(t) = 100 - 80e^{-t/5}, \quad t \ge 0$

5 : { 1 : separation of variables ; 1 : antiderivatives ; 1 : constant of integration ; 1 : uses initial condition ; 1 : solves for B }

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

   (a) For $0 \le t \le 12$, when is the particle moving to the left?

   (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

   (c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

   (d) Find the position of the particle at time $t = 4$.

STOP

END OF EXAM

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Question 6

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow t = 3,\ 9$

The particle is moving to the left when $v(t) < 0$.
This occurs when $3 < t < 9$.

2 : { 1 : considers $v(t) = 0$; 1 : interval }

(b) $\int_0^6 |v(t)|\,dt$

1 : answer

(c) $a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$

$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$

The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.

3 : { 1 : $a(t)$; 2 : conclusion with reason }

(d) $x(4) = -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right)dt$

$= -2 + \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right]_0^4$

$= -2 + \frac{6}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - 0\right]$

$= -2 + \frac{6}{\pi}\cdot\frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi}$

3 : { 1 : antiderivative ; 1 : uses initial condition ; 1 : answer }

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CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

   (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

   (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

   (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

   (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

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Question 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

(a) $G'(5) = -24.588$ (or -24.587)

The rate at which gravel is arriving is decreasing by 24.588 (or 24.587) tons per hour per hour at time $t = 5$ hours.

$2 : \begin{cases} 1 : G'(5) \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\int_0^8 G(t)\,dt = 825.551$ tons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $G(5) = 98.140764 < 100$

At time $t = 5$, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed. Therefore, the amount of unprocessed gravel at the plant is decreasing at time $t = 5$.

$2 : \begin{cases} 1 : \text{compares } G(5) \text{ to } 100 \\ 1 : \text{conclusion} \end{cases}$

(d) The amount of unprocessed gravel at time t is given by $A(t) = 500 + \int_0^t (G(s) - 100)\,ds$.

$A'(t) = G(t) - 100 = 0 \Rightarrow t = 4.923480$

t	$A(t)$
0	500
4.92348	635.376123
8	525.551089

The maximum amount of unprocessed gravel at the plant during this workday is 635.376 tons.

$3 : \begin{cases} 1 : \text{considers } A'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

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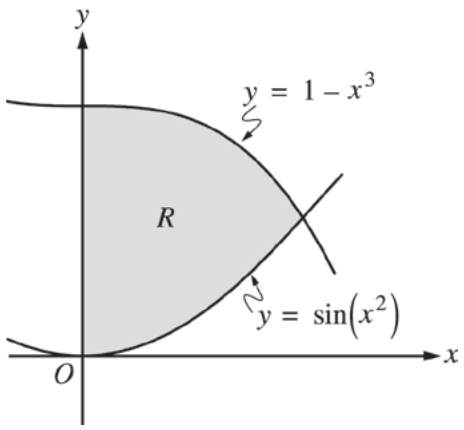
A graphing calculator is required for these problems.



1. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$, as shown in the figure above.

   (a) Find the area of R.

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Do not write beyond this border.

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Continue problem 1 on page 5.

A graphing calculator is required for these problems.

(b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

(c) Find the volume of the solid generated when R is revolved about the line $y = -3$.

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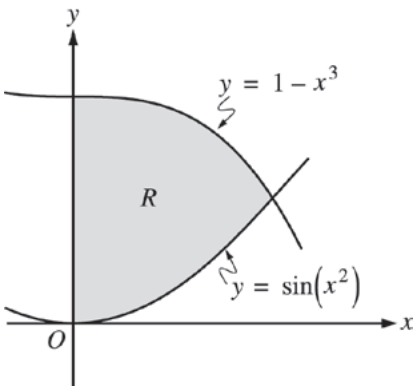
Question 1

Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$, as shown in the figure above.



(a) Find the area of R.

(b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

(c) Find the volume of the solid generated when R is revolved about the line $y = -3$.

The graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect in the first quadrant at the point $(A, B) = (0.764972, 0.552352)$.

1 : correct limits in an integral in (a), (b), or (c)

(a) $\text{Area} = \int_0^A \left(1 - x^3 - \sin\left(x^2\right)\right) dx$
$= 0.533$ (or 0.534)

2 : { 1 : integrand; 1 : answer }

(b) $k = B = 0.552352$

$\int_0^A \left(1 - x^3 - k\right) dx = 0.257$ (or 0.256)

$\int_0^A \left(k - \sin\left(x^2\right)\right) dx = 0.277$ (or 0.276)

The two regions have unequal areas.

3 : { 1 : integral(s) with k value; 1 : value(s) of integral(s); 1 : conclusion tied to part (a) or comparison of two integrals }

Note: Stating k value only does not earn a point.

(c) $\text{Volume} = \pi \int_0^A \left(\left(1 - x^3 + 3\right)^2 - \left(\sin\left(x^2\right) + 3\right)^2\right) dx$
$= 11.841$ (or 11.840)

3 : { 2 : integrand; 1 : answer }

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2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

   (a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

   (b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

   (c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

   (d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

END OF PART A OF SECTION II

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Question 2

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) Solve $|v(t)| = 2$ on $2 \le t \le 4$.
$t = 3.128$ (or 3.127) and $t = 3.473$

$2 : \begin{cases} 1 : \text{considers } |v(t)| = 2 \\ 1 : \text{answer} \end{cases}$

(b) $s(t) = 10 + \int_0^t v(x)\,dx$

$s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$

$2 : \begin{cases} 1 : s(t) \\ 1 : s(5) \end{cases}$

(c) $v(t) = 0$ when $t = 0.536033,\ 3.317756$
$v(t)$ changes sign from negative to positive at time $t = 0.536033$.
$v(t)$ changes sign from positive to negative at time $t = 3.317756$.

Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).

$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$

(d) $v(4) = -11.475758 < 0,\ a(4) = v'(4) = -22.295714 < 0$

The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.

2 : conclusion with reason

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A graphing calculator is required for these problems.

2. The penguin population on an island is modeled by a differentiable function P of time t, where $P(t)$ is the number of penguins and t is measured in years, for $0 \le t \le 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

$$B(t) = 1000e^{0.06t} \text{ penguins per year}$$

and the death rate for the penguins on the island is modeled by

$$D(t) = 250e^{0.1t} \text{ penguins per year.}$$

(a) What is the rate of change of the penguin population on the island at time $t = 0$?

(b) To the nearest whole number, what is the penguin population on the island at time $t = 40$?

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Continue problem 2 on page 7.

A graphing calculator is required for these problems.

(c) To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \le t \le 40$?

(d) To the nearest whole number, find the absolute minimum penguin population and the absolute maximum penguin population on the island for $0 \le t \le 40$. Show the analysis that leads to your answers.

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Question 2

The penguin population on an island is modeled by a differentiable function P of time t, where $P(t)$ is the number of penguins and t is measured in years, for $0 \le t \le 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

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(b) To the nearest whole number, what is the penguin population on the island at time $t = 40$?

(c) To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \le t \le 40$?

(d) To the nearest whole number, find the absolute minimum penguin population and the absolute maximum penguin population on the island for $0 \le t \le 40$. Show the analysis that leads to your answers.

(a) $P'(0) = B(0) - D(0) = 1000 - 250 = 750$ penguins per year

1 : answer

(b) $P(40) = 100000 + \int_0^{40} (B(t) - D(t))\, dt$

$= 100000 + 33057.56459$

There are 133,058 penguins on the island.

3 : { 1 : limits; 1 : integrand; 1 : answer }

(c) $\frac{1}{40}\int_0^{40} (B(t) - D(t))\, dt = 826.439$

OR

$$\frac{P(40) - P(0)}{40 - 0} = \frac{133058 - 100000}{40} = 826.45$$

The average rate of change is 826 penguins per year.

1 : answer

(d) $B(t) - D(t) = 0$

$1000e^{0.06t} = 250e^{0.1t} \Rightarrow t = A = \frac{\ln 4}{0.04} = 34.657359$

The absolute minimum and absolute maximum occur at a critical point or at an endpoint.

$P(0) = 100000$

$P(A) = 100000 + \int_0^{A} (B(t) - D(t))\, dt = 139166.667$

$P(40) = 133058$

The minimum population is 100,000 and the maximum population is 139,167 penguins.

4 : { 1 : $B(t) - D(t) = 0$; 1 : solves for t; 1 : minimum value; 1 : maximum value }

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CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

   (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

   (b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

   (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

   (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

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Question 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

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(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

(a) $C'(3.5) \approx \dfrac{C(4) - C(3)}{4 - 3} = \dfrac{12.8 - 11.2}{1} = 1.6$ ounces/min

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$

(b) C is differentiable $\Rightarrow$ C is continuous (on the closed interval)

$\dfrac{C(4) - C(2)}{4 - 2} = \dfrac{12.8 - 8.8}{2} = 2$

Therefore, by the Mean Value Theorem, there is at least one time t, $2 < t < 4$, for which $C'(t) = 2$.

$2 : \begin{cases} 1 : \dfrac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c) $\dfrac{1}{6}\int_0^6 C(t)\,dt \approx \dfrac{1}{6}[2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)]$

$= \dfrac{1}{6}(2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8)$

$= \dfrac{1}{6}(60.6) = 10.1$ ounces

$\dfrac{1}{6}\int_0^6 C(t)\,dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \le t \le 6$ minutes.

$3 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$

(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$

$B'(5) = 6.4e^{-0.4(5)} = \dfrac{6.4}{e^2}$ ounces/min

$2 : \begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$

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t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

3. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

   (a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

   (b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

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Continue problem 3 on page 15.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

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Question 3

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

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(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

(a) An equation of the tangent line is $y = 0.5(t - 30) + 125$.
$W(32) \approx 0.5(32 - 30) + 125 = 126$

1 : answer

(b) $\int_0^{30} W'(t)\,dt \approx (10)(0.6) + (12)(0.7) + (8)(1.0) = 22.4$

$W(0) = W(30) - \int_0^{30} W'(t)\,dt = 125 - 22.4 = 102.6$

3 : 1 : left Riemann sum; 1 : approximation; 1 : answer

(c) W' is differentiable $\Rightarrow$ W' is continuous.

$W'(30) = 0.5 < 0.7 < 1.0 = W'(22)$

By the Intermediate Value Theorem, there must be at least one time t, $22 \le t \le 30$, such that $W'(t) = 0.7$.

2 : explanation

(d) $\dfrac{dA}{dt} = (0.3)\dfrac{2}{3}W^{-1/3} \cdot \dfrac{dW}{dt} = \dfrac{0.2}{\sqrt[3]{W}} \cdot \dfrac{dW}{dt}$

$\left.\dfrac{dA}{dt}\right|_{t=30} = \dfrac{0.2}{\sqrt[3]{125}} \cdot 0.5 = 0.02$

3 : 2 : $\dfrac{dA}{dt}$; 1 : answer

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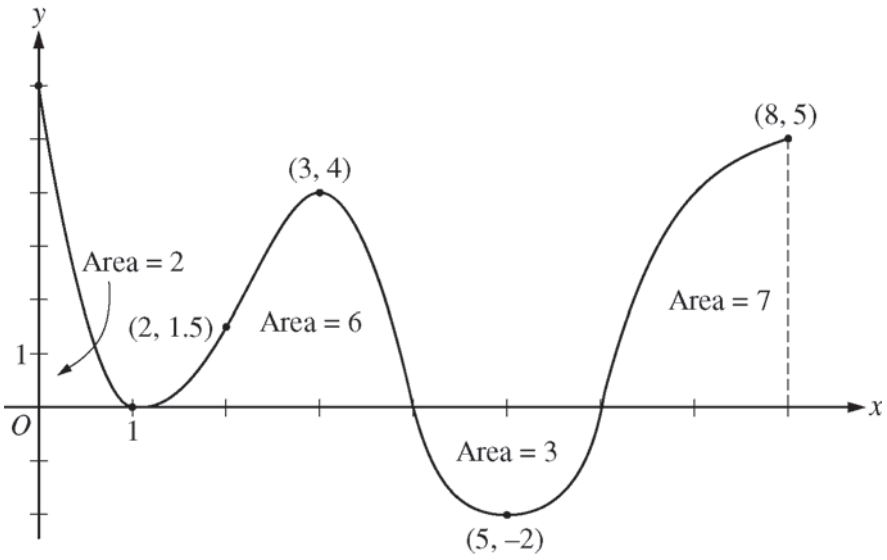

Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

   (a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

   (b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

   (c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

   (d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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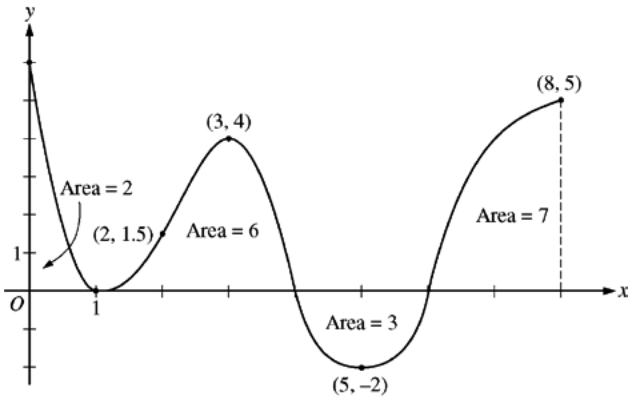
Question 4

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



Graph of f'

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

(a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.

1 : answer with justification

(b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint.

$$f(0) = f(8) + \int_8^0 f'(x)\,dx$$
$$= f(8) - \int_0^8 f'(x)\,dx = 4 - 12 = -8$$
$$f(6) = f(8) + \int_8^6 f'(x)\,dx$$
$$= f(8) - \int_6^8 f'(x)\,dx = 4 - 7 = -3$$
$$f(8) = 4$$

The absolute minimum value of f on the closed interval $[0, 8]$ is -8.

3 : { 1 : considers $x = 0$ and $x = 6$; 1 : answer; 1 : justification }

(c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.

2 : { 1 : answer; 1 : explanation }

(d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$

$g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$

3 : { 2 : $g'(x)$; 1 : answer }

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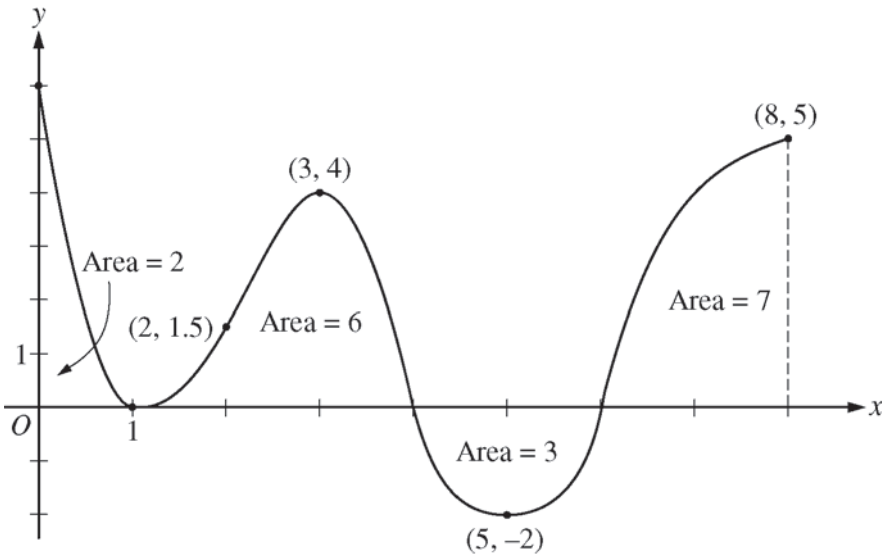

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   (a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

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Question 4

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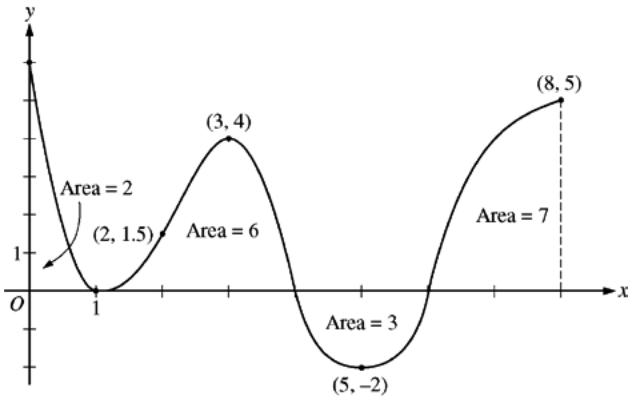

Graph of f'

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

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$$= f(8) - \int_0^8 f'(x)\,dx = 4 - 12 = -8$$
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$$= f(8) - \int_6^8 f'(x)\,dx = 4 - 7 = -3$$
$$f(8) = 4$$

The absolute minimum value of f on the closed interval $[0, 8]$ is -8.

3 : { 1 : considers $x = 0$ and $x = 6$; 1 : answer; 1 : justification }

(c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.

2 : { 1 : answer; 1 : explanation }

(d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$

$g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$

3 : { 2 : $g'(x)$; 1 : answer }

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4. Let f be the function given by $f(x) = \left(x^2 - 2x - 1\right)e^x$.

(a) Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

$\lim_{x \to \infty} f(x) =$ _____

$\lim_{x \to -\infty} f(x) =$ _____

(b) Find the intervals on which f is increasing. Show the analysis that leads to your answer.

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Continue problem 4 on page 17.

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(c) Find the intervals on which the graph of f is concave down. Show the analysis that leads to your answer.

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Question 4

Let f be the function given by $f(x) = \left(x^2 - 2x - 1\right)e^x$.

(a) Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

(b) Find the intervals on which f is increasing. Show the analysis that leads to your answer.

(c) Find the intervals on which the graph of f is concave down. Show the analysis that leads to your answer.

(a) $\lim_{x \to \infty} f(x) = \infty$ or does not exist

$\lim_{x \to -\infty} f(x) = 0$

1 : answers

(b) $f'(x) = (2x - 2)e^x + \left(x^2 - 2x - 1\right)e^x$
$= \left(x^2 - 3\right)e^x$

$f'(x) = 0$ when $x = -\sqrt{3}, x = \sqrt{3}$

$f'(x) > 0$ for $-\infty < x < -\sqrt{3}$ and $\sqrt{3} < x < \infty$.

f is increasing on the intervals $-\infty < x \le -\sqrt{3}$ and $\sqrt{3} \le x < \infty$.

4 : { 2 : $f'(x)$; 1 : analysis; 1 : intervals }

(c) $f''(x) = 2xe^x + \left(x^2 - 3\right)e^x$
$= \left(x^2 + 2x - 3\right)e^x = (x + 3)(x - 1)e^x$

$f''(x) < 0$ for $-3 < x < 1$

The graph of f is concave down on the interval $-3 < x < 1$.

4 : { 2 : $f''(x)$; 1 : analysis; 1 : interval }

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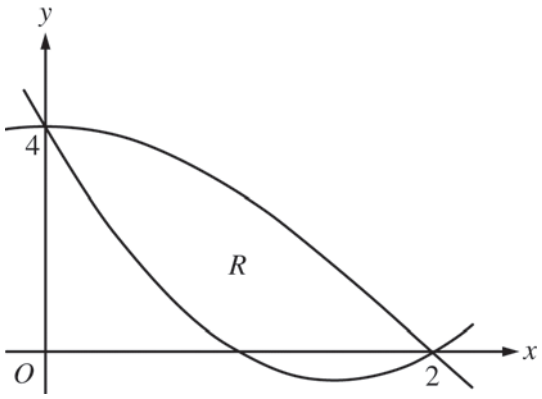

5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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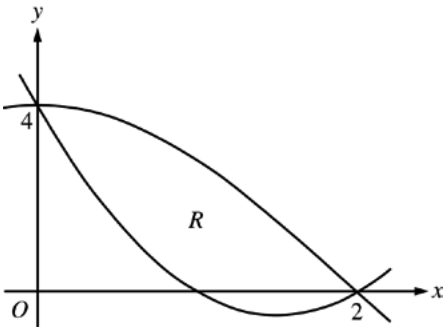
Question 5

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.



(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) $\text{Area} = \int_0^2 [g(x) - f(x)]\,dx$

$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right] dx$

$= \left[4 \cdot \frac{4}{\pi}\sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right)\right]_0^2$

$= \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3}$

4 : { 1 : integrand; 2 : antiderivative; 1 : answer }

(b) $\text{Volume} = \pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$

$= \pi\int_0^2 \left[\left(4 - \left(2x^2 - 6x + 4\right)\right)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$

3 : { 2 : integrand; 1 : limits and constant }

(c) $\text{Volume} = \int_0^2 [g(x) - f(x)]^2\,dx$

$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right]^2 dx$

2 : { 1 : integrand; 1 : limits and constant }

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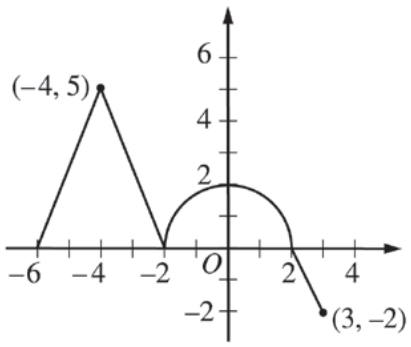

Graph of f

5. The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(-6)$ and $g(3)$.

(b) Find $g'(0)$.

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Continue problem 5 on page 19.

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(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

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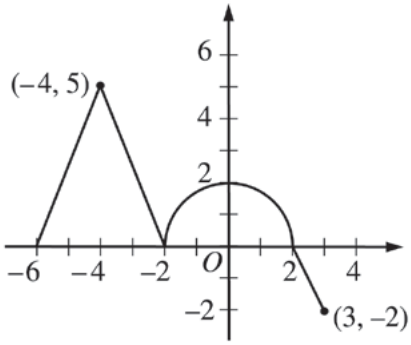
Question 5

The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(-6)$ and $g(3)$.

(b) Find $g'(0)$.

(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(-6) = \int_{-2}^{-6} f(t)\,dt = -\int_{-6}^{-2} f(t)\,dt = -\frac{1}{2}\cdot 4\cdot 5 = -10$

$g(3) = \int_{-2}^{3} f(t)\,dt = \frac{1}{2}\pi\cdot 2^2 - \frac{1}{2}\cdot 1\cdot 2 = 2\pi - 1$

$2 : \begin{cases} 1 : g(-6) \\ 1 : g(3) \end{cases}$

(b) $g'(0) = f(0) = 2$

$1 : g'(0)$

(c) The graph of g has a horizontal tangent at $x = -2$ and $x = 2$ where $g'(x) = f(x) = 0$.

The graph of g has neither a local maximum nor a local minimum at $x = -2$ because $g'(x) = f(x)$ does not change sign at $x = -2$.

The graph of g has a local maximum at $x = 2$ because $g'(x) = f(x)$ changes sign from positive to negative at $x = 2$.

$3 : \begin{cases} 1 : \text{horizontal tangent at } x = -2 \text{ and } x = 2 \\ 2 : \text{answers with justifications} \end{cases}$

(d) The graph of g has a point of inflection at $x = -4$, $x = -2$, and $x = 0$.
$g'(x) = f(x)$ changes from increasing to decreasing at $x = -4$ and $x = 0$, and changes from decreasing to increasing at $x = -2$.

OR

$g''(x) = f'(x)$ changes from positive to negative at $x = -4$ and $x = 0$, and changes from negative to positive at $x = -2$.

$3 : \begin{cases} 2 : \text{values of } x \\ 1 : \text{explanation} \end{cases}$

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6. Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

   (a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

   (b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

STOP

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Question 6

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

(a) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(1,\,0)} = e^0\left(3\cdot 1^2 - 6\cdot 1\right) = -3$

An equation for the tangent line is $y = -3(x-1)$.

$f(1.2) \approx -3(1.2-1) = -0.6$

$3 : \begin{cases} 1 : \frac{dy}{dx} \text{ at the point } (x,\,y) = (1,\,0) \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(b) $\frac{dy}{e^y} = \left(3x^2 - 6x\right)dx$

$\int \frac{dy}{e^y} = \int \left(3x^2 - 6x\right)dx$

$-e^{-y} = x^3 - 3x^2 + C$

$-e^{-0} = 1^3 - 3\cdot 1^2 + C \Rightarrow C = 1$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln\left(-x^3 + 3x^2 - 1\right)$

$y = -\ln\left(-x^3 + 3x^2 - 1\right)$

Note: This solution is valid on an interval containing $x = 1$ for which $-x^3 + 3x^2 - 1 > 0$.

$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

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6. Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

   (a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

   (b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

STOP

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Question 6

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

(a) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(1,\,0)} = e^0\left(3\cdot 1^2 - 6\cdot 1\right) = -3$

An equation for the tangent line is $y = -3(x-1)$.

$f(1.2) \approx -3(1.2-1) = -0.6$

$3 : \begin{cases} 1 : \frac{dy}{dx} \text{ at the point } (x,\,y) = (1,\,0) \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(b) $\frac{dy}{e^y} = \left(3x^2 - 6x\right)dx$

$\int \frac{dy}{e^y} = \int \left(3x^2 - 6x\right)dx$

$-e^{-y} = x^3 - 3x^2 + C$

$-e^{-0} = 1^3 - 3\cdot 1^2 + C \Rightarrow C = 1$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln\left(-x^3 + 3x^2 - 1\right)$

$y = -\ln\left(-x^3 + 3x^2 - 1\right)$

Note: This solution is valid on an interval containing $x = 1$ for which $-x^3 + 3x^2 - 1 > 0$.

$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

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6. Let f be a function with $f(2) = -8$ such that for all points (x, y) on the graph of f, the slope is given by $\frac{3x^2}{y}$.

   (a) Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.

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Continue problem 6 on page 21.

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(b) Find an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = \frac{3x^2}{y}$ with the initial condition $f(2) = -8$.

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Question 6

Let f be a function with $f(2) = -8$ such that for all points (x, y) on the graph of f, the slope is given by $\frac{3x^2}{y}$.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.

(b) Find an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = \frac{3x^2}{y}$ with the initial condition $f(2) = -8$.

(a) Slope $= \frac{(3)(4)}{-8} = -\frac{3}{2}$

An equation for the tangent line is $y = -\frac{3}{2}(x - 2) - 8$.

$f(1.8) \approx -\frac{3}{2}(1.8 - 2) - 8 = -7.7$

3 : { 1 : slope; 1 : tangent line equation; 1 : approximation }

(b) $\int y\, dy = \int 3x^2\, dx$

$\frac{1}{2}y^2 = x^3 + C$

$\frac{1}{2}(-8)^2 = 2^3 + C \Rightarrow C = 24$

$y^2 = 2\left(x^3 + 24\right) = 2x^3 + 48$

$y = -\sqrt{2x^3 + 48}$

Note: This solution is valid for $x > -\sqrt[3]{24}$.

6 : { 1 : separation of variables; 2 : antiderivatives; 1 : constant of integration; 1 : uses initial condition; 1 : solves for y }

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

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CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

   (a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

   (b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

   (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

   (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

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Question 1

Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

(a) $\dfrac{A(30) - A(0)}{30 - 0} = -0.197$ (or -0.196) lbs/day

1 : answer with units

(b) $A'(15) = -0.164$ (or -0.163)

The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.

$2 : \begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$

(c) $A(t) = \dfrac{1}{30}\displaystyle\int_0^{30} A(t)\,dt \Rightarrow t = 12.415$ (or 12.414)

$2 : \begin{cases} 1 : \dfrac{1}{30}\displaystyle\int_0^{30} A(t)\,dt \\ 1 : \text{answer} \end{cases}$

(d) $L(t) = A(30) + A'(30) \cdot (t - 30)$

$A'(30) = -0.055976$

$A(30) = 0.782928$

$L(t) = 0.5 \Rightarrow t = 35.054$

$4 : \begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

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A graphing calculator is required for these problems.

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

1. Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

   (a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

   (b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

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Continue problem 1 on page 5.

A graphing calculator is required for these problems.

(c) For $0 \le t \le 2.4$ hours, Ruth's velocity can be modeled by the function g given by $g(t) = \dfrac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 2.4$?

(d) According to the model given in part (c), is Ruth's speed increasing or decreasing at time $t = 1.3$? Give a reason for your answer.

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Question 1

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

(a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

(c) For $0 \le t \le 2.4$ hours, Ruth's velocity can be modeled by the function g given by $g(t) = \dfrac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 2.4$?

(d) According to the model given in part (c), is Ruth's speed increasing or decreasing at time $t = 1.3$? Give a reason for your answer.

(a) $a(1.4) \approx \dfrac{v(1.6) - v(1.2)}{1.6 - 1.2} = \dfrac{16.3 - 17.2}{1.6 - 1.2} = -2.25$ miles/hr^2

2 : { 1 : approximation; 1 : units }

(b) $\int_0^{2.4} v(t)\,dt$ is the total distance Ruth traveled, in miles, from time $t = 0$ to time $t = 2.4$ hours.

$\int_0^{2.4} v(t)\,dt \approx (0.8)(11.8) + (0.8)(17.2) + (0.8)(16.8)$
$= 36.64$ miles

3 : { 1 : interpretation; 1 : midpoint Riemann sum; 1 : approximation }

(c) Average velocity $= \dfrac{1}{2.4}\int_0^{2.4} g(t)\,dt = 14.064$ miles/hr

2 : { 1 : integral; 1 : answer }

(d) Velocity $= g(1.3) = 18.096358 > 0$
Acceleration $= g'(1.3) = 3.761152 > 0$

Ruth's speed is increasing at time $t = 1.3$ since velocity and acceleration have the same sign at this time.

2 : conclusion with reason

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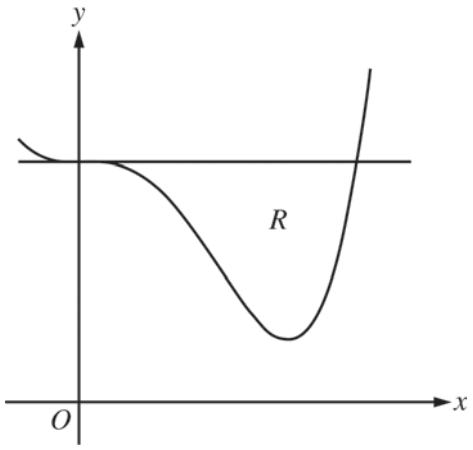

2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

   (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

   (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

END OF PART A OF SECTION II

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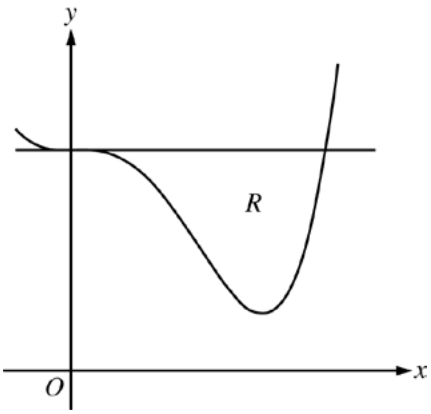
Question 2

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.



(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

(a) $f(x) = 4 \Rightarrow x = 0, 2.3$

$$\text{Volume} = \pi\int_0^{2.3}\left[(4+2)^2 - (f(x)+2)^2\right]dx$$
$$= 98.868 \text{ (or } 98.867\text{)}$$

4 : { 2 : integrand; 1 : limits; 1 : answer }

(b) $$\text{Volume} = \int_0^{2.3}\frac{1}{2}(4 - f(x))^2\,dx$$
$$= 3.574 \text{ (or } 3.573\text{)}$$

3 : { 2 : integrand; 1 : answer }

(c) $\int_0^k (4 - f(x))\,dx = \int_k^{2.3} (4 - f(x))\,dx$

2 : { 1 : area of one region; 1 : equation }

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A graphing calculator is required for these problems.

2. A store is having a 12-hour sale. The total number of shoppers who have entered the store t hours after the sale begins is modeled by the function S defined by $S(t) = 0.5t^4 - 16t^3 + 144t^2$ for $0 \le t \le 12$. At time $t = 0$, when the sale begins, there are no shoppers in the store.

   (a) At what rate are shoppers entering the store 3 hours after the start of the sale?

   (b) Find the value of $\frac{1}{3}\int_6^9 S'(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{3}\int_6^9 S'(t)\,dt$ in the context of this problem.

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Continue problem 2 on page 7.

A graphing calculator is required for these problems.

(c) The rate at which shoppers leave the store, measured in shoppers per hour, is modeled by the function L defined by $L(t) = -80 + \dfrac{4400}{t^2 - 14t + 55}$ for $0 \le t \le 12$. According to the model, how many shoppers are in the store at the end of the sale (time $t = 12$)? Give your answer to the nearest whole number.

(d) Using the given models, find the time t, $0 \le t \le 12$, at which the number of shoppers in the store is the greatest. Justify your answer.

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Question 2

A store is having a 12-hour sale. The total number of shoppers who have entered the store t hours after the sale begins is modeled by the function S defined by $S(t) = 0.5t^4 - 16t^3 + 144t^2$ for $0 \le t \le 12$. At time $t = 0$, when the sale begins, there are no shoppers in the store.

(a) At what rate are shoppers entering the store 3 hours after the start of the sale?

(b) Find the value of $\frac{1}{3}\int_6^9 S'(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{3}\int_6^9 S'(t)\,dt$ in the context of this problem.

(c) The rate at which shoppers leave the store, measured in shoppers per hour, is modeled by the function L defined by $L(t) = -80 + \frac{4400}{t^2 - 14t + 55}$ for $0 \le t \le 12$. According to the model, how many shoppers are in the store at the end of the sale (time $t = 12$)? Give your answer to the nearest whole number.

(d) Using the given models, find the time t, $0 \le t \le 12$, at which the number of shoppers in the store is the greatest. Justify your answer.

(a) $S'(3) = 486$ shoppers/hour

1 : answer

(b) $\frac{1}{3}\int_6^9 S'(t)\,dt = \frac{1}{3}(S(9) - S(6))$

$= \frac{1}{3}(3280.5 - 2376) = 301.5$

$\frac{1}{3}\int_6^9 S'(t)\,dt$ is the average rate at which shoppers are entering the store in shoppers/hr between times $t = 6$ and $t = 9$ hours.

2 : { 1 : value of $\frac{1}{3}\int_6^9 S'(t)\,dt$; 1 : meaning of $\frac{1}{3}\int_6^9 S'(t)\,dt$ }

(c) $S(12) - \int_0^{12} L(t)\,dt = 195.701684$

Therefore, there are approximately 196 shoppers in the store at the end of the sale.

3 : { 1 : integral; 1 : uses $S(12)$; 1 : answer }

(d) The number of shoppers in the store at time t is given by the function N defined by

$N(t) = S(t) - \int_0^t L(x)\,dx$

$N'(t) = S'(t) - L(t) = 0$ when $t = 5.545066$

t	$N(t)$
0	0
5.545066	1361.832842
12	195.702

The number of shoppers in the store is greatest at time $t = 5.545$ hours.

3 : { 1 : considers $S'(t) - L(t) = 0$; 1 : answer; 1 : justification }

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CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

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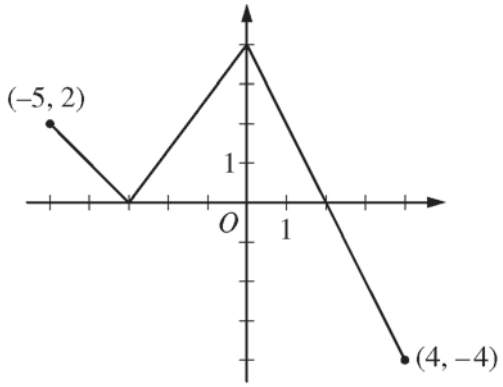

Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

   (a) Find $g(3)$.

   (b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

   (c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

   (d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

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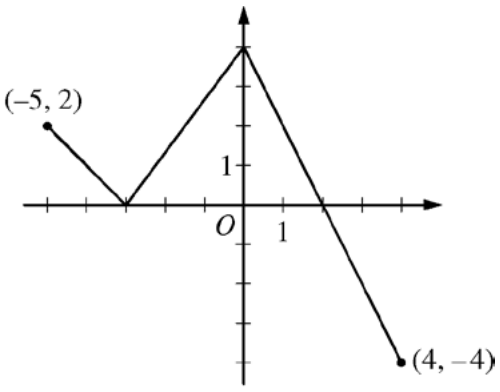
Question 3

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

(c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

(a) $g(3) = \int_{-3}^{3} f(t)\,dt = 6 + 4 - 1 = 9$

1 : answer

(b) $g'(x) = f(x)$

The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.

2 : { 1 : answer; 1 : reason }

(c) $h'(x) = \dfrac{5xg'(x) - g(x)5}{(5x)^2} = \dfrac{5xg'(x) - 5g(x)}{25x^2}$

$h'(3) = \dfrac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2}$

$= \dfrac{15(-2) - 5(9)}{225} = \dfrac{-75}{225} = -\dfrac{1}{3}$

3 : { 2 : $h'(x)$; 1 : answer }

(d) $p'(x) = f'\left(x^2 - x\right)(2x - 1)$

$p'(-1) = f'(2)(-3) = (-2)(-3) = 6$

3 : { 2 : $p'(x)$; 1 : answer }

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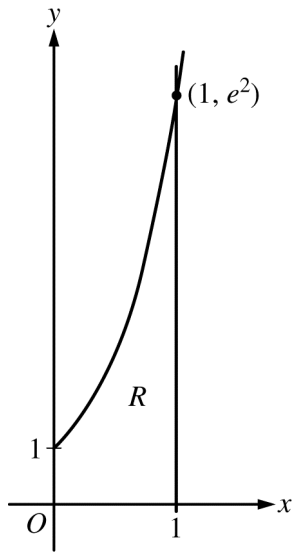

3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of $y = f(x)$ and the vertical line $x = 1$, as shown in the figure above.

   (a) Write an equation for the line tangent to the graph of f at $x = 1$.

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Continue problem 3 on page 15.

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(b) Find the area of R.

(c) Region R forms the base of a solid whose cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

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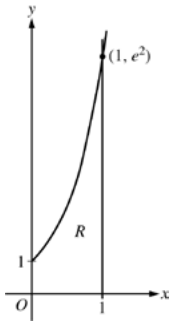
Question 3

Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of $y = f(x)$ and the vertical line $x = 1$, as shown in the figure above.



(a) Write an equation for the line tangent to the graph of f at $x = 1$.

(b) Find the area of R.

(c) Region R forms the base of a solid whose cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(a) $f(1) = e^2$

$f'(x) = 2e^{2x} \Rightarrow f'(1) = 2e^2$

An equation for the tangent line is $y = e^2 + 2e^2(x - 1)$.

$2 : \begin{cases} 1 : f'(1) \\ 1 : \text{answer} \end{cases}$

(b) $\text{Area} = \int_0^1 e^{2x}\,dx = \left[\frac{1}{2}e^{2x}\right]_{x=0}^{x=1} = \frac{1}{2}\left(e^2 - 1\right)$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) $\text{Volume} = 1 + \int_1^{e^2} \left(1 - \frac{1}{2}\ln y\right)^2 dy$

$4 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

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t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	−120	−150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

   (a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

   (b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

   (c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

   (d) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

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Question 4

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	–120	–150

(a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

(b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

(c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

(d) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

(a) average accel $= \dfrac{v_A(8) - v_A(2)}{8 - 2} = \dfrac{-120 - 100}{6} = -\dfrac{110}{3}$ m/min^2

1 : average acceleration

(b) v_A is differentiable $\Rightarrow$ v_A is continuous
$v_A(8) = -120 < -100 < 40 = v_A(5)$

Therefore, by the Intermediate Value Theorem, there is a time t, $5 < t < 8$, such that $v_A(t) = -100$.

2 : { 1 : $v_A(8) < -100 < v_A(5)$; 1 : conclusion, using IVT }

(c) $s_A(12) = s_A(2) + \int_2^{12} v_A(t)\,dt = 300 + \int_2^{12} v_A(t)\,dt$

$\int_2^{12} v_A(t)\,dt \approx 3 \cdot \dfrac{100 + 40}{2} + 3 \cdot \dfrac{40 - 120}{2} + 4 \cdot \dfrac{-120 - 150}{2}$
$= -450$

$s_A(12) \approx 300 - 450 = -150$

The position of Train A at time $t = 12$ minutes is approximately 150 meters west of Origin Station.

3 : { 1 : position expression; 1 : trapezoidal sum; 1 : position at time $t = 12$ }

(d) Let x be train A's position, y train B's position, and z the distance between train A and train B.

$z^2 = x^2 + y^2 \Rightarrow 2z\dfrac{dz}{dt} = 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt}$

$x = 300,\ y = 400 \Rightarrow z = 500$

$v_B(2) = -20 + 120 + 25 = 125$

$500\dfrac{dz}{dt} = (300)(100) + (400)(125)$

$\dfrac{dz}{dt} = \dfrac{80000}{500} = 160$ meters per minute

3 : { 2 : implicit differentiation of distance relationship; 1 : answer }

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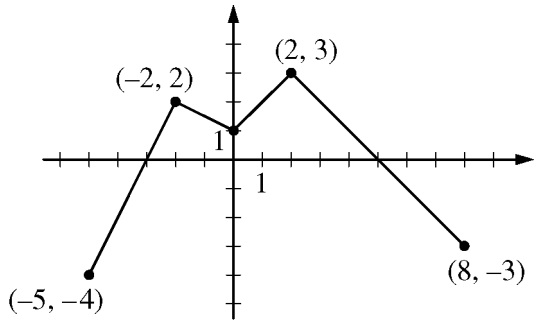

Graph of f

4. The continuous function f is defined on the interval $-5 \le x \le 8$. The graph of f, which consists of four line segments, is shown in the figure above. Let g be the function given by $g(x) = 2x + \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(-5)$.

(b) Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

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Continue problem 4 on page 17.

(c) On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

(d) The function h is given by $h(x) = g\left(x^3 + 1\right)$. Find $h'(1)$. Show the work that leads to your answer.

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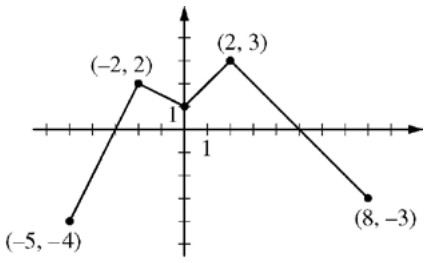
Question 4

The continuous function f is defined on the interval $-5 \le x \le 8$. The graph of f, which consists of four line segments, is shown in the figure above.

Let g be the function given by $g(x) = 2x + \int_{-2}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g(-5)$.

(b) Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

(c) On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

(d) The function h is given by $h(x) = g\left(x^3 + 1\right)$. Find $h'(1)$. Show the work that leads to your answer.

(a) $g(0) = 2 \cdot 0 + \int_{-2}^{0} f(t)\,dt = 3$

$g(-5) = 2 \cdot (-5) + \int_{-2}^{-5} f(t)\,dt = -10 + 3 = -7$

$2 : \begin{cases} 1 : g(0) \\ 1 : g(-5) \end{cases}$

(b) $g'(x) = 2 + f(x)$
$g''(x) = f'(x)$

$g''(4) = f'(4) = -1$

$g''(-2) = f'(-2)$ does not exist.

$3 : \begin{cases} 1 : g'(x) \\ 1 : g''(4) \\ 1 : g''(-2) \text{ does not exist} \end{cases}$

(c) The graph of g is concave down on the intervals $(-2, 0)$ and $(2, 8)$ since $g'(x) = 2 + f(x)$ decreases on those intervals.

1 : intervals and reason

(d) $h'(x) = g'\left(x^3 + 1\right) \cdot 3x^2$

$h'(1) = g'(2) \cdot 3 = (2 + f(2)) \cdot 3$
$= (2 + 3) \cdot 3 = 15$

$3 : \begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

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x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

   (a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

   (b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

   (c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

   (d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

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Question 5

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

(a) $x = 1$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a relative minimum at $x = 1$.

1 : answer with justification

(b) f' is differentiable $\Rightarrow$ f' is continuous on the interval $-1 \le x \le 1$

$$\frac{f'(1) - f'(-1)}{1 - (-1)} = \frac{0 - 0}{2} = 0$$

Therefore, by the Mean Value Theorem, there is at least one value c, $-1 < c < 1$, such that $f''(c) = 0$.

2 : $\begin{cases} 1 : f'(1) - f'(-1) = 0 \\ 1 : \text{explanation, using Mean Value Theorem} \end{cases}$

(c) $h'(x) = \frac{1}{f(x)} \cdot f'(x)$

$$h'(3) = \frac{1}{f(3)} \cdot f'(3) = \frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14}$$

3 : $\begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

(d) $\int_{-2}^{3} f'(g(x))g'(x)\,dx = \Big[f(g(x)) \Big]_{x=-2}^{x=3}$

$= f(g(3)) - f(g(-2))$

$= f(1) - f(-1)$

$= 2 - 8 = -6$

3 : $\begin{cases} 2 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

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5. Particle X moves along the positive x-axis so that its position at time $t \geq 0$ is given by $x(t) = 5t^3 - 9t^2 + 7$.

(a) Is particle X moving toward the left or toward the right at time $t = 1$? Give a reason for your answer.

(b) At what time $t \geq 0$ is particle X farthest to the left? Justify your answer.

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Continue problem 5 on page 19.

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(c) A second particle, Y, moves along the positive y-axis so that its position at time t is given by $y(t) = 7t + 3$. At any time t, $t \geq 0$, the origin and the positions of the particles X and Y are the vertices of a triangle in the first quadrant. Find the rate of change of the area of the triangle at time $t = 1$. Show the work that leads to your answer.

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Question 5

Particle X moves along the positive x-axis so that its position at time $t \geq 0$ is given by $x(t) = 5t^3 - 9t^2 + 7$.

(a) Is particle X moving toward the left or toward the right at time $t = 1$? Give a reason for your answer.

(b) At what time $t \geq 0$ is particle X farthest to the left? Justify your answer.

(c) A second particle, Y, moves along the positive y-axis so that its position at time t is given by $y(t) = 7t + 3$. At any time t, $t \geq 0$, the origin and the positions of the particles X and Y are the vertices of a triangle in the first quadrant. Find the rate of change of the area of the triangle at time $t = 1$. Show the work that leads to your answer.

(a) $x'(t) = 15t^2 - 18t$

$x'(1) = 15 - 18 = -3$

Since $x'(1) < 0$, the particle is moving to the left at time $t = 1$.

$2 : \begin{cases} 1 : \text{considers } x'(1) \\ 1 : \text{answer with reason} \end{cases}$

(b) $x'(t) = 3t(5t - 6) = 0 \Rightarrow t = 0,\ t = \frac{6}{5}$

Since $x'(t) < 0$ for $0 < t < \frac{6}{5}$ and $x'(t) > 0$ for $t > \frac{6}{5}$,

the particle is farthest to the left at time $t = \frac{6}{5}$.

$3 : \begin{cases} 1 : \text{considers } x'(t) = 0 \\ 1 : \text{identifies } t = \frac{6}{5} \\ 1 : \text{answer with reason} \end{cases}$

(c) Area $= A(t) = \frac{1}{2}x(t)\,y(t)$

$= \frac{1}{2}\left(5t^3 - 9t^2 + 7\right)(7t + 3)$

$A'(t) = \frac{1}{2}\left[\left(15t^2 - 18t\right)(7t + 3) + \left(5t^3 - 9t^2 + 7\right)(7)\right]$

$A'(1) = \frac{1}{2}[(-3)(10) + (3)(7)] = \frac{1}{2}[-30 + 21] = -\frac{9}{2}$

$4 : \begin{cases} 1 : \text{area function} \\ 2 : \text{derivative} \\ 1 : \text{answer} \end{cases}$

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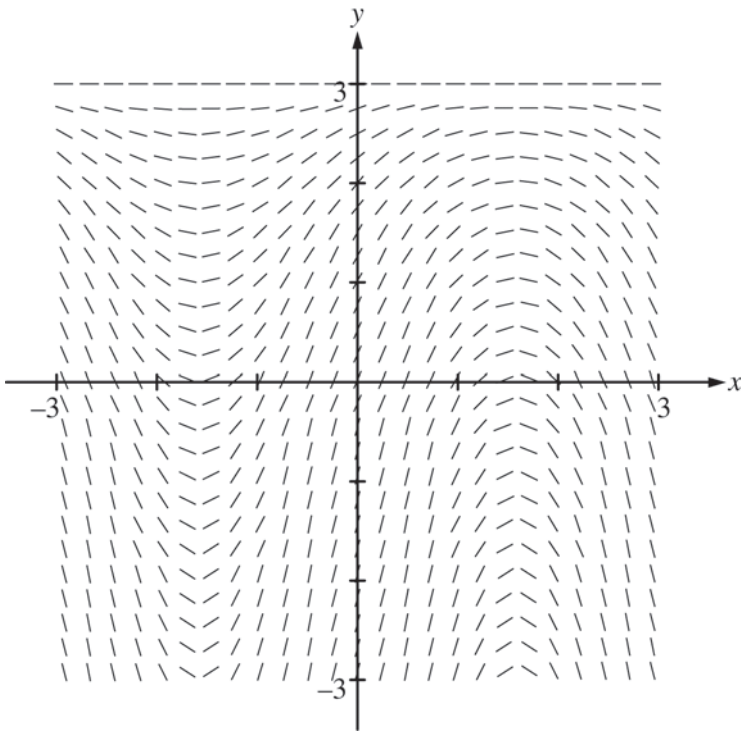
6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

   (a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



   (b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

   (c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

STOP

END OF EXAM

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Question 6

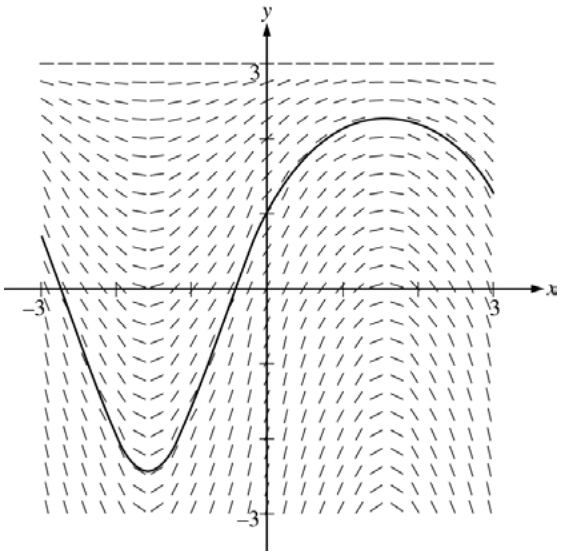
Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.

(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

(a)



1 : solution curve

(b) $\left.\frac{dy}{dx}\right|_{(x, y)=(0, 1)} = 2\cos 0 = 2$

An equation for the tangent line is $y = 2x + 1$.

$f(0.2) \approx 2(0.2) + 1 = 1.4$

2 : { 1 : tangent line equation; 1 : approximation }

(c) $\frac{dy}{dx} = (3 - y)\cos x$

$\int \frac{dy}{3 - y} = \int \cos x \, dx$

$-\ln|3 - y| = \sin x + C$

$-\ln 2 = \sin 0 + C \Rightarrow C = -\ln 2$

$-\ln|3 - y| = \sin x - \ln 2$

Because $y(0) = 1$, $y < 3$, so $|3 - y| = 3 - y$

$3 - y = 2e^{-\sin x}$

$y = 3 - 2e^{-\sin x}$

Note: this solution is valid for all real numbers.

6 : { 1 : separation of variables; 2 : antiderivatives; 1 : constant of integration; 1 : uses initial condition; 1 : solves for y }

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

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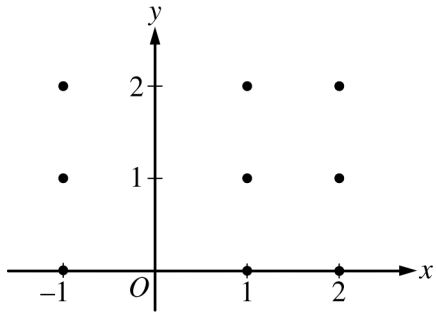
6. Consider the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$, where $x \neq 0$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(1) = 2$.

(a) Find the slope of the line tangent to the graph of f at the point $(1, 2)$.

(b) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



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Continue problem 6 on page 21.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$ with initial condition $f(1) = 2$.

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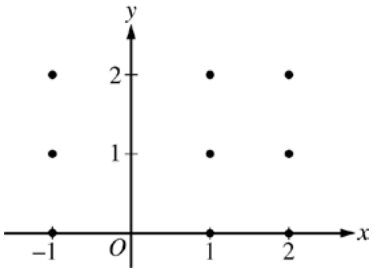
Question 6

Consider the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$, where $x \neq 0$.

Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(1) = 2$.



(a) Find the slope of the line tangent to the graph of f at the point $(1, 2)$.

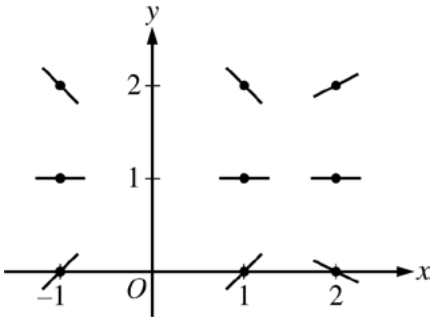
(b) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$ with initial condition $f(1) = 2$.

(a) $\left.\frac{dy}{dx}\right|_{(x, y)=(1, 2)} = \left(1 - \frac{2}{1}\right)(2 - 1) = -1$

1 : answer

(b)



$2 : \begin{cases} 1 : \text{zero slope at each point } (x, y) \text{ where } y = 1 \\ 1 : \text{remaining slopes} \end{cases}$

(c) $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$

$\int \frac{dy}{y - 1} = \int \left(1 - \frac{2}{x^2}\right) dx$

$\ln|y - 1| = x + \frac{2}{x} + C$

$\ln|2 - 1| = 1 + \frac{2}{1} + C \Rightarrow C = -3$

$\ln|y - 1| = x + \frac{2}{x} - 3$

Note that $y - 1 > 0$ since the solution curve includes the point $(1, 2)$.

$\ln(y - 1) = x + \frac{2}{x} - 3$

$y = f(x) = e^{\left(x + \frac{2}{x} - 3\right)} + 1$

Note: This solution is valid for $x > 0$.

$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

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CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

   (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

   (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

   (c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

   (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

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Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t)\,dt = 76.570$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$
Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

$2 : \begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t, $0 \le t \le 8$, is $30 + \int_0^t [R(x) - D(x)]\,dx$.

$R(t) - D(t) = 0 \Rightarrow t = 0,\ 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

$3 : \begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $30 + \int_0^w [R(t) - D(t)]\,dt = 50$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

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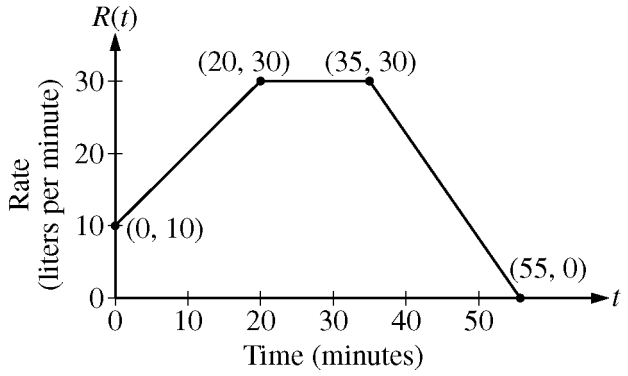
A graphing calculator is required for these problems.



1. At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.

   (a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

   (b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

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Continue problem 1 on page 5.

A graphing calculator is required for these problems.

(c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

(d) Using the functions R and D, determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.

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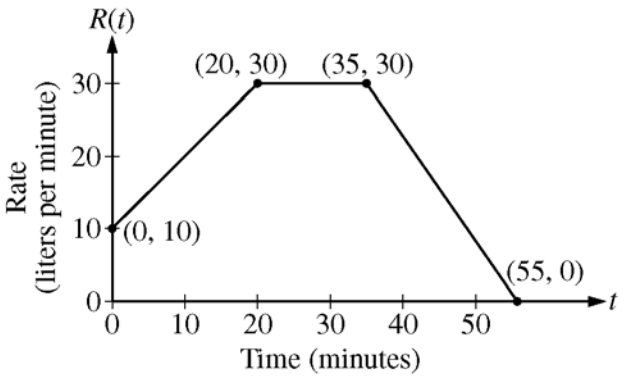
Question 1

At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.



(a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

(b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

(c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

(d) Using the functions R and D, determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.

(a) $R'(45) = \dfrac{30 - 0}{35 - 55} = -\dfrac{3}{2}$

The rate at which water is being pumped into the tank is decreasing at $\dfrac{3}{2}$ liters/min^2 at $t = 45$ minutes.

$2 : \begin{cases} 1 : R'(45) \\ 1 : \text{explanation} \end{cases}$

(b) $\displaystyle\int_0^{55} R(t)\,dt = 20 \cdot \frac{10 + 30}{2} + 15 \cdot 30 + \frac{1}{2} \cdot 20 \cdot 30$

$= 400 + 450 + 300 = 1150$

$2 : \begin{cases} 1 : \text{sum of areas} \\ 1 : \text{answer} \end{cases}$

(c) $\text{Amt} = 100 + 1150 - \displaystyle\int_{10}^{55} 10e^{(\sin t)/10}\,dt$

$= 1250 - 450.275371 = 799.725$ (or 799.724)

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{expression for water in the tank} \\ 1 : \text{answer} \end{cases}$

(d) $R(45) = 15$

$D(45) = 10.88815$

At time $t = 45$ minutes, the rate of water pumped into the tank is greater than the rate of water draining from the tank. Therefore, the amount of water in the tank is increasing at time $t = 45$ minutes.

$2 :$ answer with justification

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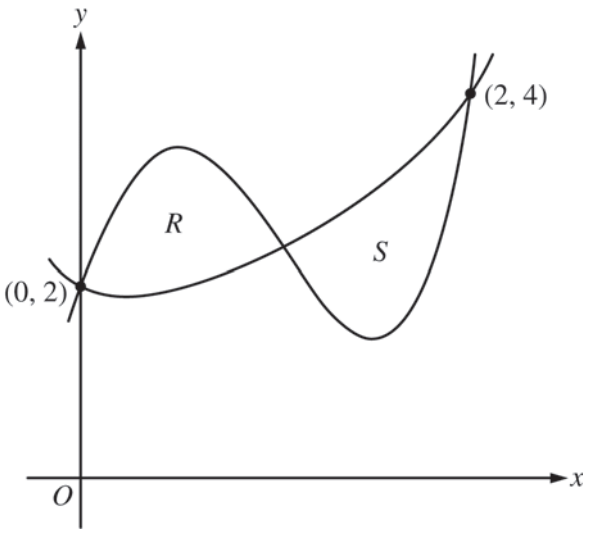

2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

   (a) Find the sum of the areas of regions R and S.

   (b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

   (c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

END OF PART A OF SECTION II

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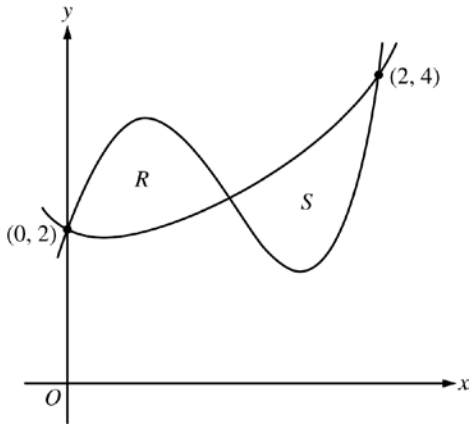
Question 2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

(a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.



(a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$.

$$\text{Area} = \int_0^A [g(x) - f(x)]\,dx + \int_A^2 [f(x) - g(x)]\,dx$$
$$= 0.997427 + 1.006919 = 2.004$$

4 : { 1 : limits; 2 : integrands; 1 : answer }

(b) $\text{Volume} = \int_A^2 [f(x) - g(x)]^2\,dx = 1.283$

3 : { 2 : integrand; 1 : answer }

(c) $h(x) = f(x) - g(x)$
$h'(x) = f'(x) - g'(x)$
$h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)

2 : { 1 : considers h'; 1 : answer }

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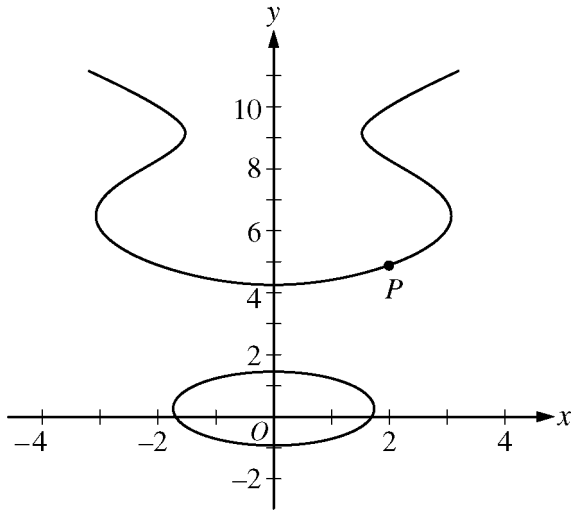
A graphing calculator is required for these problems.



2. The graph of the equation $x^2 = -2 + y + 5\cos y$ is shown above for $y \le 11$. It is known that $\frac{dy}{dx} = \frac{2x}{1 - 5\sin y}$.

The x-coordinate of point P shown on the graph is 2.

(a) Write an equation for the line tangent to the graph at point P.

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Continue problem 2 on page 7.

A graphing calculator is required for these problems.

(b) For $y \leq 11$, find the y-coordinate of each point on the graph where the line tangent to the graph at that point is vertical.

(c) Find the average value of the x-coordinates of the points on the graph in the first quadrant between $y = 5$ and $y = 9$.

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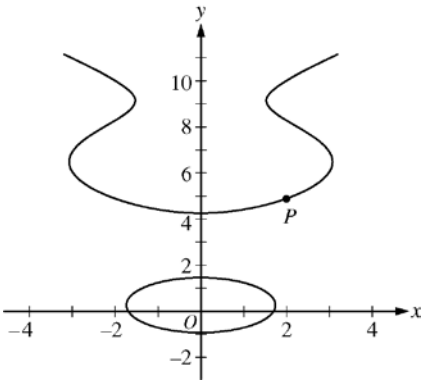
Question 2

The graph of the equation $x^2 = -2 + y + 5\cos y$ is shown above for $y \le 11$. It is known that $\frac{dy}{dx} = \frac{2x}{1 - 5\sin y}$. The x-coordinate of point P shown on the graph is 2.



(a) Write an equation for the line tangent to the graph at point P.

(b) For $y \le 11$, find the y-coordinate of each point on the graph where the line tangent to the graph at that point is vertical.

(c) Find the average value of the x-coordinates of the points on the graph in the first quadrant between $y = 5$ and $y = 9$.

(a) $x = 2 \Rightarrow y + 5\cos y = 6 \Rightarrow y = 4.928387$

$\left.\frac{dy}{dx}\right|_{(x, y)=(2, 4.928387)} = 0.679831$

An equation for the line tangent to the graph at point P is $y = 0.680(x - 2) + 4.928$ (or $y = 0.679(x - 2) + 4.928$).

3 : { 1 : y-coordinate; 1 : slope; 1 : tangent line equation }

(b) $1 - 5\sin y = 0$

$y = 0.201, 6.485$ (or 6.484), 9.223

3 : { 1 : sets $1 - 5\sin y = 0$; 2 : y-coordinates }

(c) $x = \sqrt{y + 5\cos y - 2}$

Average value $= \frac{1}{4}\int_5^9 \sqrt{y + 5\cos y - 2}\, dy = 2.550$

3 : { 1 : integrand; 1 : limits and constant; 1 : answer }

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CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

3. Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

   (a) Use the data in the table to estimate the value of $v'(16)$.

   (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem.

   Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

   (c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute.

   Find Bob's acceleration at time $t = 5$.

   (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

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Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\,dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)|\,dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute.

Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

(a) $v'(16) \approx \dfrac{240 - 200}{20 - 12} = 5$ meters/min^2

1 : approximation

(b) $\int_0^{40} |v(t)|\,dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \le t \le 40$ minutes.

$$\begin{aligned}\int_0^{40} |v(t)|\,dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters}\end{aligned}$$

3 : { 1 : explanation; 1 : right Riemann sum; 1 : approximation }

(c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.

$B'(5) = 3(25) - 12(5) = 15$ meters/min^2

2 : { 1 : uses $B'(t)$; 1 : answer }

(d) $\text{Avg vel} = \dfrac{1}{10}\displaystyle\int_0^{10} \left(t^3 - 6t^2 + 300\right) dt$

$$= \frac{1}{10}\left[\frac{t^4}{4} - 2t^3 + 300t\right]_0^{10}$$

$$= \frac{1}{10}\left[\frac{10000}{4} - 2000 + 3000\right] = 350 \text{ meters/min}$$

3 : { 1 : integral; 1 : antiderivative; 1 : answer }

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t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

3. Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \le 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

   (a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

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Continue problem 3 on page 15.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \frac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

(d) Write an expression for Nathan's acceleration in terms of t.

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Question 3

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \le 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

(a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \frac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

(d) Write an expression for Nathan's acceleration in terms of t.

(a) $a(4) \approx \frac{10-5}{5-3} = \frac{5}{2}$ ft/sec^2

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) $\int_0^{12} k(t)\,dt \approx (5)(3) + (10)(2) + (20)(3) + (24)(4) = 191$ feet

This approximation is an overestimate since a right Riemann sum is used and the function k is increasing.

$3 : \begin{cases} 1 : \text{right Riemann sum} \\ 1 : \text{approximation with units} \\ 1 : \text{overestimate with reason} \end{cases}$

(c) $s(12) = 5 + \int_0^{12} n(t)\,dt$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $n'(t) = (150)(-1)(t+3)^{-2} - 50e^{-t}(-1)$

$= -\frac{150}{(t+3)^2} + 50e^{-t}$

$2 : n'(t)$

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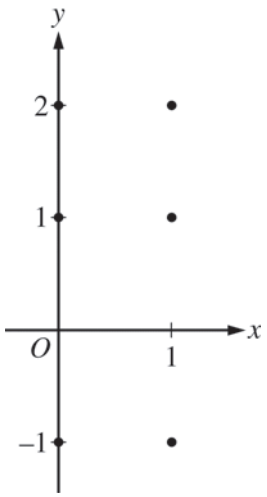
4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

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Question 4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

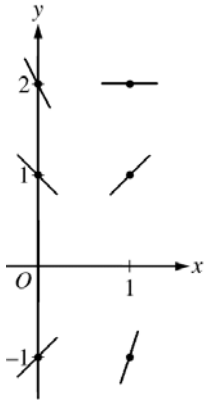
(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

(a)



$2 : \begin{cases} 1 : \text{slopes where } x = 0 \\ 1 : \text{slopes where } x = 1 \end{cases}$

(b) $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$

In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$.
Therefore, all solution curves are concave up in Quadrant II.

$2 : \begin{cases} 1 : \frac{d^2y}{dx^2} \\ 1 : \text{concave up with reason} \end{cases}$

(c) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,3)} = 2(2) - 3 = 1 \neq 0$

Therefore, f has neither a relative minimum nor a relative maximum at $x = 2$.

$2 : \begin{cases} 1 : \text{considers } \left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,3)} \\ 1 : \text{conclusion with justification} \end{cases}$

(d) $y = mx + b \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(mx + b) = m$

$2x - y = m$

$2x - (mx + b) = m$

$(2 - m)x - (m + b) = 0$

$2 - m = 0 \Rightarrow m = 2$

$b = -m \Rightarrow b = -2$

Therefore, $m = 2$ and $b = -2$.

$3 : \begin{cases} 1 : \frac{d}{dx}(mx + b) = m \\ 1 : 2x - y = m \\ 1 : \text{answer} \end{cases}$

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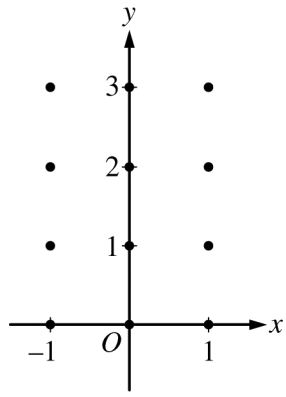
4. Consider the differential equation $\frac{dy}{dx} = \frac{x(y-1)}{4}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 3$. Write an equation for the line tangent to the graph of f at the point $(1, 3)$ and use it to approximate $f(1.4)$.

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Continue problem 4 on page 17.

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(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 3$.

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Question 4

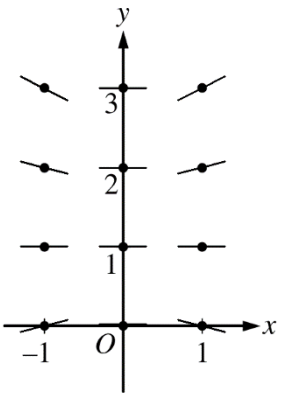
Consider the differential equation $\frac{dy}{dx} = \frac{x(y-1)}{4}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 3$. Write an equation for the line tangent to the graph of f at the point $(1, 3)$ and use it to approximate $f(1.4)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 3$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{other slopes} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{(x, y)=(1, 3)} = \frac{(1)(2)}{4} = \frac{1}{2}$

An equation for the line tangent to the graph of f at the point $(1, 3)$ is $y = \frac{1}{2}(x-1) + 3$.

$f(1.4) \approx \frac{1}{2}(0.4) + 3 = 3.2$

$2 : \begin{cases} 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{dy}{dx} = \frac{x(y-1)}{4}$

$\int \frac{dy}{y-1} = \int \frac{x}{4}\,dx$

$\ln|y-1| = \frac{x^2}{8} + C$

$\ln 2 = \frac{1}{8} + C \Rightarrow C = \ln 2 - \frac{1}{8}$

Because $f(1) = 3,\ y > 1,$ so $|y-1| = y-1$.

$\ln(y-1) = \frac{x^2}{8} + \ln 2 - \frac{1}{8}$

$y = 2e^{(x^2-1)/8} + 1$ for all x

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/5$ [1-2-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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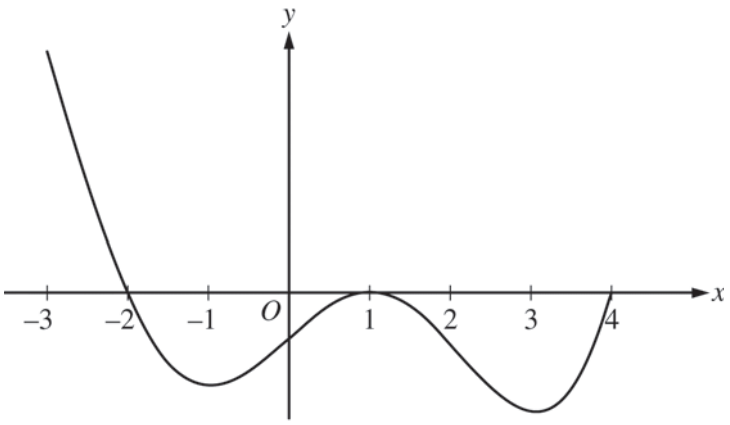

Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

   (a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

   (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

   (c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

   (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

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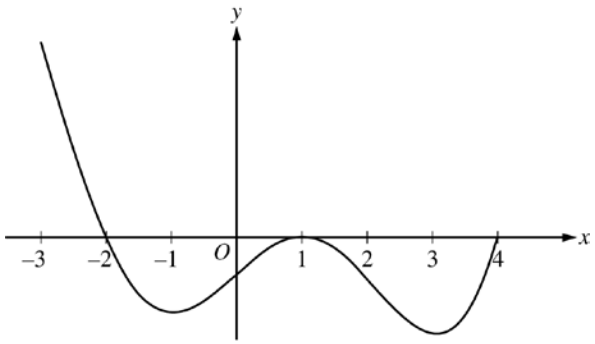
Question 5

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



Graph of f'

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

(a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$.
$f'(x)$ changes from positive to negative at $x = -2$.
Therefore, f has a relative maximum at $x = -2$.

$2 : \begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$

(b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points.

The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.

$2 : \begin{cases} 1 : \text{identifies } x = -1, 1, \text{ and } 3 \\ 1 : \text{reason} \end{cases}$

(d) $f(x) = 3 + \int_1^x f'(t)\,dt$

$f(4) = 3 + \int_1^4 f'(t)\,dt = 3 + (-12) = -9$

$f(-2) = 3 + \int_1^{-2} f'(t)\,dt = 3 - \int_{-2}^1 f'(t)\,dt$
$= 3 - (-9) = 12$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$

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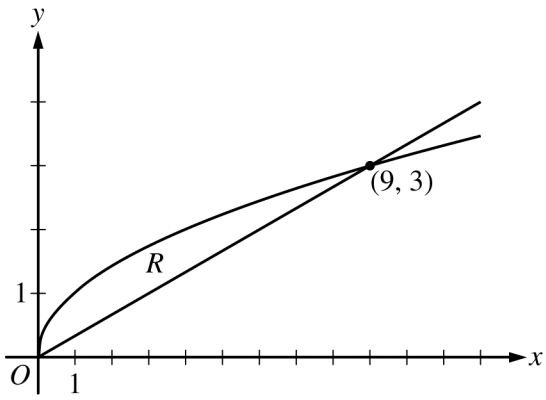

5. Let R be the region in the first quadrant enclosed by the graphs of $g(x) = \sqrt{x}$ and $h(x) = \frac{x}{3}$, as shown in the figure above.

   (a) Find the area of region R.

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Continue problem 5 on page 19.

(b) Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when R is revolved about the horizontal line $y = 4$.

(c) Find the maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$. Justify your answer.

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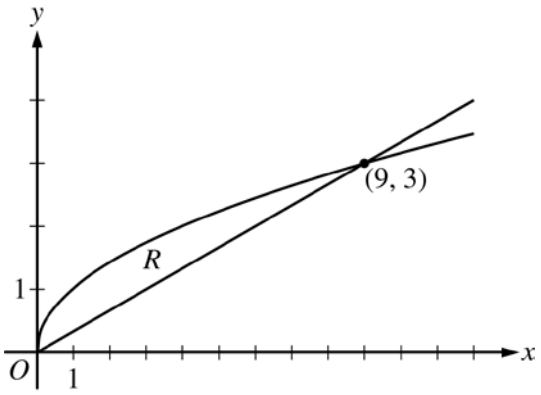
Question 5

Let R be the region in the first quadrant enclosed by the graphs of $g(x) = \sqrt{x}$ and $h(x) = \frac{x}{3}$, as shown in the figure above.

(a) Find the area of region R.

(b) Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when R is revolved about the horizontal line $y = 4$.

(c) Find the maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$. Justify your answer.



(a) Area $= \int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{6}x^2\right]_0^9$

$= \frac{2}{3} \cdot 27 - \frac{1}{6} \cdot 81 = \frac{9}{2}$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(b) Volume $= \pi \int_0^9 \left[\left(4 - \frac{x}{3}\right)^2 - \left(4 - \sqrt{x}\right)^2\right] dx$

3 : { 2 : integrand; 1 : limits and constant }

(c) Consider the function $D(x) = \sqrt{x} - \frac{x}{3}$.

$D'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{3} = \frac{1}{2\sqrt{x}} - \frac{1}{3}$

$D'(x) = 0 \Rightarrow x = \frac{9}{4}$

x	$D(x)$	Distance between graphs
0	0	0
$\frac{9}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
16	$-\frac{4}{3}$	$\frac{4}{3}$

The maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$ is $\frac{4}{3}$.

3 : { 1 : sets $D'(x) = 0$; 1 : identifies $x = \frac{9}{4}$ as a candidate; 1 : answer and justification }

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6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

   (a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

   (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

   (c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

STOP

END OF EXAM

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Question 6

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

(a) $\left.\frac{dy}{dx}\right|_{(x, y)=(-1, 1)} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{4}$

An equation for the tangent line is $y = \frac{1}{4}(x + 1) + 1$.

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{equation for tangent line} \end{cases}$

(b) $3y^2 - x = 0 \Rightarrow x = 3y^2$

So, $y^3 - xy = 2 \Rightarrow y^3 - \left(3y^2\right)(y) = 2 \Rightarrow y = -1$

$(-1)^3 - x(-1) = 2 \Rightarrow x = 3$

The tangent line to the curve is vertical at the point $(3, -1)$.

$3 : \begin{cases} 1 : \text{sets } 3y^2 - x = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{coordinates} \end{cases}$

(c) $\frac{d^2y}{dx^2} = \frac{\left(3y^2 - x\right)\frac{dy}{dx} - y\left(6y\frac{dy}{dx} - 1\right)}{\left(3y^2 - x\right)^2}$

$\left.\frac{d^2y}{dx^2}\right|_{(x, y)=(-1, 1)} = \frac{\left(3 \cdot 1^2 - (-1)\right) \cdot \frac{1}{4} - 1 \cdot \left(6 \cdot 1 \cdot \frac{1}{4} - 1\right)}{\left(3 \cdot 1^2 - (-1)\right)^2}$

$= \frac{1 - \frac{1}{2}}{16} = \frac{1}{32}$

$4 : \begin{cases} 2 : \text{implicit differentiation} \\ 1 : \text{substitution for } \frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$

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6. Let $g(x) = 4(x+1)^{-2/3}$ and let f be the function defined by $f(x) = \int_0^x g(t)\,dt$ for $x \geq 0$.

(a) Find $f(26)$.

(b) Determine the concavity of the graph of $y = f(x)$ for $x > 0$. Justify your answer.

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Continue problem 6 on page 21.

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(c) Let h be the function defined by $h(x) = x - f(x)$. Find the minimum value of h on the interval $0 \le x \le 26$.

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Question 6

Let $g(x) = 4(x+1)^{-2/3}$ and let f be the function defined by $f(x) = \int_0^x g(t)\,dt$ for $x \geq 0$.

(a) Find $f(26)$.

(b) Determine the concavity of the graph of $y = f(x)$ for $x > 0$. Justify your answer.

(c) Let h be the function defined by $h(x) = x - f(x)$. Find the minimum value of h on the interval $0 \leq x \leq 26$.

(a) $f(26) = \int_0^{26} 4(x+1)^{-2/3}\,dx = \left[4 \cdot 3(x+1)^{1/3}\right]_{x=0}^{x=26}$
$= 36 - 12 = 24$

3 : { 2 : antiderivative; 1 : answer }

(b) $f'(x) = g(x) = 4(x+1)^{-2/3}$

$f''(x) = 4\left(-\frac{2}{3}\right)(x+1)^{-5/3} = -\frac{8}{3(x+1)^{5/3}}$

$f''(x) < 0$ for $x > 0$.

The graph of $y = f(x)$ is concave down for $x > 0$.

3 : { 1 : $f'(x)$; 1 : $f''(x)$; 1 : answer with justification }

(c) $h'(x) = 1 - f'(x) = 1 - 4(x+1)^{-2/3}$

$h'(x) = 0 \Rightarrow 1 = \frac{4}{(x+1)^{2/3}} \Rightarrow x = 7$

$h(7) = 7 - \int_0^7 4(x+1)^{-2/3}\,dx = 7 - \left[4 \cdot 3(x+1)^{1/3}\right]_{x=0}^{x=7}$
$= 7 - [24 - 12] = -5$
$h(26) = 26 - 24 = 2$

x	$h(x)$
0	0
7	−5
26	2

The minimum value of h on the interval $0 \leq x \leq 26$ is -5.

3 : { 1 : sets $h'(x) = 0$; 1 : identifies $x = 7$ as a candidate; 1 : answer with reason }

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CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

   (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

   (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

   (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

   (d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

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Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

(a) $R'(2) \approx \dfrac{R(3) - R(1)}{3 - 1} = \dfrac{950 - 1190}{3 - 1} = -120$ liters/hr^2

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The total amount of water removed is given by $\int_0^8 R(t)\,dt$.

$$\begin{aligned} \int_0^8 R(t)\,dt &\approx 1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6) \\ &= 1(1340) + 2(1190) + 3(950) + 2(740) \\ &= 8050 \text{ liters} \end{aligned}$$

This is an overestimate since R is a decreasing function.

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{estimate} \\ 1 : \text{overestimate with reason} \end{cases}$

(c) $\text{Total} \approx 50000 + \int_0^8 W(t)\,dt - 8050$

$= 50000 + 7836.195325 - 8050 \approx 49786$ liters

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$

(d) $W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous.

Therefore, the Intermediate Value Theorem guarantees at least one time t, $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

For this value of t, the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

$2 : \begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$

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A graphing calculator is required for these problems.

1. A company produces and sells chili powder. The company's weekly profit on the sale of x kilograms of chili powder is modeled by the function P given by $P(x) = 48x + 1.4x^2 - 0.05x^{2.8} - 270$, where $P(x)$ is in dollars and $0 \le x \le 80$.

   (a) Find the rate, in dollars per kilogram, at which the company's weekly profit is changing when it sells 32 kilograms of chili powder. Is the company's weekly profit increasing or decreasing when it sells 32 kilograms of chili powder? Give a reason for your answer.

   (b) How many kilograms of chili powder must the company sell to maximize its weekly profit? Justify your answer.

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Continue problem 1 on page 5.

A graphing calculator is required for these problems.

(c) The company plans to have a one-day sale on chili powder. Management estimates that t hours after the company store opens, chili powder will sell at a rate modeled by the function S given by $S(t) = 2 + \cos\left(\frac{\pi}{10}t^2\right)$ kilograms per hour. Based on this model, estimate the amount of chili powder, in kilograms, that will be sold during the first 5 hours of the sale.

(d) Using the function S from part (c), find the value of $S'(3)$. Interpret the meaning of this value in the context of the problem.

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Question 1

A company produces and sells chili powder. The company's weekly profit on the sale of x kilograms of chili powder is modeled by the function P given by $P(x) = 48x + 1.4x^2 - 0.05x^{2.8} - 270$, where $P(x)$ is in dollars and $0 \le x \le 80$.

(a) Find the rate, in dollars per kilogram, at which the company's weekly profit is changing when it sells 32 kilograms of chili powder. Is the company's weekly profit increasing or decreasing when it sells 32 kilograms of chili powder? Give a reason for your answer.

(b) How many kilograms of chili powder must the company sell to maximize its weekly profit? Justify your answer.

(c) The company plans to have a one-day sale on chili powder. Management estimates that t hours after the company store opens, chili powder will sell at a rate modeled by the function S given by $S(t) = 2 + \cos\left(\frac{\pi}{10}t^2\right)$ kilograms per hour. Based on this model, estimate the amount of chili powder, in kilograms, that will be sold during the first 5 hours of the sale.

(d) Using the function S from part (c), find the value of $S'(3)$. Interpret the meaning of this value in the context of the problem.

(a) $P'(32) = 65.920$
Since $P'(32) > 0$, the company's profit is increasing when it sells 32 kilograms of chili powder.

$2 : \begin{cases} 1 : P'(32) \\ 1 : \text{increasing with reason} \end{cases}$

(b) $P'(x) = 48 + 2.8x - 0.14x^{1.8} = 0$ when $x = 58.358152$

x	$P(x)$
0	−270
58.358152	2893.04
80	1873.32

The company must sell 58.358 kilograms of chili powder to maximize its profit.

$3 : \begin{cases} 1 : \text{sets } P'(x) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) Based on this model, the estimate, in kilograms, is $\int_0^5 S(t)\,dt = 11.433$ (or 11.432).

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $S'(3) = -0.582$

The rate of sale of chili powder is decreasing at a rate of 0.582 kilogram per hour per hour at time $t = 3$ hours.

$2 : \begin{cases} 1 : S'(3) \\ 1 : \text{interpretation} \end{cases}$

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t (minutes)	0	2	5	7	10
$h(t)$ (inches)	3.5	10.0	15.5	18.5	20.0

1. The depth of water in tank A, in inches, is modeled by a differentiable and increasing function h for $0 \le t \le 10$, where t is measured in minutes. Values of $h(t)$ for selected values of t are given in the table above.

   (a) Use the data in the table to find an approximation for $h'(6)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Approximate the value of $\int_0^{10} h(t)\,dt$ using a right Riemann sum with the four subintervals indicated by the data in the table. Is this approximation greater than or less than $\int_0^{10} h(t)\,dt$? Give a reason for your answer.

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(c) The depth of water in tank B, in inches, is modeled by the function $g(t) = 3.2 + 17.5\sqrt{\sin(0.16t)}$ for $0 \le t \le 10$, where t is measured in minutes. Find the average depth of the water in tank B over the interval $0 \le t \le 10$. Is this value greater than or less than the average depth of the water in tank A over the interval $0 \le t \le 10$? Give a reason for your answer.

(d) According to the model given in part (c), is the depth of the water in tank B increasing or decreasing at time $t = 6$? Give a reason for your answer.

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GO ON TO THE NEXT PAGE.

Question 1

(a) $h'(6) \approx \dfrac{h(7) - h(5)}{7 - 5} = \dfrac{18.5 - 15.5}{2} = 1.5$ in/min

1 : answer with units

(b) $\displaystyle\int_0^{10} h(t)\,dt \approx (2 - 0) \cdot h(2) + (5 - 2) \cdot h(5) + (7 - 5) \cdot h(7) + (10 - 7) \cdot h(10)$
$= 2(10.0) + 3(15.5) + 2(18.5) + 3(20.0) = 163.5$

Because h is an increasing function, the right Riemann sum approximation is greater than $\displaystyle\int_0^{10} h(t)\,dt$.

3 : { 1 : right Riemann sum; 1 : approximation; 1 : overestimate with reason }

(c) Average depth in tank $B = \dfrac{1}{10}\displaystyle\int_0^{10} g(t)\,dt = 16.624$ in

Average depth in tank $A = \dfrac{1}{10}\displaystyle\int_0^{10} h(t)\,dt < \dfrac{1}{10}(163.5) = 16.35$ in < 16.624 in

Therefore, the average depth of the water in tank B is greater than the average depth of the water in tank A.

3 : { 1 : integral; 1 : average depth in tank B; 1 : answer with reason }

(d) $g'(6) = 0.887$

The depth of the water in tank B is increasing at time $t = 6$ because $g'(6) > 0$.

2 : { 1 : uses $g'(6)$; 1 : answer with reason }

A graphing calculator is required for these problems.

t (weeks)	0	3	6	10	12
$G(t)$ (games per week)	160	450	900	2100	2400

2. A store tracks the sales of one of its popular board games over a 12-week period. The rate at which games are being sold is modeled by the differentiable function G, where $G(t)$ is measured in games per week and t is measured in weeks for $0 \le t \le 12$. Values of $G(t)$ are given in the table above for selected values of t.

   (a) Approximate the value of $G'(8)$ using the data in the table. Show the computations that lead to your answer.

   (b) Approximate the value of $\int_0^{12} G(t)\,dt$ using a right Riemann sum with the four subintervals indicated by the table. Explain the meaning of $\int_0^{12} G(t)\,dt$ in the context of this problem.

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Continue problem 2 on page 7.

A graphing calculator is required for these problems.

(c) One salesperson believes that, starting with 2400 games per week at time $t = 12$, the rate at which games will be sold will increase at a constant rate of 100 games per week per week. Based on this model, how many total games will be sold in the 8 weeks between time $t = 12$ and $t = 20$?

(d) Another salesperson believes the best model for the rate at which games will be sold in the 8 weeks between time $t = 12$ and $t = 20$ is $M(t) = 2400e^{-0.01(t-12)^2}$ games per week. Based on this model, how many total games, to the nearest whole number, will be sold during this period?

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Question 2

t (weeks)	0	3	6	10	12
$G(t)$ (games per week)	160	450	900	2100	2400

A store tracks the sales of one of its popular board games over a 12-week period. The rate at which games are being sold is modeled by the differentiable function G, where $G(t)$ is measured in games per week and t is measured in weeks for $0 \le t \le 12$. Values of $G(t)$ are given in the table above for selected values of t.

(a) Approximate the value of $G'(8)$ using the data in the table. Show the computations that lead to your answer.

(b) Approximate the value of $\int_0^{12} G(t)\,dt$ using a right Riemann sum with the four subintervals indicated by the table. Explain the meaning of $\int_0^{12} G(t)\,dt$ in the context of this problem.

(c) One salesperson believes that, starting with 2400 games per week at time $t = 12$, the rate at which games will be sold will increase at a constant rate of 100 games per week per week. Based on this model, how many total games will be sold in the 8 weeks between time $t = 12$ and $t = 20$?

(d) Another salesperson believes the best model for the rate at which games will be sold in the 8 weeks between time $t = 12$ and $t = 20$ is $M(t) = 2400e^{-0.01(t-12)^2}$ games per week. Based on this model, how many total games, to the nearest whole number, will be sold during this period?

(a) $G'(8) \approx \dfrac{G(10) - G(6)}{10 - 6} = \dfrac{2100 - 900}{10 - 6} = 300$ games per week per week

1 : approximation

(b) $\int_0^{12} G(t)\,dt \approx 3 \cdot G(3) + 3 \cdot G(6) + 4 \cdot G(10) + 2 \cdot G(12)$
$= 3 \cdot 450 + 3 \cdot 900 + 4 \cdot 2100 + 2 \cdot 2400$
$= 17250$

$\int_0^{12} G(t)\,dt$ represents the total number of games sold over the 12-week period $0 \le t \le 12$.

3 : { 1 : right Riemann sum; 1 : approximation; 1 : explanation }

(c) The rate of sales of the game for $12 \le t \le 20$ is $R(t) = 2400 + 100(t - 12)$.

Based on this model, the total number of games that will be sold between time $t = 12$ and $t = 20$ is $\int_{12}^{20} R(t)\,dt = 22400$.

3 : { 1 : rate function; 1 : integral; 1 : answer }

(d) $\int_{12}^{20} M(t)\,dt = 15784.07655$

Based on this model, the total number of games that will be sold between time $t = 12$ and $t = 20$ is 15784.

2 : { 1 : integral; 1 : answer }

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2. Particle Q moves along the x-axis so that its velocity at any time t is given by $v_Q(t) = 1 - 3\cos\left(\frac{t^2}{5}\right)$, and its acceleration at any time t is given by $a_Q(t) = \frac{6t}{5}\sin\left(\frac{t^2}{5}\right)$. The particle is at position $x = 2$ at time $t = 0$.

(a) In the interval $0 < t < 5$, when is the velocity of particle Q increasing? Give a reason for your answer.

(b) Find the position of particle Q at time $t = 3$.

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(c) A second particle, R, moves along the x-axis so that its position at any time t is given by a differentiable function $x_R(t)$, where $x_R(1) = 4$ and $x_R(3) = 8$. Explain why there must be a time t, for $1 < t < 3$, at which the velocity of particle R is 2.

(d) At time $t = 3$, the velocity of particle R described in part (c) is -2. Are particles Q and R moving toward each other or away from each other at time $t = 3$? Explain your reasoning.

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Question 2

(a) $a_Q(t) > 0$ for $0 < t < 3.963$.

The velocity of particle Q is increasing on the interval $(0, 3.963]$.

2 : { 1 : interval; 1 : reason }

(b) $x_Q(3) = x_Q(0) + \int_0^3 v_Q(t)\,dt$
$= 2 + (-3.490102)$
$= -1.490$

3 : { 1 : integral; 1 : uses $x_Q(0)$; 1 : answer }

(c) $x_R(t)$ is differentiable. $\Rightarrow$ $x_R(t)$ is continuous on $1 \le t \le 3$.

$\frac{x_R(3) - x_R(1)}{3 - 1} = \frac{8 - 4}{2} = 2$

Therefore, by the Mean Value Theorem, there is at least one time t, $1 < t < 3$, for which $x_R'(t) = v_R(t) = 2$.

2 : { 1 : $\frac{x_R(3) - x_R(1)}{3 - 1}$; 1 : conclusion, using MVT }

(d) $x_Q(3) = -1.490$ and $v_Q(3) = 1.682$ (or 1.681)
Particle Q is moving to the right from $x = -1.490$.

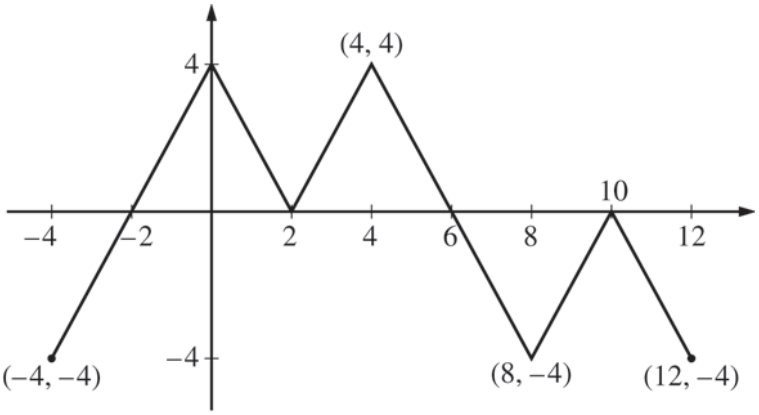
$x_R(3) = 8$ and $v_R(3) = -2$
Particle R is moving to the left from $x = 8$.

Therefore, the particles are moving toward each other.

2 : { 1 : answer; 1 : explanation }

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

   (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

   (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

   (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

   (d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

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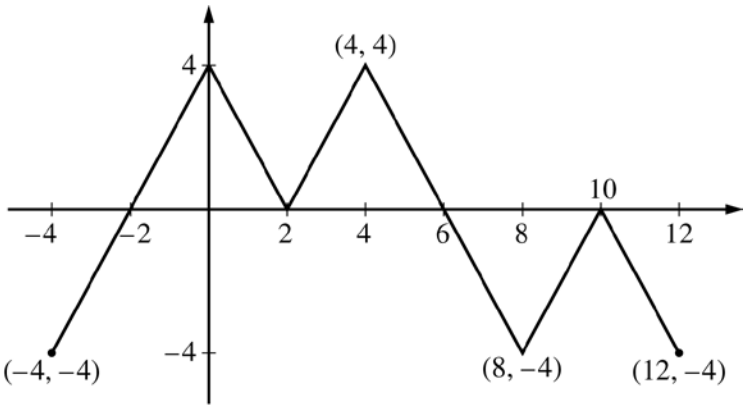
Question 3

The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.



Graph of f

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

$1 : g'(x) = f(x)$ in (a), (b), (c), or (d)

(a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \le 0$ for $8 \le x \le 12$.

1 : answer with justification

(b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \le x \le 4$ and decreasing for $4 \le x \le 8$.

1 : answer with justification

(c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
-4	-4
-2	-8
6	8
12	-4

On the interval $-4 \le x \le 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

4 : 1 : considers $x = -2$ and $x = 6$ as candidates
1 : considers $x = -4$ and $x = 12$
2 : answers with justification

(d) $g(x) \le 0$ for $-4 \le x \le 2$ and $10 \le x \le 12$.

2 : intervals

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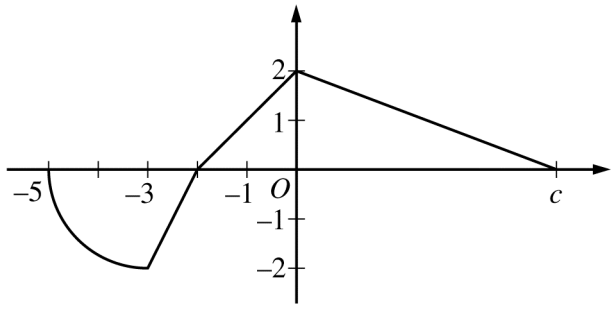

Graph of f

3. The function f is defined on the interval $-5 \le x \le c$, where $c > 0$ and $f(c) = 0$. The graph of f, which consists of three line segments and a quarter of a circle with center $(-3, 0)$ and radius 2, is shown in the figure above.

   (a) Find the average rate of change of f over the interval $[-5, 0]$. Show the computations that lead to your answer.

   (b) For $-5 \le x \le c$, let g be the function defined by $g(x) = \int_{-1}^{x} f(t)\,dt$. Find the x-coordinate of each point of inflection of the graph of g. Justify your answer.

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Continue problem 3 on page 15.

(c) Find the value of c for which the average value of f over the interval $-5 \le x \le c$ is $\frac{1}{2}$.

(d) Assume $c > 3$. The function h is defined by $h(x) = f\left(\frac{x}{2}\right)$. Find $h'(6)$ in terms of c.

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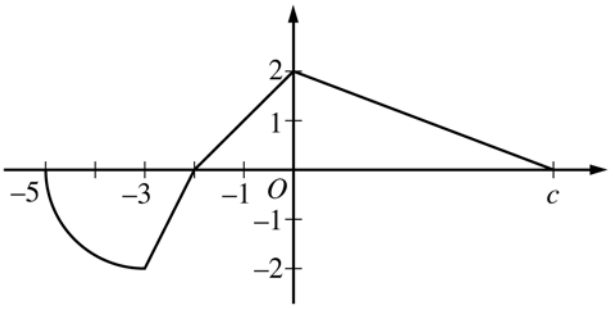
Question 3

The function f is defined on the interval $-5 \le x \le c$, where $c > 0$ and $f(c) = 0$. The graph of f, which consists of three line segments and a quarter of a circle with center $(-3, 0)$ and radius 2, is shown in the figure above.



Graph of f

(a) Find the average rate of change of f over the interval $[-5, 0]$. Show the computations that lead to your answer.

(b) For $-5 \le x \le c$, let g be the function defined by $g(x) = \int_{-1}^{x} f(t)\,dt$. Find the x-coordinate of each point of inflection of the graph of g. Justify your answer.

(c) Find the value of c for which the average value of f over the interval $-5 \le x \le c$ is $\frac{1}{2}$.

(d) Assume $c > 3$. The function h is defined by $h(x) = f\left(\frac{x}{2}\right)$. Find $h'(6)$ in terms of c.

(a) The average rate of change of f over the interval $[-5, 0]$ is $\frac{f(0) - f(-5)}{0 - (-5)} = \frac{2}{5}$.

1 : answer

(b) $g'(x) = f(x)$

The graph of g has a point of inflection at $x = -3$ because $g' = f$ changes from decreasing to increasing at this point.

The graph of g has a point of inflection at $x = 0$ because $g' = f$ changes from increasing to decreasing at this point.

3 : { 1 : $g'(x) = f(x)$; 1 : identifies $x = -3$ and $x = 0$; 1 : justification }

(c) $\frac{1}{c+5}\int_{-5}^{c} f(x)\,dx = \frac{1}{2}$

$\frac{1}{c+5}(-\pi + (-1) + 2 + c) = \frac{1}{2}$

$c = 3 + 2\pi$

3 : { 1 : integral; 1 : equation; 1 : answer }

(d) $h'(x) = \frac{1}{2} f'\left(\frac{x}{2}\right)$

$h'(6) = \frac{1}{2} f'(3) = \frac{1}{2} \cdot \frac{-2}{c} = -\frac{1}{c}$

2 : { 1 : $h'(x)$; 1 : answer }

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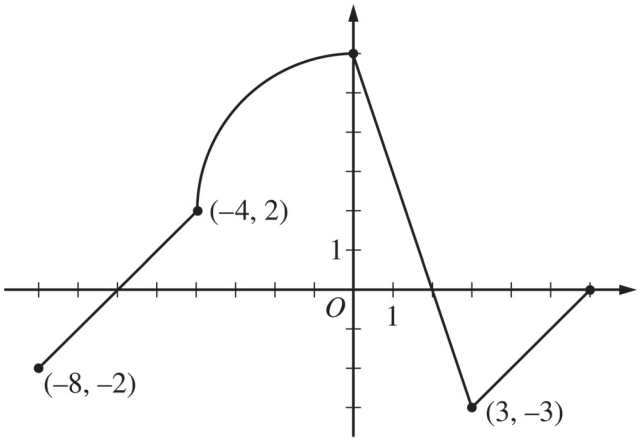

Graph of g

3. A continuous function g is defined on the closed interval $-8 \le x \le 6$. The graph of g, shown above, consists of three line segments and a quarter of a circle centered at the point $(0, 2)$. Let f be the function given by $f(x) = \int_{-8}^{x} g(t)\,dt$.

   (a) Find all values of x in the interval $-8 < x < 6$ at which f has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

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(b) Find $f(0)$.

(c) Find $\lim_{x \to -4} \dfrac{f(x)}{x^2 + 4x}$.

(d) Let h be the function defined by $h(x) = \dfrac{g(x)}{x^2 + 1}$. Find $h'(1)$.

GO ON TO THE NEXT PAGE.

Question 3

(a) $f'(x) = g(x)$

The function f has critical points at $x = -6$ and $x = 2$.

f has a local minimum at $x = -6$ because f' changes from negative to positive at that value.

f has a local maximum at $x = 2$ because f' changes from positive to negative at that value.

3 : { 1 : $f'(x) = g(x)$; 1 : critical points; 1 : classifications with justification }

(b) $f(0) = \int_{-8}^{0} g(t)\,dt = 8 + 4\pi$

1 : answer

(c) $\lim_{x \to -4} f(x) = f(-4) = \int_{-8}^{-4} g(t)\,dt = 0$

$\lim_{x \to -4} \left(x^2 + 4x\right) = 0$

Using L'Hospital's Rule,

$\lim_{x \to -4} \frac{f(x)}{x^2 + 4x} = \lim_{x \to -4} \frac{f'(x)}{2x + 4} = \lim_{x \to -4} \frac{g(x)}{2x + 4} = \frac{2}{-4} = -\frac{1}{2}$

2 : { 1 : L'Hospital's Rule; 1 : answer }

(d) $h'(x) = \frac{\left(x^2 + 1\right)g'(x) - g(x)2x}{\left(x^2 + 1\right)^2}$

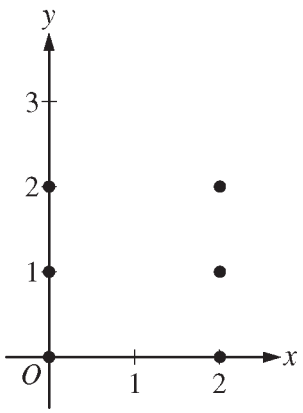
$h'(1) = \frac{\left(1^2 + 1\right)g'(1) - g(1)(2)(1)}{\left(1^2 + 1\right)^2}$

$= \frac{2(-3) - 3(2)}{4} = -3$

3 : { 2 : $h'(x)$; 1 : answer }

4. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

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Question 4

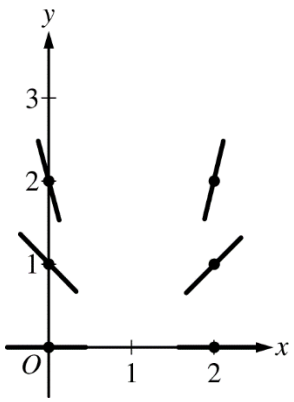
Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,3)} = \frac{3^2}{2-1} = 9$

An equation for the tangent line is $y = 9(x-2) + 3$.

$f(2.1) \approx 9(2.1-2) + 3 = 3.9$

$2 : \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{1}{y^2}\,dy = \frac{1}{x-1}\,dx$

$\int \frac{1}{y^2}\,dy = \int \frac{1}{x-1}\,dx$

$-\frac{1}{y} = \ln|x-1| + C$

$-\frac{1}{3} = \ln|2-1| + C \Rightarrow C = -\frac{1}{3}$

$-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$

$y = \frac{1}{\frac{1}{3} - \ln(x-1)}$

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/5$ [1-2-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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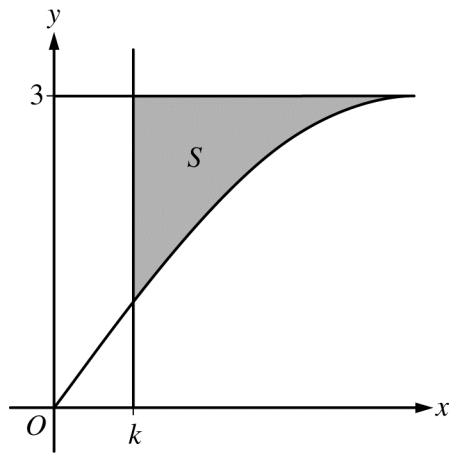

4. Let S be the shaded region in the first quadrant bounded above by the horizontal line $y = 3$, below by the graph of $y = 3\sin x$, and on the left by the vertical line $x = k$, where $0 < k < \frac{\pi}{2}$, as shown in the figure above.

   (a) Find the area of S when $k = \frac{\pi}{3}$.

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Continue problem 4 on page 17.

(b) The area of S is a function of k. Find the rate of change of the area of S with respect to k when $k = \frac{\pi}{6}$.

(c) Region S is revolved about the horizontal line $y = 5$ to form a solid. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid when $k = \frac{\pi}{4}$.

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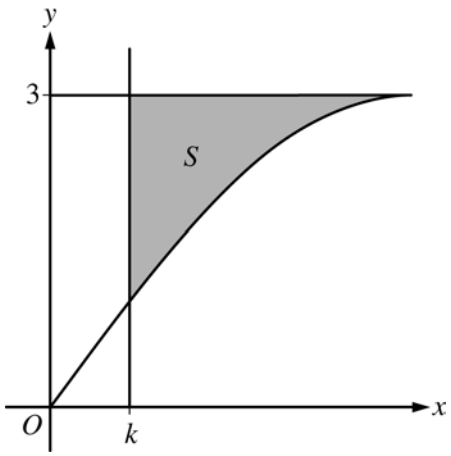
Question 4

Let S be the shaded region in the first quadrant bounded above by the horizontal line $y = 3$, below by the graph of $y = 3\sin x$, and on the left by the vertical line $x = k$, where $0 < k < \frac{\pi}{2}$, as shown in the figure above.



(a) Find the area of S when $k = \frac{\pi}{3}$.

(b) The area of S is a function of k. Find the rate of change of the area of S with respect to k when $k = \frac{\pi}{6}$.

(c) Region S is revolved about the horizontal line $y = 5$ to form a solid. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid when $k = \frac{\pi}{4}$.

(a) $\text{Area} = \int_{\pi/3}^{\pi/2} (3 - 3\sin x)\,dx = \left[3x + 3\cos x\right]_{\pi/3}^{\pi/2}$

$= 3\left[\left(\frac{\pi}{2} + 0\right) - \left(\frac{\pi}{3} + \frac{1}{2}\right)\right] = \frac{\pi}{2} - \frac{3}{2}$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(b) Let $A(k)$ be the area of S.

$A(k) = \int_{k}^{\pi/2} (3 - 3\sin x)\,dx$

$A'(k) = -3 + 3\sin k$

$A'\left(\frac{\pi}{6}\right) = -3 + 3\sin\left(\frac{\pi}{6}\right) = -\frac{3}{2}$

3 : { 1 : expression for area; 1 : expression for $A'(k)$; 1 : answer }

(c) $\text{Volume} = \pi\int_{\pi/4}^{\pi/2}\left[(5 - 3\sin x)^2 - (5 - 3)^2\right]dx$

$= \pi\int_{\pi/4}^{\pi/2}\left[(5 - 3\sin x)^2 - 4\right]dx$

3 : { 2 : integrand; 1 : limits and constant }

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4. Consider the differential equation $\frac{dy}{dx} = (y - 2)(x^2 + 1)$.

(a) Find $y = g(x)$, the particular solution to the given differential equation with initial condition $g(0) = 5$.

(b) For the particular solution $y = g(x)$ found in part (a), find $\lim_{x \to -\infty} g(x)$.

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(c) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 3$.

Find the value of $\frac{d^2y}{dx^2}$ at the point $(1, 3)$. Is the graph of $y = f(x)$ concave up or concave down at the point $(1, 3)$? Give a reason for your answer.

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Question 4

(a) $\dfrac{dy}{dx} = (y-2)\left(x^2+1\right)$

$\displaystyle\int \frac{dy}{y-2} = \int \left(x^2+1\right) dx$

$\ln|y-2| = \dfrac{x^3}{3} + x + C$

$\ln 3 = \dfrac{0^3}{3} + 0 + C \Rightarrow C = \ln 3$

Because $y(0) = 5,\ y > 2,$ so $|y-2| = y-2.$

$y - 2 = 3e^{\frac{x^3}{3}+x}$

$y = 2 + 3e^{\frac{x^3}{3}+x}$

Note: this solution is valid for all real numbers.

5 : { 1 : separation of variables; 2 : antiderivatives; 1 : constant of integration and uses initial condition; 1 : solves for y }

Note: max $3/5$ [1-2-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

(b) $\displaystyle\lim_{x\to-\infty}\left(2 + 3e^{\frac{x^3}{3}+x}\right) = 2$

1 : answer

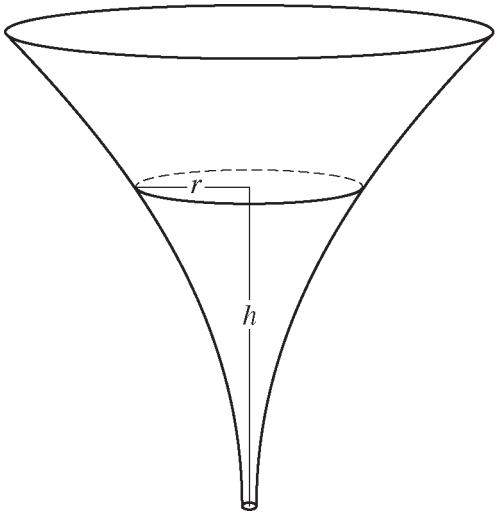
(c) $\left.\dfrac{dy}{dx}\right|_{(1,\,3)} = (3-2)\left(1^2+1\right) = 2$

$\dfrac{d^2y}{dx^2} = \dfrac{dy}{dx}\left(x^2+1\right) + (y-2)(2x)$

$\left.\dfrac{d^2y}{dx^2}\right|_{(1,\,3)} = (2)\left(1^2+1\right) + (3-2)(2) = 6$

Because $\left.\dfrac{d^2y}{dx^2}\right|_{(1,\,3)} > 0$ and $\dfrac{d^2y}{dx^2}$ is continuous, the graph of $y = f(x)$ is concave up at the point $(1, 3)$.

3 : { 2 : $\left.\dfrac{d^2y}{dx^2}\right|_{(1,\,3)}$; 1 : concave up with reason }



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

   (a) Find the average value of the radius of the funnel.

   (b) Find the volume of the funnel.

   (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

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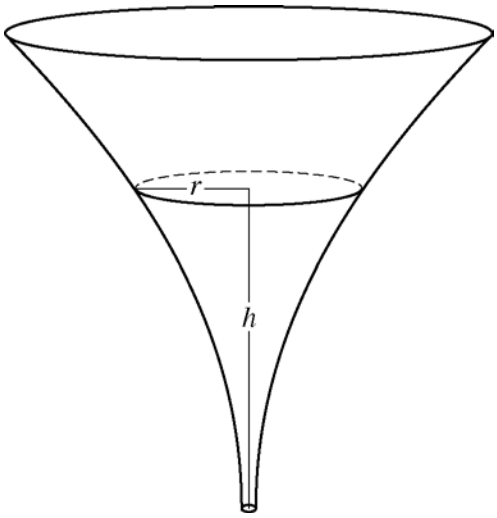
Question 5



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(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

(a) Average radius $= \frac{1}{10}\int_0^{10} \frac{1}{20}\left(3 + h^2\right)\,dh = \frac{1}{200}\left[3h + \frac{h^3}{3}\right]_0^{10}$

$= \frac{1}{200}\left(\left(30 + \frac{1000}{3}\right) - 0\right) = \frac{109}{60}$ in

3 : { 1 : integral; 1 : antiderivative; 1 : answer }

(b) Volume $= \pi\int_0^{10} \left(\left(\frac{1}{20}\right)\left(3 + h^2\right)\right)^2\,dh = \frac{\pi}{400}\int_0^{10}\left(9 + 6h^2 + h^4\right)\,dh$

$= \frac{\pi}{400}\left[9h + 2h^3 + \frac{h^5}{5}\right]_0^{10}$

$= \frac{\pi}{400}\left(\left(90 + 2000 + \frac{100000}{5}\right) - 0\right) = \frac{2209\pi}{40}$ in^3

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(c) $\frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt}$

$-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{5}\cdot\frac{10}{3} = -\frac{2}{3}$ in/sec

3 : { 2 : chain rule; 1 : answer }

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5. For $0 \le t \le 24$ hours, the temperature inside a refrigerator in a kitchen is given by the function W that satisfies the differential equation $\frac{dW}{dt} = \frac{3\cos t}{2W}$. $W(t)$ is measured in degrees Celsius (°C), and t is measured in hours. At time $t = 0$ hours, the temperature inside the refrigerator is 3°C.

   (a) Write an equation for the line tangent to the graph of $y = W(t)$ at the point where $t = 0$. Use the equation to approximate the temperature inside the refrigerator at $t = 0.4$ hour.

   (b) Find $y = W(t)$, the particular solution to the differential equation with initial condition $W(0) = 3$.

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Continue problem 5 on page 19.

(c) The temperature in the kitchen remains constant at 20°C for $0 \le t \le 24$. The cost of operating the refrigerator accumulates at the rate of \$0.001 per hour for each degree that the temperature in the kitchen exceeds the temperature inside the refrigerator. Write, but do not evaluate, an expression involving an integral that can be used to find the cost of operating the refrigerator for the 24-hour interval.

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Question 5

For $0 \le t \le 24$ hours, the temperature inside a refrigerator in a kitchen is given by the function W that satisfies the differential equation $\frac{dW}{dt} = \frac{3\cos t}{2W}$. $W(t)$ is measured in degrees Celsius (°C), and t is measured in hours. At time $t = 0$ hours, the temperature inside the refrigerator is 3°C.

(a) Write an equation for the line tangent to the graph of $y = W(t)$ at the point where $t = 0$. Use the equation to approximate the temperature inside the refrigerator at $t = 0.4$ hour.

(b) Find $y = W(t)$, the particular solution to the differential equation with initial condition $W(0) = 3$.

(c) The temperature in the kitchen remains constant at 20°C for $0 \le t \le 24$. The cost of operating the refrigerator accumulates at the rate of \$0.001 per hour for each degree that the temperature in the kitchen exceeds the temperature inside the refrigerator. Write, but do not evaluate, an expression involving an integral that can be used to find the cost of operating the refrigerator for the 24-hour interval.

(a) $\left.\frac{dW}{dt}\right|_{(t, W)=(0, 3)} = \frac{3\cos 0}{2(3)} = \frac{1}{2}$

An equation for the tangent line is $y = \frac{1}{2}t + 3$.

$W(0.4) \approx \frac{1}{2}(0.4) + 3 = 3.2$°C

$2 : \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(b) $2W\,dW = 3\cos t\,dt$

$\int 2W\,dW = \int 3\cos t\,dt$

$W^2 = 3\sin t + C$

$3^2 = 3\sin 0 + C \Rightarrow C = 9$

$W^2 = 3\sin t + 9$

Since $W(0) = 3$, $W = \sqrt{3\sin t + 9}$ for $0 \le t \le 24$.

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } W \end{cases}$

Note: max $3/5$ [1-2-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

(c) $0.001\int_0^{24}(20 - W(t))\,dt$ dollars

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

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5. The function f is defined by

$$f(x) = \begin{cases} 3x^2 + 2x & \text{for } x \le 0 \\ e^{2x} + 2 & \text{for } x > 0. \end{cases}$$

(a) Is f continuous at $x = 0$? Justify your answer.

(b) Find $f'(-2)$ and $f'(3)$.

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(c) Explain why $f'(0)$ does not exist.

(d) Let g be the function given by $g(x) = \int_{-1}^{x} f(t)\,dt$. Find $g(1)$.

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Question 5

(a) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \left(3x^2 + 2x\right) = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(e^{2x} + 2\right) = 3$

Therefore, $\lim_{x\to 0} f(x)$ does not exist, and f is not continuous at $x = 0$.

2 : { 1 : considers one-sided limits; 1 : answer with justification }

(b) $f'(-2) = (6x + 2)\big|_{x=-2} = 6(-2) + 2 = -10$

$f'(3) = 2e^{2x}\Big|_{x=3} = 2e^{(2)(3)} = 2e^6$

2 : { 1 : $f'(-2)$; 1 : $f'(3)$ }

(c) $f'(0)$ does not exist because f is not continuous at $x = 0$.

1 : explanation

(d) $g(1) = \int_{-1}^{1} f(t)\,dt = \int_{-1}^{0} \left(3t^2 + 2t\right) dt + \int_{0}^{1} \left(e^{2t} + 2\right) dt$

$= \left[t^3 + t^2\right]_{-1}^{0} + \left[\frac{e^{2t}}{2} + 2t\right]_{0}^{1}$

$= [(0 + 0) - (-1 + 1)] + \left[\left(\frac{e^2}{2} + 2\right) - \left(\frac{1}{2} + 0\right)\right]$

$= \frac{e^2}{2} + \frac{3}{2}$

4 : { 1 : integrals; 2 : antiderivatives; 1 : answer }

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

   (a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

   (b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

   (c) Evaluate $\displaystyle\int_1^3 f''(2x)\,dx$.

STOP

END OF EXAM

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Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

(a) $k(3) = f(g(3)) = f(6) = 4$
$k'(3) = f'(g(3)) \cdot g'(3) = f'(6) \cdot 2 = 5 \cdot 2 = 10$

An equation for the tangent line is $y = 10(x - 3) + 4$.

3 : { 2 : slope at $x = 3$; 1 : equation for tangent line }

(b) $h'(1) = \dfrac{f(1) \cdot g'(1) - g(1) \cdot f'(1)}{(f(1))^2}$
$= \dfrac{(-6) \cdot 8 - 2 \cdot 3}{(-6)^2} = \dfrac{-54}{36} = -\dfrac{3}{2}$

3 : { 2 : expression for $h'(1)$; 1 : answer }

(c) $\int_1^3 f''(2x)\,dx = \dfrac{1}{2}\left[f'(2x)\right]_1^3 = \dfrac{1}{2}[f'(6) - f'(2)]$
$= \dfrac{1}{2}[5 - (-2)] = \dfrac{7}{2}$

3 : { 2 : antiderivative ; 1 : answer }

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$$f(x) = \begin{cases} 10 - 2x - x^2 & \text{for } x \le 1 \\ 3 + 4e^{x-1} & \text{for } x > 1 \end{cases}$$

6. Let f be the function defined above.

(a) Is f continuous at $x = 1$? Why or why not?

(b) Find the absolute minimum value and the absolute maximum value of f on the closed interval $-2 \le x \le 2$. Show the analysis that leads to your conclusion.

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Continue problem 6 on page 21.

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(c) Find the value of $\int_0^2 f(x)\,dx$.

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Question 6

$$f(x) = \begin{cases} 10 - 2x - x^2 & \text{for } x \le 1 \\ 3 + 4e^{x-1} & \text{for } x > 1 \end{cases}$$

Let f be the function defined above.

(a) Is f continuous at $x = 1$? Why or why not?

(b) Find the absolute minimum value and the absolute maximum value of f on the closed interval $-2 \le x \le 2$. Show the analysis that leads to your conclusion.

(c) Find the value of $\int_0^2 f(x)\,dx$.

(a) $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \left(10 - 2x - x^2\right) = 7$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \left(3 + 4e^{x-1}\right) = 7$

Therefore, $\lim\limits_{x \to 1} f(x) = 7$.

Since $\lim\limits_{x \to 1} f(x) = f(1)$, f is continuous at $x = 1$.

$2 : \begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with explanation} \end{cases}$

(b) For $x < 1$, $f'(x) = -2 - 2x$.

$f'(x) = 0 \Rightarrow x = -1$

For $x > 1$, $f'(x) = 4e^{x-1} \neq 0$.

At $x = 1$, $f'(x)$ is not defined.

x	$f(x)$
-2	10
-1	11
1	7
2	$3 + 4e$

The absolute minimum value of f on the interval $-2 \le x \le 2$ is 7. The absolute maximum value of f on the interval $-2 \le x \le 2$ is $3 + 4e$.

$4 : \begin{cases} 1 : \frac{d}{dx}\left(10 - 2x - x^2\right) \text{ and } \frac{d}{dx}\left(3 + 4e^{x-1}\right) \\ 1 : \text{identifies } x = -1 \text{ and } x = 1 \text{ as critical points} \\ 1 : \text{evaluates } f \text{ at endpoints} \\ 1 : \text{answers with analysis} \end{cases}$

(c) $\int_0^2 f(x)\,dx = \int_0^1 \left(10 - 2x - x^2\right) dx + \int_1^2 \left(3 + 4e^{x-1}\right) dx$

$= \left[10x - x^2 - \frac{1}{3}x^3\right]_0^1 + \left[3x + 4e^{x-1}\right]_1^2$

$= \left(10 - 1 - \frac{1}{3}\right) + \left[(6 + 4e) - (3 + 4)\right]$

$= \frac{23}{3} + 4e$

$3 : \begin{cases} 1 : \text{sum of integrals} \\ 1 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

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6. A hive contains 35 hundred bees at time $t = 0$. During the time interval $0 \le t \le 4$ hours, bees enter the hive at a rate modeled by $E(t) = 16t - 3t^2$, where $E(t)$ is measured in hundreds of bees per hour. During the same time interval, bees leave the hive at a rate modeled by $L(t) = -2t + 15$, where $L(t)$ is measured in hundreds of bees per hour.

(a) How many bees leave the hive during the time interval $0 \le t \le 2$?

(b) Write an expression involving one or more integrals for the total number of bees, in hundreds, in the hive at time t for $0 \le t \le 4$. Find the total number of bees in the hive at $t = 4$.

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(c) Find the minimum number of bees in the hive for $0 \le t \le 4$. Justify your answer.

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Question 6

(a) $\int_0^2 L(t)\,dt = \int_0^2 (-2t + 15)\,dt = \left[-t^2 + 15t\right]_0^2$
$= -4 + 30 = 26$

26 hundred bees leave the hive during the time interval $0 \le t \le 2$.

3 : { 1 : integral; 1 : antiderivative; 1 : answer }

(b) The total number of bees, in hundreds, in the hive at time t is $35 + \int_0^t (E(x) - L(x))\,dx$.

$$\begin{aligned} 35 + \int_0^4 (E(x) - L(x))\,dx &= 35 + \int_0^4 \left(\left(16x - 3x^2\right) - (-2x + 15)\right) dx \\ &= 35 + \int_0^4 \left(-3x^2 + 18x - 15\right) dx \\ &= 35 + \left[-x^3 + 9x^2 - 15x\right]_0^4 \\ &= 35 + (-64 + 144 - 60) \\ &= 55 \end{aligned}$$

55 hundred bees are in the hive at time $t = 4$.

3 : { 1 : expression for total; 1 : antiderivative; 1 : answer }

(c) Let $B(t)$ be the total number of bees, in hundreds, in the hive at time t, for $0 \le t \le 4$.

$B(t) = 35 + \int_0^t (E(x) - L(x))\,dx$

$B'(t) = E(t) - L(t) = -3t^2 + 18t - 15 = -3(t - 1)(t - 5)$

$B'(t) = 0 \Rightarrow t = 1, t = 5$

t	$B(t)$
0	35
1	28
4	55

The minimum numbers of bees in the hive for $0 \le t \le 4$ is 28 hundred bees.

3 : { 1 : sets $E(t) - L(t) = 0$; 1 : answer; 1 : justification }

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

   (a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

   (b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

   (c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

   (d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

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Question 1

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1 : units in parts (a), (c), and (d)

(a) Volume $= \int_0^{10} A(h)\,dh$

$\approx (2-0)\cdot A(0) + (5-2)\cdot A(2) + (10-5)\cdot A(5)$

$= 2\cdot 50.3 + 3\cdot 14.4 + 5\cdot 6.5$

$= 176.3$ cubic feet

2 : { 1 : left Riemann sum; 1 : approximation }

(b) The approximation in part (a) is an overestimate because a left Riemann sum is used and A is decreasing.

1 : overestimate with reason

(c) $\int_0^{10} f(h)\,dh = 101.325338$

The volume is 101.325 cubic feet.

2 : { 1 : integral; 1 : answer }

(d) Using the model, $V(h) = \int_0^h f(x)\,dx$.

$$\left.\frac{dV}{dt}\right|_{h=5} = \left[\frac{dV}{dh}\cdot\frac{dh}{dt}\right]_{h=5}$$

$$= \left[f(h)\cdot\frac{dh}{dt}\right]_{h=5}$$

$$= f(5)\cdot 0.26 = 1.694419$$

When $h = 5$, the volume of water is changing at a rate of 1.694 cubic feet per minute.

3 : { 2 : $\frac{dV}{dt}$; 1 : answer }

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1. A particle moves along the x-axis so that its velocity at time t is given by $v(t) = \dfrac{t^6 - 13t^4 + 12}{10t^3 + 3}$. At time $t = 0$, the initial position of the particle is $x = 7$.

(a) Find the acceleration of the particle at time $t = 5.1$.

(b) Find all values of t in the interval $0 \le t \le 2$ for which the speed of the particle is 1.

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Continue question 1 on page 5.

(c) Find the position of the particle at time $t = 4$. Is the particle moving toward the origin or away from the origin at time $t = 4$? Justify your answer.

(d) During the time interval $0 < t \leq 4$, does the particle return to its initial position? Give a reason for your answer.

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Question 1

(a) $v'(5.1) = 6.491816$

The acceleration of the particle at time $t = 5.1$ is 6.492 (or 6.491).

1 : answer

(b) $|v(t)| = 1$ for $0 \le t \le 2 \Rightarrow t = 0.771829, t = 1.400556$

The speed of the particle is 1 at times $t = 0.772$ (or 0.771) and $t = 1.401$ (or 1.400).

2 : { 1 : considers $|v(t)| = 1$; 1 : answer }

(c) Let $x(t)$ be the position of the particle at time t.

$x(4) = 7 + \int_0^4 v(t)\,dt = 6.711558$

The position of the particle at time $t = 4$ is 6.712 (or 6.711).

The particle is moving away from the origin because $x(4) > 0$ and $v(4) = 1.213064 > 0$.

4 : { 1 : integral; 1 : uses initial condition; 1 : position; 1 : movement relative to the origin, with justification }

(d) $x'(t) = v(t) \Rightarrow x(t)$ is continuous on $0 < t \le 4$.

$x(1) = 7 + \int_0^1 v(t)\,dt = 9.403593$

From part (c), $x(4) = 6.712$.

From time $t = 1$ to $t = 4$, the particle moves from $x = 9.404$ to $x = 6.712$. By the Intermediate Value Theorem, the particle must return to $x = 7$ during the time interval.

2 : { 1 : sandwiches initial position; 1 : answer with reason }

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2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

(d) How many pounds of bananas are on the display table at time $t = 8$?

END OF PART A OF SECTION II

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Question 2

(a) $\int_0^2 f(t)\,dt = 20.051175$

20.051 pounds of bananas are removed from the display table during the first 2 hours the store is open.

2 : { 1 : integral; 1 : answer }

(b) $f'(7) = -8.120$ (or -8.119)

After the store has been open 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 (or 8.119) pounds per hour per hour.

2 : { 1 : value; 1 : meaning }

(c) $g(5) - f(5) = -2.263103 < 0$

Since $g(5) - f(5) < 0$, the number of pounds of bananas on the display table is decreasing at time $t = 5$.

2 : { 1 : considers $f(5)$ and $g(5)$; 1 : answer with reason }

(d) $50 + \int_3^8 g(t)\,dt - \int_0^8 f(t)\,dt = 23.347396$

23.347 pounds of bananas are on the display table at time $t = 8$.

3 : { 2 : integrals; 1 : answer }

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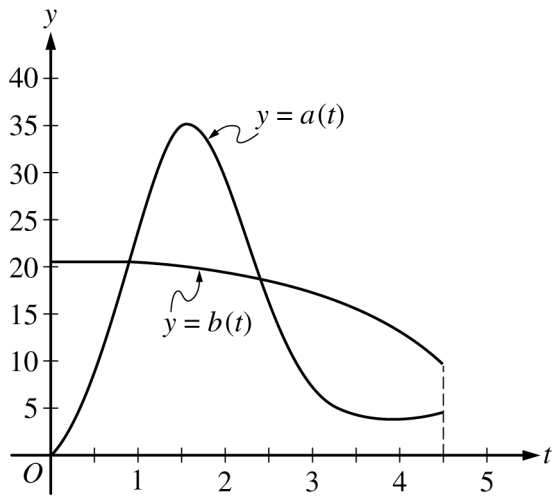

2. During the time interval $0 \le t \le 4.5$ hours, water flows into tank A at a rate of $a(t) = (2t - 5) + 5e^{2 \sin t}$ liters per hour. During the same time interval, water flows into tank B at a rate of $b(t)$ liters per hour. Both tanks are empty at time $t = 0$. The graphs of $y = a(t)$ and $y = b(t)$, shown in the figure above, intersect at $t = k$ and $t = 2.416$.

(a) How much water will be in tank A at time $t = 4.5$?

(b) During the time interval $0 \le t \le k$ hours, water flows into tank B at a constant rate of 20.5 liters per hour. What is the difference between the amount of water in tank A and the amount of water in tank B at time $t = k$?

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Continue question 2 on page 7.

(c) The area of the region bounded by the graphs of $y = a(t)$ and $y = b(t)$ for $k \le t \le 2.416$ is 14.470. How much water is in tank B at time $t = 2.416$?

(d) During the time interval $2.7 \le t \le 4.5$ hours, the rate at which water flows into tank B is modeled by $w(t) = 21 - \dfrac{30t}{(t-8)^2}$ liters per hour. Is the difference $w(t) - a(t)$ increasing or decreasing at time $t = 3.5$? Show the work that leads to your answer.

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Question 2

(a) $\int_0^{4.5} a(t)\,dt = 66.532128$

At time $t = 4.5$, tank A contains 66.532 liters of water.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $a(k) = 20.5 \Rightarrow k = 0.892040$

$\int_0^k (20.5 - a(t))\,dt = 10.599191$

At time $t = k$, the difference in the amounts of water in the tanks is 10.599 liters.

$3 : \begin{cases} 1 : \text{sets } a(k) = 20.5 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{2.416} b(t)\,dt = \int_0^k b(t)\,dt + \int_k^{2.416} b(t)\,dt$

$\int_0^k b(t)\,dt = 20.5 \cdot k = 18.286826$

On $k < t < 2.416$, tank A receives $\int_k^{2.416} a(t)\,dt = 44.497051$ liters of water, which is 14.470 more liters of water than tank B.
Therefore, $\int_k^{2.416} b(t)\,dt = \int_k^{2.416} a(t)\,dt - 14.470 = 30.027051$.

$\int_0^k b(t)\,dt + \int_k^{2.416} b(t)\,dt = 48.313876$

At time $t = 2.416$, tank B contains 48.314 (or 48.313) liters of water.

$2 : \begin{cases} 1 : \int_k^{2.416} a(t)\,dt \\ 1 : \text{answer} \end{cases}$

(d) $w'(3.5) - a'(3.5) = -1.14298 < 0$

The difference $w(t) - a(t)$ is decreasing at $t = 3.5$.

$2 : \begin{cases} 1 : w'(3.5) - a'(3.5) < 0 \\ 1 : \text{conclusion} \end{cases}$

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CALCULUS BC

SECTION II, Part B

Time—1 hour

Number of questions—4

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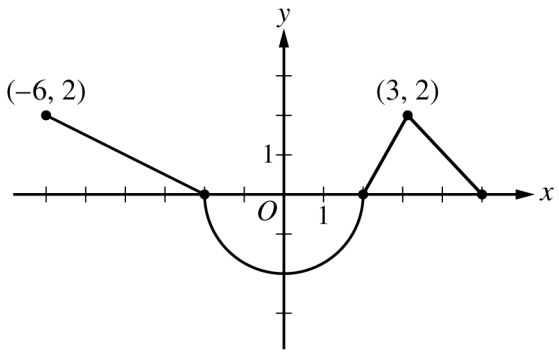

Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

   (a) Find the values of $f(-6)$ and $f(5)$.

   (b) On what intervals is f increasing? Justify your answer.

   (c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

   (d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

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Question 3

(a) $f(-6) = f(-2) + \int_{-2}^{-6} f'(x)\,dx = 7 - \int_{-6}^{-2} f'(x)\,dx = 7 - 4 = 3$

$f(5) = f(-2) + \int_{-2}^{5} f'(x)\,dx = 7 - 2\pi + 3 = 10 - 2\pi$

3 : $\begin{cases} 1 : \text{uses initial condition} \\ 1 : f(-6) \\ 1 : f(5) \end{cases}$

(b) $f'(x) > 0$ on the intervals $[-6, -2)$ and $(2, 5)$.
Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

2 : answer with justification

(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint.

$f'(x) = 0 \Rightarrow x = -2, x = 2$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$.

2 : $\begin{cases} 1 : \text{considers } x = 2 \\ 1 : \text{answer with justification} \end{cases}$

(d) $f''(-5) = \dfrac{2 - 0}{-6 - (-2)} = -\dfrac{1}{2}$

$\lim\limits_{x \to 3^-} \dfrac{f'(x) - f'(3)}{x - 3} = 2$ and $\lim\limits_{x \to 3^+} \dfrac{f'(x) - f'(3)}{x - 3} = -1$

$f''(3)$ does not exist because
$\lim\limits_{x \to 3^-} \dfrac{f'(x) - f'(3)}{x - 3} \neq \lim\limits_{x \to 3^+} \dfrac{f'(x) - f'(3)}{x - 3}$.

2 : $\begin{cases} 1 : f''(-5) \\ 1 : f''(3) \text{ does not exist, with explanation} \end{cases}$

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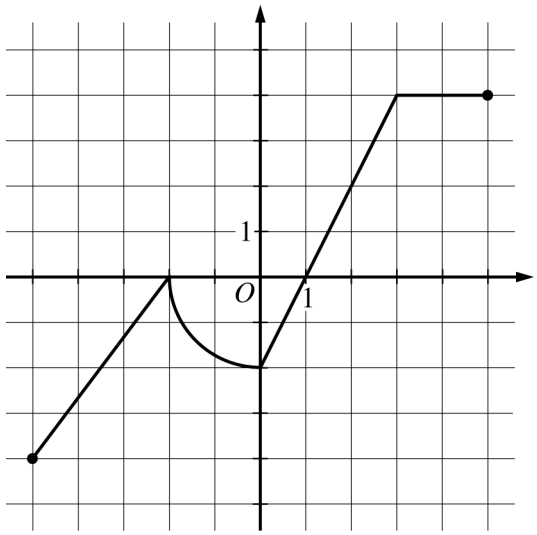

Graph of f

3. The graph of the function f, consisting of three line segments and a quarter of a circle, is shown above. Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the average rate of change of g from $x = -5$ to $x = 5$.

(b) Find the instantaneous rate of change of g with respect to x at $x = 3$, or state that it does not exist.

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Continue question 3 on page 15.

(c) On what open intervals, if any, is the graph of g concave up? Justify your answer.

(d) Find all x-values in the interval $-5 < x < 5$ at which g has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

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Question 3

(a) $\frac{g(5)-g(-5)}{5-(-5)} = \frac{12-(\pi+7)}{10} = \frac{5-\pi}{10}$	3 : $\begin{cases} 1 : \text{difference quotient} \\ 2 : \text{answer} \end{cases}$
(b) $g'(x) = f(x)$ $g'(3) = f(3) = 4$ The instantaneous rate of change of g at $x = 3$ is 4.	1 : answer
(c) The graph of g is concave up on $-5 < x < -2$ and $0 < x < 3$, because $g'(x) = f(x)$ is increasing on these intervals.	2 : intervals with justification
(d) $g'(x) = f(x)$ is defined at all x with $-5 < x < 5$. $g'(x) = f(x) = 0$ at $x = -2$ and $x = 1$. Therefore, g has critical points at $x = -2$ and $x = 1$. g has neither a local maximum nor a local minimum at $x = -2$ because g' does not change sign there. g has a local minimum at $x = 1$ because g' changes from negative to positive there.	3 : $\begin{cases} 1 : \text{considers } f(x) = 0 \\ 1 : \text{critical points at } x = -2 \text{ and } x = 1 \\ 1 : \text{answers with justifications} \end{cases}$

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4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

   (a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

   (b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

   (c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

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Question 4

(a) $H'(0) = -\frac{1}{4}(91 - 27) = -16$

$H(0) = 91$

An equation for the tangent line is $y = 91 - 16t$.

The internal temperature of the potato at time $t = 3$ minutes is approximately $91 - 16 \cdot 3 = 43$ degrees Celsius.

$3 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

(b) $\frac{d^2H}{dt^2} = -\frac{1}{4}\frac{dH}{dt} = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H - 27) = \frac{1}{16}(H - 27)$

$H > 27$ for $t > 0 \Rightarrow \frac{d^2H}{dt^2} = \frac{1}{16}(H - 27) > 0$ for $t > 0$

Therefore, the graph of H is concave up for $t > 0$. Thus, the answer in part (a) is an underestimate.

1 : underestimate with reason

(c) $\frac{dG}{(G - 27)^{2/3}} = -dt$

$\int \frac{dG}{(G - 27)^{2/3}} = \int (-1)\, dt$

$3(G - 27)^{1/3} = -t + C$

$3(91 - 27)^{1/3} = 0 + C \Rightarrow C = 12$

$3(G - 27)^{1/3} = 12 - t$

$G(t) = 27 + \left(\frac{12 - t}{3}\right)^3$ for $0 \le t < 10$

The internal temperature of the potato at time $t = 3$ minutes is $27 + \left(\frac{12 - 3}{3}\right)^3 = 54$ degrees Celsius.

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{equation involving } G \text{ and } t \\ 1 : G(t) \text{ and } G(3) \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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x	0	1	2	3	4	5	6
$f'(x)$	4	3.5	2	0.8	1.7	5.8	7

4. The function f satisfies $f(0) = 20$. The first derivative of f satisfies the inequality $0 \le f'(x) \le 7$ for all x in the closed interval $[0, 6]$. Selected values of f' are shown in the table above. The function f has a continuous second derivative for all real numbers.

   (a) Use a midpoint Riemann sum with three subintervals of equal length indicated by the data in the table to approximate the value of $f(6)$.

   (b) Determine whether the actual value of $f(6)$ could be 70. Explain your reasoning.

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Continue question 4 on page 17.

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(c) Evaluate $\int_2^4 f''(x)\,dx$.

(d) Find $\lim_{x \to 0} \dfrac{f(x) - 20e^x}{0.5f(x) - 10}$.

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Question 4

(a) $\int_0^6 f'(x)\,dx \approx 2 \cdot 3.5 + 2 \cdot 0.8 + 2 \cdot 5.8 = 20.2$

$f(6) - f(0) = \int_0^6 f'(x)\,dx$

$f(6) = f(0) + \int_0^6 f'(x)\,dx \approx 20 + 20.2 = 40.2$

3 : { 1 : midpoint sum; 1 : Fundamental Theorem of Calculus; 1 : answer }

(b) Since $f'(x) \le 7$, $\int_0^6 f'(x)\,dx \le 6 \cdot 7 = 42$.

$f(6) - f(0) \le 42 \Rightarrow f(6) \le 20 + 42 = 62$

Therefore, the actual value of $f(6)$ could not be 70.

2 : { 1 : integral bound; 1 : answer with reasoning }

(c) $\int_2^4 f''(x)\,dx = f'(4) - f'(2) = 1.7 - 2 = -0.3$

2 : { 1 : Fundamental Theorem of Calculus; 1 : answer }

(d) $\lim_{x \to 0} \left(f(x) - 20e^x \right) = 0$

$\lim_{x \to 0} (0.5f(x) - 10) = 0$

Using L'Hospital's Rule,

$\lim_{x \to 0} \frac{f(x) - 20e^x}{0.5f(x) - 10} = \lim_{x \to 0} \frac{f'(x) - 20e^x}{0.5f'(x)} = \frac{4 - 20}{0.5(4)} = -8$

2 : { 1 : L'Hospital's Rule; 1 : answer }

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5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

   (a) For $0 \le t \le 8$, when is particle P moving to the left?

   (b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

   (c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

   (d) Find the position of particle Q the first time it changes direction.

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Question 5

(a) $x_P'(t) = \dfrac{2t-2}{t^2-2t+10} = \dfrac{2(t-1)}{t^2-2t+10}$

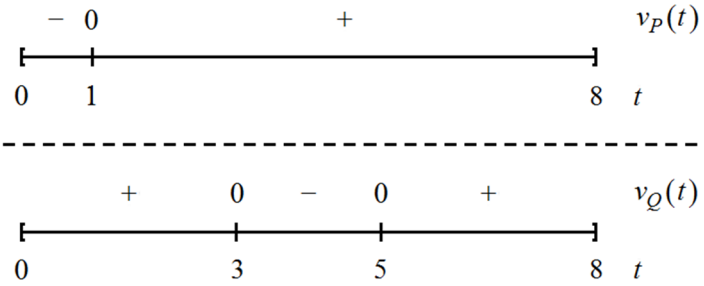
$t^2 - 2t + 10 > 0$ for all t.

$x_P'(t) = 0 \Rightarrow t = 1$
$x_P'(t) < 0$ for $0 \le t < 1$.

Therefore, the particle is moving to the left for $0 \le t < 1$.

$2 : \begin{cases} 1 : x_P'(t) \\ 1 : \text{interval} \end{cases}$

(b) $v_Q(t) = (t-5)(t-3)$
$v_Q(t) = 0 \Rightarrow t = 3, t = 5$



Both particles move in the same direction for $1 < t < 3$ and $5 < t \le 8$ since $v_P(t) = x_P'(t)$ and $v_Q(t)$ have the same sign on these intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{analysis using } v_P(t) \text{ and } v_Q(t) \end{cases}$

Note: $1/2$ if only one interval with analysis

Note: $0/2$ if no analysis

(c) $a_Q(t) = v_Q'(t) = 2t - 8$
$a_Q(2) = 2 \cdot 2 - 8 = -4$

$a_Q(2) < 0$ and $v_Q(2) = 3 > 0$

At time $t = 2$, the speed of the particle is decreasing because velocity and acceleration have opposite signs.

$2 : \begin{cases} 1 : a_Q(2) \\ 1 : \text{speed decreasing with reason} \end{cases}$

(d) Particle Q first changes direction at time $t = 3$.

$$x_Q(3) = x_Q(0) + \int_0^3 v_Q(t)\,dt = 5 + \int_0^3 \left(t^2 - 8t + 15\right) dt$$
$$= 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t\right]_{t=0}^{t=3} = 5 + (9 - 36 + 45) = 23$$

$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

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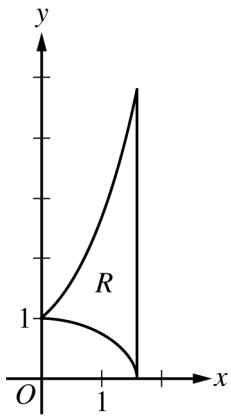

5. Let R be the region in the first quadrant enclosed by the graph of $f(x) = \sqrt{\cos x}$, the graph of $g(x) = e^x$, and the vertical line $x = \dfrac{\pi}{2}$, as shown in the figure above.

   (a) Write, but do not evaluate, an integral expression that gives the area of R.

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Continue question 5 on page 19.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Region R is the base of a solid whose cross sections perpendicular to the x-axis are semicircles with diameters on the xy-plane. Write, but do not evaluate, an integral expression that gives the volume of this solid.

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Question 5

(a) Area $= \int_0^{\pi/2} (g(x) - f(x))\, dx$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \end{cases}$

(b) Volume $= \pi \int_0^{\pi/2} \left((g(x))^2 - (f(x))^2 \right) dx$

$= \pi \int_0^{\pi/2} \left(\left(e^x\right)^2 - \left(\sqrt{\cos x}\right)^2 \right) dx$

$= \pi \int_0^{\pi/2} \left(e^{2x} - \cos x\right) dx = \pi \left[\frac{1}{2} e^{2x} - \sin x \right]_{x=0}^{x=\pi/2}$

$= \pi \left(\frac{1}{2} e^{\pi} - \sin \frac{\pi}{2} - \left(\frac{1}{2} - 0 \right) \right) = \pi \left(\frac{1}{2} e^{\pi} - \frac{3}{2} \right)$

$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \frac{1}{2} \int_0^{\pi/2} \pi \left(\frac{g(x) - f(x)}{2} \right)^2 dx$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{constant} \end{cases}$

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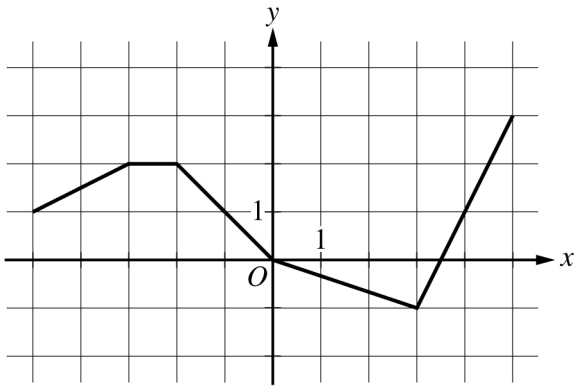
x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

STOP

END OF EXAM

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Question 6

(a) $f'(x) = -2\sin(2x) + \cos x\, e^{\sin x}$

$f'(\pi) = -2\sin(2\pi) + \cos\pi\, e^{\sin\pi} = -1$

$2 : f'(\pi)$

(b) $k'(x) = h'(f(x)) \cdot f'(x)$

$k'(\pi) = h'(f(\pi)) \cdot f'(\pi) = h'(2) \cdot (-1)$

$= \left(-\frac{1}{3}\right)(-1) = \frac{1}{3}$

$2 : \begin{cases} 1 : k'(x) \\ 1 : k'(\pi) \end{cases}$

(c) $m'(x) = -2g'(-2x) \cdot h(x) + g(-2x) \cdot h'(x)$

$m'(2) = -2g'(-4) \cdot h(2) + g(-4) \cdot h'(2)$

$= -2(-1)\left(-\frac{2}{3}\right) + 5\left(-\frac{1}{3}\right) = -3$

$3 : \begin{cases} 2 : m'(x) \\ 1 : m'(2) \end{cases}$

(d) g is differentiable. $\Rightarrow$ g is continuous on the interval $[-5, -3]$.

$\frac{g(-3) - g(-5)}{-3 - (-5)} = \frac{2 - 10}{2} = -4$

Therefore, by the Mean Value Theorem, there is a least one value c, $-5 < c < -3$, such that $g'(c) = -4$.

$2 : \begin{cases} 1 : \frac{g(-3) - g(-5)}{-3 - (-5)} \\ 1 : \text{justification, using Mean Value Theorem} \end{cases}$

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6. For $0 \le t \le 6$ seconds, a screen saver on a computer screen shows two circles that start as dots and expand outward.

(a) At the instant that the first circle has a radius of 9 centimeters, the radius is increasing at a rate of $\frac{3}{2}$ centimeters per second. Find the rate at which the area of the circle is changing at that instant. Indicate units of measure.

(b) The radius of the first circle is modeled by $w(t) = 12 - 12e^{-0.5t}$ for $0 \le t \le 6$, where $w(t)$ is measured in centimeters and t is measured in seconds. At what time t is the radius of the circle increasing at a rate of 3 centimeters per second?

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Continue question 6 on page 21.

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(c) A model for the radius of the second circle is given by the function f for $0 \le t \le 6$, where $f(t)$ is measured in centimeters and t is measured in seconds. The rate of change of the radius of the second circle is given by $f'(t) = t^2 - 4t + 4$. Based on this model, by how many centimeters does the radius of the second circle increase from time $t = 0$ to $t = 3$?

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Question 6

(a) $A = \pi r^2$

$\dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$

$\left.\dfrac{dA}{dt}\right|_{r=9} = 2\pi \cdot 9 \cdot \dfrac{3}{2} = 27\pi$

When the radius is 9 centimeters, the area is changing at a rate of 27π cm^2/sec.

$3 : \begin{cases} 1 : \dfrac{dA}{dt} \\ 1 : \text{answer} \\ 1 : \text{units} \end{cases}$

(b) $w'(t) = (-12)(-0.5)e^{-0.5t} = 6e^{-0.5t}$

$6e^{-0.5t} = 3 \Rightarrow e^{-0.5t} = \dfrac{1}{2} \Rightarrow -0.5t = \ln\left(\dfrac{1}{2}\right) \Rightarrow t = 2\ln 2$

The radius is increasing at a rate of 3 centimeters per second at time $t = 2\ln 2$ seconds.

$3 : \begin{cases} 1 : w'(t) \\ 1 : \text{sets } w'(t) = 3 \\ 1 : \text{answer} \end{cases}$

(c) $\displaystyle\int_0^3 \left(t^2 - 4t + 4\right) dt = \left[\frac{1}{3}t^3 - 2t^2 + 4t\right]_{t=0}^{t=3}$

$= \left(\dfrac{1}{3} \cdot 3^3 - 2 \cdot 3^2 + 4 \cdot 3\right) - \left(\dfrac{1}{3} \cdot 0^3 - 2 \cdot 0^2 + 4 \cdot 0\right)$

$= 9 - 18 + 12 = 3$

The radius increases by 3 centimeters from time $t = 0$ to time $t = 3$ seconds.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

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Question 1

(a) $\int_0^{300} r(t)\,dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \le t \le 300$.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $20 + \int_0^{300} (r(t) - 0.7)\,dt = 20 + \int_0^{300} r(t)\,dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

$2 : \begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is $300 + \frac{80}{0.7} = 414.286$ (or 414.285) seconds.

1 : answer

(d) The total number of people in line at time t, $0 \le t \le 300$, is modeled by $20 + \int_0^t r(x)\,dx - 0.7t$.

$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298,\ t_2 = 166.574719$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

$4 : \begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

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t (minutes)	0	1	5	6	8
$g(t)$ (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

1. Grain is being added to a silo. At time $t = 0$, the silo is empty. The rate at which grain is being added is modeled by the differentiable function g, where $g(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Selected values of $g(t)$ are given in the table above.

   (a) Using the data in the table, approximate $g'(3)$. Using correct units, interpret the meaning of $g'(3)$ in the context of the problem.

   (b) Write an integral expression that represents the total amount of grain added to the silo from time $t = 0$ to time $t = 8$. Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the integral.

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Continue question 1 on page 5.

(c) The grain in the silo is spoiling at a rate modeled by $w(t) = 32 \cdot \sqrt{\sin\left(\frac{\pi t}{74}\right)}$, where $w(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Using the result from part (b), approximate the amount of unspoiled grain remaining in the silo at time $t = 8$.

(d) Based on the model in part (c), is the amount of unspoiled grain in the silo increasing or decreasing at time $t = 6$? Show the work that leads to your answer.

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Question 1

(a) $g'(3) \approx \dfrac{g(5) - g(1)}{5 - 1} = \dfrac{20.5 - 15.1}{4} = 1.35$

At time $t = 3$ minutes, the rate at which grain is being added to the silo is increasing at a rate of 1.35 cubic feet per minute per minute.

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{interpretation with units} \end{cases}$

(b) The total amount of grain added to the silo from time $t = 0$ to time $t = 8$ is $\int_0^8 g(t)\,dt$ cubic feet.

$$\int_0^8 g(t)\,dt \approx g(1)\cdot(1-0) + g(5)\cdot(5-1) + g(6)\cdot(6-5) + g(8)\cdot(8-6)$$
$$= 15.1\cdot 1 + 20.5\cdot 4 + 18.3\cdot 1 + 22.7\cdot 2 = 160.8$$

$3 : \begin{cases} 1 : \text{integral expression} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

(c) $\int_0^8 w(t)\,dt = 99.051497$

The approximate amount of unspoiled grain remaining in the silo at time $t = 8$ is $160.8 - \int_0^8 w(t)\,dt = 61.749$ (or 61.748) cubic feet.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $g(6) - w(6) = 18.3 - 16.063173 = 2.236827 > 0$

Because $g(6) - w(6) > 0$, the amount of unspoiled grain is increasing at time $t = 6$.

$2 : \begin{cases} 1 : \text{considers } g(6) - w(6) \\ 1 : \text{answer} \end{cases}$

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} \left|v(t)\right|\,dt$. Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

END OF PART A OF SECTION II

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Question 2

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118.

1 : answer

(b) $x(3) = x(0) + \int_0^3 v(t)\,dt = -5 + \int_0^3 v(t)\,dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760.

3 : 1 : $\int_0^3 v(t)\,dt$
1 : uses initial condition
1 : answer

(c) $\int_0^{3.5} v(t)\,dt = 2.844$ (or 2.843)

$\int_0^{3.5} |v(t)|\,dt = 3.737$

The integral $\int_0^{3.5} v(t)\,dt$ is the displacement of the particle over the time interval $0 \le t \le 3.5$.

The integral $\int_0^{3.5} |v(t)|\,dt$ is the total distance traveled by the particle over the time interval $0 \le t \le 3.5$.

3 : 1 : answers
2 : interpretations of $\int_0^{3.5} v(t)\,dt$ and $\int_0^{3.5} |v(t)|\,dt$

(d) $v(t) = x_2'(t)$

$v(t) = 2t - 1 \Rightarrow t = 1.57054$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

2 : 1 : sets $v(t) = x_2'(t)$
1 : answer

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2. A snail is traveling along a straight path. The snail's velocity can be modeled by $v(t) = 1.4 \ln\left(1 + t^2\right)$ inches per minute for $0 \le t \le 15$ minutes.

(a) Find the acceleration of the snail at time $t = 5$ minutes.

(b) What is the displacement of the snail over the interval $0 \le t \le 15$ minutes?

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Continue question 2 on page 7.

(c) At what time t, $0 \le t \le 15$, is the snail's instantaneous velocity equal to its average velocity over the interval $0 \le t \le 15$?

(d) An ant arrives at the snail's starting position at time $t = 12$ minutes and follows the snail's path. During the interval $12 \le t \le 15$ minutes, the ant travels in the same direction as the snail with a constant acceleration of 2 inches per minute per minute. The ant catches up to the snail at time $t = 15$ minutes. The ant's velocity at time $t = 12$ is B inches per minute. Find the value of B.

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Question 2

(a) $v'(5) = 0.538462$

The acceleration of the snail at time $t = 5$ minutes is 0.538 inches per minute per minute.

1 : answer

(b) $\int_0^{15} v(t)\,dt = 76.043074$

The displacement of the snail over the interval $0 \le t \le 15$ minutes is 76.043 inches.

2 : { 1 : integral; 1 : answer }

(c) $\frac{1}{15}\int_0^{15} v(t)\,dt = 5.069538$

$1.4\ln\left(1+t^2\right) = 5.069538 \Rightarrow t = 6.031$ minutes

2 : { 1 : average value expression; 1 : answer }

(d) The velocity of the ant at time t, $12 \le t \le 15$, is $\int 2\,dt = 2t + c$ inches per minute for some constant c.

For $12 \le t \le 15$, the displacement of the ant is
$\int_{12}^{15} (2t + c)\,dt = \left(t^2 + ct\right)\Big|_{t=12}^{t=15} = 81 + 3c$ inches.

Thus, $81 + 3c = 76.043074 \Rightarrow c = -1.652309$.

The velocity of the ant at time $t = 12$ is
$B = 2 \cdot 12 - 1.652309 = 22.348$ (or 22.347) inches per minute.

— OR —

The velocity of the ant at time t, $12 \le t \le 15$, is $2(t-12) + B$ inches per minute.

For $12 \le t \le 15$, the displacement of the ant is
$\int_{12}^{15} (2(t-12) + B)\,dt = \left((t-12)^2 + Bt\right)\Big|_{t=12}^{t=15} = 9 + 3B$ inches.

$9 + 3B = 76.043074 \Rightarrow B = 22.348$ (or 22.347) inches per minute

4 : { 1 : ant's velocity; 1 : ant's displacement; 1 : equation; 1 : answer }

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CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

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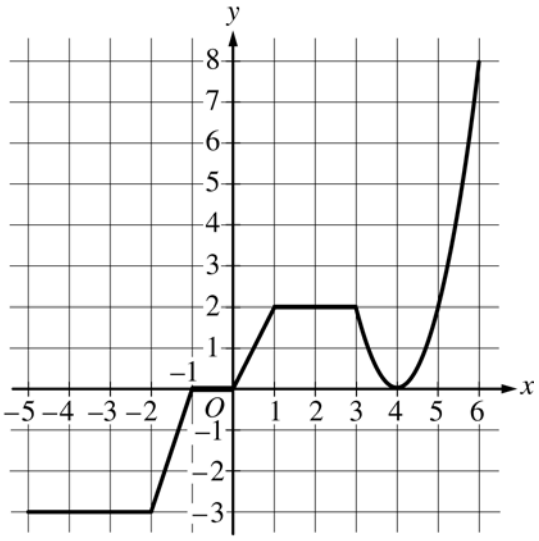

Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

   (a) If $f(1) = 3$, what is the value of $f(-5)$?

   (b) Evaluate $\int_1^6 g(x)\,dx$.

   (c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

   (d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

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Question 3

(a) $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^1 g(x)\,dx$
$= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$
$= \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx$
$= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$

$3 : \begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

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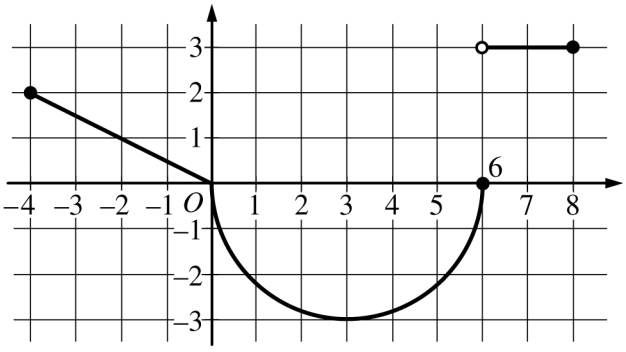

Graph of g

3. The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above. Let f be the function defined by $f(x) = 3x + \int_0^x g(t)\,dt$.

(a) Find $f(7)$ and $f'(7)$.

(b) Find the value of x in the closed interval $[-4, 3]$ at which f attains its maximum value. Justify your answer.

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Continue question 3 on page 15.

(c) For each of $\lim_{x \to 0^-} g'(x)$ and $\lim_{x \to 0^+} g'(x)$, find the value or state that it does not exist.

(d) Find $\lim_{x \to -2} \dfrac{f(x) + 7}{e^{3x+6} - 1}$.

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Question 3

(a) $f(7) = 3 \cdot 7 + \int_0^7 g(t)\,dt = 21 - \frac{9\pi}{2} + 3 = 24 - \frac{9\pi}{2}$ $f'(7) = 3 + g(7) = 3 + 3 = 6$	$2 : \begin{cases} 1 : f(7) \\ 1 : f'(7) \end{cases}$
(b) On the interval $-4 \le x \le 3,\ f'(x) = 3 + g(x)$. Because $f'(x) \ge 0$ for $-4 \le x \le 3,\ f$ is nondecreasing over the entire interval, and the maximum must occur when $x = 3$.	2 : answer with justification
(c) $\lim_{x \to 0^-} g'(x) = -\frac{1}{2}$ $\lim_{x \to 0^+} g'(x)$ does not exist.	$2 : \begin{cases} 1 : \text{left-hand limit} \\ 1 : \text{right-hand limit} \end{cases}$
(d) $\lim_{x \to -2} (f(x) + 7) = -6 + \int_0^{-2} g(t)\,dt + 7 = 0$ $\lim_{x \to -2} \left(e^{3x+6} - 1\right) = 0$ Using L'Hospital's Rule, $\lim_{x \to -2} \frac{f(x) + 7}{e^{3x+6} - 1} = \lim_{x \to -2} \frac{f'(x)}{3e^{3x+6}} = \frac{3 + g(-2)}{3} = \frac{3+1}{3} = \frac{4}{3}$.	$3 : \begin{cases} 1 : \text{limits equal } 0 \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$ Note: max $1/3$ [1-0-0] if no limit notation attached to a ratio of derivatives

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

   (a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

   (b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

   (c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

   (d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

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Question 4

(a) $H'(6) \approx \dfrac{H(7) - H(5)}{7 - 5} = \dfrac{11 - 6}{2} = \dfrac{5}{2}$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\dfrac{H(5) - H(3)}{5 - 3} = \dfrac{6 - 2}{2} = 2$

Because H is differentiable on $3 \le t \le 5$, H is continuous on $3 \le t \le 5$.

By the Mean Value Theorem, there exists a value c, $3 < c < 5$, such that $H'(c) = 2$.

$2 : \begin{cases} 1 : \dfrac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$

(c) The average height of the tree over the time interval $2 \le t \le 10$ is given by $\dfrac{1}{10 - 2}\displaystyle\int_2^{10} H(t)\, dt$.

$$\frac{1}{8}\int_2^{10} H(t)\, dt \approx \frac{1}{8}\left(\frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3\right)$$
$$= \frac{1}{8}(65.75) = \frac{263}{32}$$

The average height of the tree over the time interval $2 \le t \le 10$ is $\dfrac{263}{32}$ meters.

$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

(d) $G(x) = 50 \Rightarrow x = 1$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1 + x)100 - 100x \cdot 1}{(1 + x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1 + x)^2} \cdot \frac{dx}{dt}$$

$$\left.\frac{d}{dt}(G(x))\right|_{x=1} = \frac{100}{(1 + 1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\dfrac{3}{4}$ meter per year.

$3 : \begin{cases} 2 : \dfrac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

Note: max $1/3$ [1-0] if no chain rule

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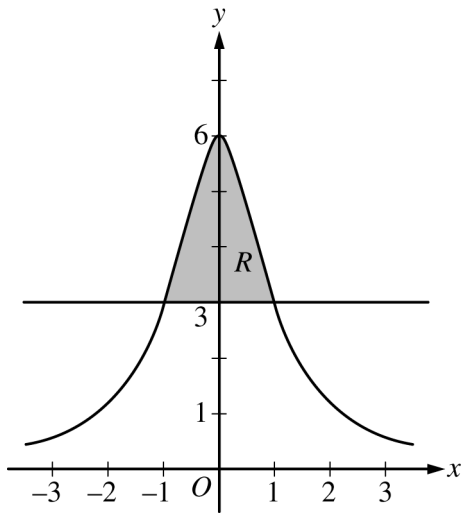

4. Let f be the function defined by $f(x) = \dfrac{6}{1+x^2}$. Let R be the shaded region bounded by the graph of f and the horizontal line $y = 3$, as shown in the figure above.

   (a) Find the area of R.

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Continue question 4 on page 17.

(b) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Let $h(x)$ be the vertical distance between the point $(x, f(x))$ and the horizontal line $y = 3$. Find the rate of change of $h(x)$ with respect to x at $x = 2$.

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Question 4

(a) $f(x) = 3 \Rightarrow x = -1$ and $x = 1$

$$\int_{-1}^{1} (f(x) - 3)\,dx = \int_{-1}^{1} \left(\frac{6}{1 + x^2} - 3 \right) dx$$

$$= \left(6\tan^{-1} x - 3x \right) \Big|_{-1}^{1}$$

$$= \left(6\tan^{-1} 1 - 3 \right) - \left(6\tan^{-1}(-1) + 3 \right)$$

$$= \left(6 \cdot \frac{\pi}{4} - 3 \right) - \left(6 \cdot \left(-\frac{\pi}{4} \right) + 3 \right)$$

$$= 3\pi - 6$$

The area of R is $3\pi - 6$.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $\text{Volume} = \pi \int_{-1}^{1} \left((7 - 3)^2 - (7 - f(x))^2 \right) dx$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c) $h(x) = 3 - f(x) = 3 - \dfrac{6}{1 + x^2}$ for $x > 1$

$$h'(x) = \frac{12x}{\left(1 + x^2\right)^2} \text{ for } x > 1$$

$$h'(2) = \frac{12 \cdot 2}{5^2} = \frac{24}{25}$$

$3 : \begin{cases} 1 : h(x) \\ 1 : h'(x) \\ 1 : h'(2) \end{cases}$

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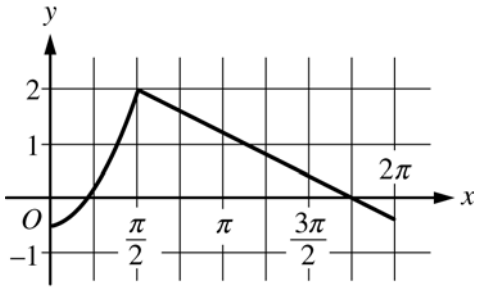
5. Let f be the function defined by $f(x) = e^x \cos x$.

   (a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

   (b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

   (c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

   (d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

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Question 5

(a) The average rate of change of f on the interval $0 \le x \le \pi$ is $\dfrac{f(\pi) - f(0)}{\pi - 0} = \dfrac{-e^{\pi} - 1}{\pi}$.

1 : answer

(b) $f'(x) = e^x \cos x - e^x \sin x$

$f'\left(\dfrac{3\pi}{2}\right) = e^{3\pi/2}\cos\left(\dfrac{3\pi}{2}\right) - e^{3\pi/2}\sin\left(\dfrac{3\pi}{2}\right) = e^{3\pi/2}$

The slope of the line tangent to the graph of f at $x = \dfrac{3\pi}{2}$ is $e^{3\pi/2}$.

2 : { 1 : $f'(x)$; 1 : slope }

(c) $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \dfrac{\pi}{4}, x = \dfrac{5\pi}{4}$

x	$f(x)$
0	1
$\dfrac{\pi}{4}$	$\dfrac{1}{\sqrt{2}}e^{\pi/4}$
$\dfrac{5\pi}{4}$	$-\dfrac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

The absolute minimum value of f on $0 \le x \le 2\pi$ is $-\dfrac{1}{\sqrt{2}}e^{5\pi/4}$.

3 : { 1 : sets $f'(x) = 0$; 1 : identifies $x = \dfrac{\pi}{4}, x = \dfrac{5\pi}{4}$ as candidates ; 1 : answer with justification }

(d) $\lim\limits_{x \to \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$\lim\limits_{x \to \pi/2} g(x) = g\left(\dfrac{\pi}{2}\right) = 0$

By L'Hospital's Rule,

$\lim\limits_{x \to \pi/2} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to \pi/2} \dfrac{f'(x)}{g'(x)} = \dfrac{-e^{\pi/2}}{2}$.

3 : { 1 : g is continuous at $x = \dfrac{\pi}{2}$ and limits equal 0 ; 1 : applies L'Hospital's Rule ; 1 : answer }

Note: max $1/3$ [1-0-0] if no limit notation attached to a ratio of derivatives

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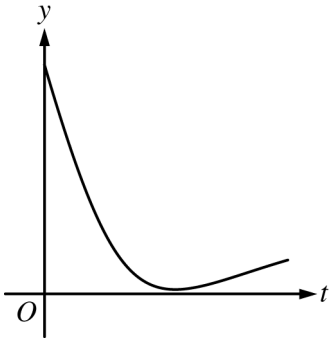
5. During a chemical reaction, the function $y = f(t)$ models the amount of a substance present, in grams, at time t seconds. At the start of the reaction $(t = 0)$, there are 10 grams of the substance present. The function $y = f(t)$ satisfies the differential equation $\frac{dy}{dt} = -0.02y^2$.

(a) Use the line tangent to the graph of $y = f(t)$ at $t = 0$ to approximate the amount of the substance remaining at time $t = 2$ seconds.

(b) Using the given differential equation, determine whether the graph of f could resemble the following graph. Give a reason for your answer.



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Continue question 5 on page 19.

(c) Find an expression for $y = f(t)$ by solving the differential equation $\frac{dy}{dt} = -0.02y^2$ with the initial condition $f(0) = 10$.

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(d) Determine whether the amount of the substance is changing at an increasing or a decreasing rate. Explain your reasoning.

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Question 5

(a) $y'(0) = -0.02\left(10^2\right) = -2$

An equation for the line tangent to the graph of $y = f(t)$ at $t = 0$ is $y = 10 - 2t$.

$y(2) \approx 10 - 2(2) = 6$ grams

$2 : \begin{cases} 1 : y'(0) \\ 1 : \text{approximation} \end{cases}$

(b) $\frac{dy}{dt} = -0.02y^2 \le 0,$ so the graph of f is nonincreasing.

The graph of f cannot resemble the given graph because the given graph is increasing on a portion of its domain.

1 : answer with reason

(c) $\int\left(-\frac{1}{y^2}\right)dy = \int 0.02\,dt$

$\frac{1}{y} = 0.02t + C$

$\frac{1}{10} = 0.02(0) + C \Rightarrow C = 0.1$

$\frac{1}{y} = 0.02t + 0.1 \Rightarrow y = \frac{1}{0.02t + 0.1} = \frac{50}{t + 5}$

Note: this solution is valid for $t > -5$.

$4 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{answer} \end{cases}$

Note: max $2/4$ [1-1-0-0] if no constant of integration

Note: $0/4$ if no separation of variables

(d) $\frac{d^2y}{dt^2} = -0.04y\frac{dy}{dt}$

$= -0.04y\left(-0.02y^2\right)$

$= 0.0008y^3$

Because $y > 0$, $0.0008y^3 > 0$.
The amount of substance is changing at an increasing rate.

— OR —

From part (c), $f(t) = \frac{50}{t + 5},$ and from context, $t \ge 0$.

$f'(t) = \frac{-50}{(t + 5)^2}$ and $f''(t) = \frac{100}{(t + 5)^3} > 0$ for $t \ge 0$.

The amount of substance is changing at an increasing rate.

$2 : \begin{cases} 1 : \frac{d^2y}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$

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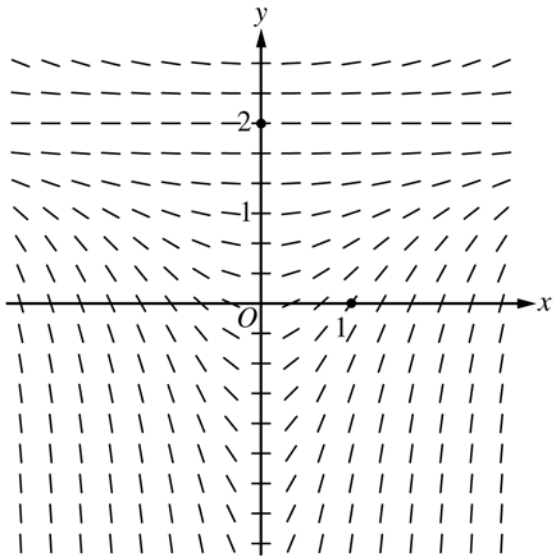
6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

(a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

STOP

END OF EXAM

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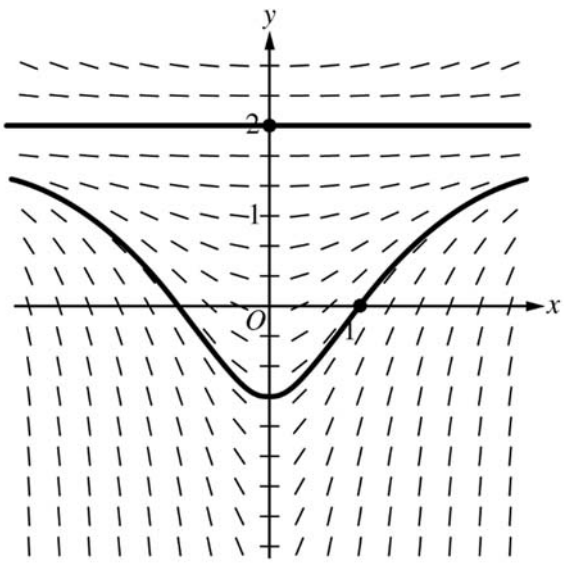
Question 6

(a)



$2 : \begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b) $\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(1,\,0)} = \dfrac{4}{3}$

An equation for the line tangent to the graph of $y = f(x)$ at $x = 1$ is $y = \dfrac{4}{3}(x - 1)$.

$f(0.7) \approx \dfrac{4}{3}(0.7 - 1) = -0.4$

$2 : \begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$

(c) $\dfrac{dy}{dx} = \dfrac{1}{3}x(y-2)^2$

$\displaystyle\int \frac{dy}{(y-2)^2} = \int \frac{1}{3}x\,dx$

$\dfrac{-1}{y-2} = \dfrac{1}{6}x^2 + C$

$\dfrac{1}{2} = \dfrac{1}{6} + C \Rightarrow C = \dfrac{1}{3}$

$\dfrac{-1}{y-2} = \dfrac{1}{6}x^2 + \dfrac{1}{3} = \dfrac{x^2+2}{6}$

$y = 2 - \dfrac{6}{x^2+2}$

Note: this solution is valid for $-\infty < x < \infty$.

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: $0/5$ if no separation of variables

Note: max $3/5$ [1-2-0-0] if no constant of integration

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6. Consider the curve given by the equation $2(x - y) = 3 + \cos y$. For all points on the curve, $\frac{2}{3} \le \frac{dy}{dx} \le 2$.

(a) Show that $\frac{dy}{dx} = \frac{2}{2 - \sin y}$.

(b) For $-\frac{\pi}{2} < y < \frac{\pi}{2}$, there is a point P on the curve through which the line tangent to the curve has slope 1. Find the coordinates of the point P.

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Continue question 6 on page 21.

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(c) Determine the concavity of the curve at points for which $-\frac{\pi}{2} < y < \frac{\pi}{2}$. Give a reason for your answer.

(d) Let $y = f(x)$ be a function, defined implicitly by $2(x - y) = 3 + \cos y$, that is continuous on the closed interval $[2, 2.1]$ and differentiable on the open interval $(2, 2.1)$. Use the Mean Value Theorem on the interval $[2, 2.1]$ to show that $\frac{1}{15} \le f(2.1) - f(2) \le \frac{1}{5}$.

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Question 6

(a) $\frac{d}{dx}(2(x-y)) = \frac{d}{dx}(3+\cos y)$

$2 - 2\frac{dy}{dx} = (-\sin y)\frac{dy}{dx}$

$2 = (2-\sin y)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2}{2-\sin y}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$

(b) $\frac{dy}{dx} = \frac{2}{2-\sin y} = 1 \Rightarrow \sin y = 0 \Rightarrow y = 0$

$2(x-0) = 3 + \cos 0 \Rightarrow 2x = 4 \Rightarrow x = 2$

Point P has coordinates $(2, 0)$.

$2 : \begin{cases} 1 : \frac{dy}{dx} = 1 \\ 1 : \text{answer} \end{cases}$

(c) $\frac{d^2y}{dx^2} = \frac{-2}{(2-\sin y)^2}(-\cos y)\frac{dy}{dx} = \frac{4\cos y}{(2-\sin y)^3}$

$\frac{d^2y}{dx^2} > 0$ for all y in the interval $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Therefore, the curve is concave up for $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

$3 : \begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{answer with reason} \end{cases}$

(d) By the Mean Value Theorem, for some value c in the interval $(2, 2.1)$, $f'(c) = \frac{f(2.1) - f(2)}{0.1}$.

For all points on the curve, $\frac{2}{3} \le f'(x) \le 2$.

Thus, $\frac{2}{3} \le \frac{f(2.1) - f(2)}{0.1} \le 2 \Rightarrow \frac{1}{15} \le f(2.1) - f(2) \le \frac{1}{5}$.

$2 : \begin{cases} 1 : \text{applies Mean Value Theorem} \\ 1 : \text{verification} \end{cases}$

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CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

   (a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ? Give your answer to the nearest whole number.

   (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ?

   (c) At what time t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

   (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$) ? Explain your reasoning.

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Question 1

(a) $\int_0^5 E(t)\,dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

2 : { 1 : integral; 1 : answer }

(b) $\frac{1}{5-0}\int_0^5 L(t)\,dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

2 : { 1 : integral; 1 : answer }

(c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$E(t) - L(t) = 0 \Rightarrow t = 6.20356$

$E(t) - L(t) > 0$ for $0 \le t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \le 8$. Therefore, the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

— OR —

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$A(t) = \int_0^t (E(s) - L(s))\,ds$

$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore, the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

3 : { 1 : sets $E(t) - L(t) = 0$; 1 : answer; 1 : justification }

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

2 : { 1 : considers $E'(5)$ and $L'(5)$; 1 : answer with explanation }

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A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (hours)	2	5	9	11	12
$L(t)$ (cars per hour)	15	40	24	68	18

1. The rate at which cars enter a parking lot is modeled by $E(t) = 30 + 5(t - 2)(t - 5)e^{-0.2t}$. The rate at which cars leave the parking lot is modeled by the differentiable function L. Selected values of $L(t)$ are given in the table above. Both $E(t)$ and $L(t)$ are measured in cars per hour, and time t is measured in hours after 5 A.M. ($t = 0$). Both functions are defined for $0 \le t \le 12$.

   (a) What is the rate of change of $E(t)$ at time $t = 7$? Indicate units of measure.

   (b) How many cars enter the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole number.

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Continue question 1 on page 5.

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate $\int_2^{12} L(t)\,dt$. Using correct units, explain the meaning of $\int_2^{12} L(t)\,dt$ in the context of this problem.

(d) For $0 \le t < 6$, 5 dollars are collected from each car entering the parking lot. For $6 \le t \le 12$, 8 dollars are collected from each car entering the parking lot. How many dollars are collected from the cars entering the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole dollar.

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Question 1

t (hours)	2	5	9	11	12
$L(t)$ (cars per hour)	15	40	24	68	18

1. The rate at which cars enter a parking lot is modeled by $E(t) = 30 + 5(t-2)(t-5)e^{-0.2t}$. The rate at which cars leave the parking lot is modeled by the differentiable function L. Selected values of $L(t)$ are given in the table above. Both $E(t)$ and $L(t)$ are measured in cars per hour, and time t is measured in hours after 5 A.M. ($t = 0$). Both functions are defined for $0 \le t \le 12$.
   (a) What is the rate of change of $E(t)$ at time $t = 7$? Indicate units of measure.
   (b) How many cars enter the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole number.
   (c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate $\int_2^{12} L(t)\,dt$. Using correct units, explain the meaning of $\int_2^{12} L(t)\,dt$ in the context of this problem.
   (d) For $0 \le t < 6$, 5 dollars are collected from each car entering the parking lot. For $6 \le t \le 12$, 8 dollars are collected from each car entering the parking lot. How many dollars are collected from the cars entering the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole dollar.

(a) $E'(7) = 6.164924$

The rate of change of $E(t)$ at time $t = 7$ is 6.165 (or 6.164) cars per hour per hour.

1 : answer with units

(b) $\int_0^{12} E(t)\,dt = 520.070489$

To the nearest whole number, 520 cars enter the parking lot from time $t = 0$ to time $t = 12$.

2 : { 1 : integrand; 1 : answer }

(c) $$\int_2^{12} L(t)\,dt \approx (5-2)\cdot\frac{L(2)+L(5)}{2} + (9-5)\cdot\frac{L(5)+L(9)}{2} + (11-9)\cdot\frac{L(9)+L(11)}{2} + (12-11)\cdot\frac{L(11)+L(12)}{2}$$
$$= 3\cdot\frac{15+40}{2} + 4\cdot\frac{40+24}{2} + 2\cdot\frac{24+68}{2} + 1\cdot\frac{68+18}{2}$$
$$= 345.5$$

$\int_2^{12} L(t)\,dt$ is the number of cars that leave the parking lot in the 10 hours between 7 A.M. ($t = 2$) and 5 P.M. ($t = 12$).

3 : { 1 : trapezoidal sum; 1 : approximation; 1 : explanation }

(d) $5\int_0^6 E(t)\,dt + 8\int_6^{12} E(t)\,dt = 3530.1396$

To the nearest dollar, 3530 dollars are collected from time $t = 0$ to time $t = 12$.

3 : { 1 : integrand; 1 : limits and constants; 1 : answer }

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t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	–29	55	48

2. The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

   (a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

   (b) Use a trapezoidal sum with the three subintervals [0, 0.3], [0.3, 1.7], and [1.7, 2.8] to approximate the value of $\int_0^{2.8} v_P(t)\,dt$.

   (c) A second particle, Q, also moves along the x-axis so that its velocity for $0 \le t \le 4$ is given by $v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.

   (d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

END OF PART A OF SECTION II

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Question 2

(a) v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

By the Mean Value Theorem, there is a value c, $0.3 < c < 2.8$, such that $v_P'(c) = 0$.

— OR —

v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

By the Extreme Value Theorem, v_p has a minimum on $[0.3, 2.8]$.
$v_P(0.3) = 55 > -29 = v_P(1.7)$ and $v_P(1.7) = -29 < 55 = v_P(2.8)$.
Thus, v_P has a minimum on the interval $(0.3, 2.8)$.

Because v_P is differentiable, $v_P'(t)$ must equal 0 at this minimum.

2 : { 1 : $v_P(2.8) - v_P(0.3) = 0$; 1 : justification, using Mean Value Theorem }

— OR —

2 : { 1 : $v_P(0.3) > v_P(1.7)$ and $v_P(1.7) < v_P(2.8)$; 1 : justification, using Extreme Value Theorem }

(b)
$$\begin{aligned}\int_0^{2.8} v_P(t)\,dt &\approx 0.3\left(\frac{v_P(0) + v_P(0.3)}{2}\right) + 1.4\left(\frac{v_P(0.3) + v_P(1.7)}{2}\right) \\ &\quad + 1.1\left(\frac{v_P(1.7) + v_P(2.8)}{2}\right) \\ &= 0.3\left(\frac{0 + 55}{2}\right) + 1.4\left(\frac{55 + (-29)}{2}\right) + 1.1\left(\frac{-29 + 55}{2}\right) \\ &= 40.75\end{aligned}$$

1 : answer, using trapezoidal sum

(c) $v_Q(t) = 60 \Rightarrow t = A = 1.866181$ or $t = B = 3.519174$
$v_Q(t) \ge 60$ for $A \le t \le B$

$$\int_A^B \left|v_Q(t)\right| dt = 106.108754$$

The distance traveled by particle Q during the interval $A \le t \le B$ is 106.109 (or 106.108) meters.

3 : { 1 : interval; 1 : definite integral; 1 : distance }

(d) From part (b), the position of particle P at time $t = 2.8$ is
$x_P(2.8) = \int_0^{2.8} v_P(t)\,dt \approx 40.75$.

$$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t)\,dt = -90 + 135.937653 = 45.937653$$

Therefore, at time $t = 2.8$, particles P and Q are approximately $45.937653 - 40.75 = 5.188$ (or 5.187) meters apart.

3 : { 1 : $\int_0^{2.8} v_Q(t)\,dt$; 1 : position of particle Q; 1 : answer }

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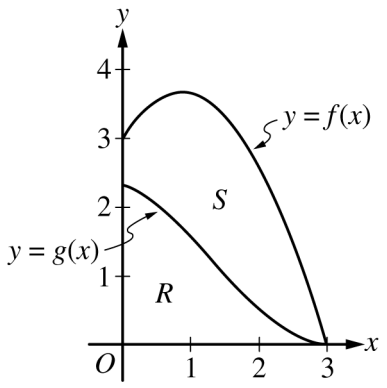

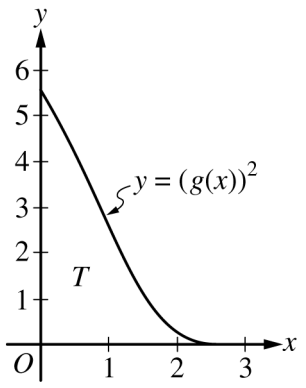

2. The function f is defined by $f(x) = 3(1 + x)^{0.5}\cos\left(\frac{\pi x}{6}\right)$ for $0 \le x \le 3$. The function g is continuous and decreasing for $0 \le x \le 3$ with $g(3) = 0$.

The figure above on the left shows the graphs of f and g and the regions R and S. R is the region bounded by the graph of g and the x- and y-axes. Region R has area 3.24125. S is the region bounded by the y-axis and the graphs of f and g.

The figure above on the right shows the graph of $y = (g(x))^2$ and the region T. T is the region bounded by the graph of $y = (g(x))^2$ and the x- and y-axes. Region T has area 5.32021.

(a) Find the area of region S.

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Continue question 2 on page 7.

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

(b) Find the volume of the solid generated when region S is revolved about the horizontal line $y = -3$.

(c) Region S is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 7 times the length of its base in region S. Write, but do not evaluate, an integral expression for the volume of this solid.

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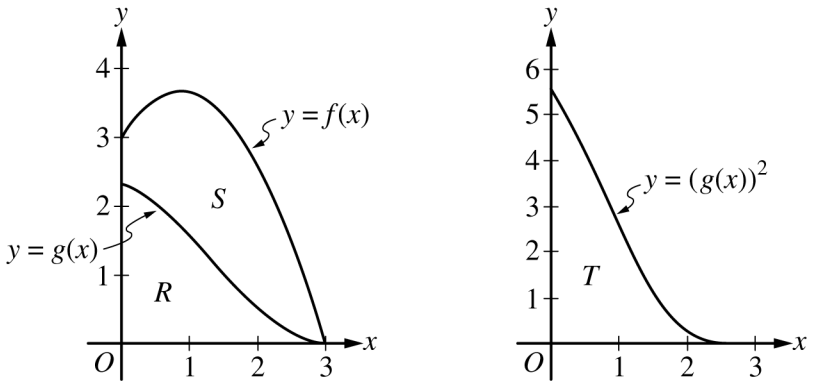
Question 2



2. The function f is defined by $f(x) = 3(1+x)^{0.5}\cos\left(\frac{\pi x}{6}\right)$ for $0 \le x \le 3$. The function g is continuous and decreasing for $0 \le x \le 3$ with $g(3) = 0$.

The figure above on the left shows the graphs of f and g and the regions R and S. R is the region bounded by the graph of g and the x- and y-axes. Region R has area 3.24125. S is the region bounded by the y-axis and the graphs of f and g.

The figure above on the right shows the graph of $y = (g(x))^2$ and the region T. T is the region bounded by the graph of $y = (g(x))^2$ and the x- and y-axes. Region T has area 5.32021.

(a) Find the area of region S.
(b) Find the volume of the solid generated when region S is revolved about the horizontal line $y = -3$.
(c) Region S is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 7 times the length of its base in region S. Write, but do not evaluate, an integral expression for the volume of this solid.

(a) $\int_0^3 (f(x) - g(x))\,dx = \int_0^3 f(x)\,dx - \int_0^3 g(x)\,dx$
$= \int_0^3 f(x)\,dx - 3.24125 = 4.919585$

The area of region S is 4.920 (or 4.919).

3 : { 1 : definite integral of f ; 1 : uses area of R ; 1 : answer }

(b) $\pi\int_0^3 \left((f(x)+3)^2 - (g(x)+3)^2\right)dx$
$= \pi\left(\int_0^3 (f(x)+3)^2\,dx - \int_0^3 \left((g(x))^2 + 6g(x) + 9\right)dx\right)$
$= \pi\left(\int_0^3 (f(x)+3)^2\,dx - \int_0^3 (g(x))^2\,dx - 6\int_0^3 g(x)\,dx - \int_0^3 9\,dx\right)$
$= \pi\left(\int_0^3 (f(x)+3)^2\,dx - 5.32021 - 6\cdot 3.24125 - 9\cdot 3\right)$
$= 156.263709$

The volume of the solid is 156.264 (or 156.263).

4 : { 1 : form of integrand ; 1 : integrand ; 1 : uses areas of R and T ; 1 : limits, constant, and answer }

(c) Volume $= \int_0^3 7(f(x) - g(x))^2\,dx$

2 : { 1 : integrand ; 1 : expression }

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CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

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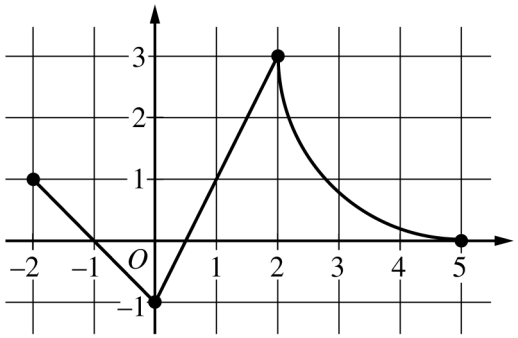

Graph of f

3. The continuous function f is defined on the closed interval $-6 \le x \le 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f.

   (a) If $\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

   (b) Evaluate $\int_{3}^{5} (2f'(x) + 4)\,dx$.

   (c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 \le x \le 5$. Justify your answer.

   (d) Find $\lim\limits_{x \to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x}$.

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Question 3

(a) $\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$

$\Rightarrow 7 = \int_{-6}^{-2} f(x)\,dx + 2 + \left(9 - \frac{9\pi}{4}\right)$

$\Rightarrow \int_{-6}^{-2} f(x)\,dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4$

$3 : \begin{cases} 1 : \int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx \\ 1 : \int_{-2}^{5} f(x)\,dx \\ 1 : \text{answer} \end{cases}$

(b) $\int_{3}^{5} (2f'(x) + 4)\,dx = 2\int_{3}^{5} f'(x)\,dx + \int_{3}^{5} 4\,dx$

$= 2(f(5) - f(3)) + 4(5 - 3)$

$= 2\left(0 - \left(3 - \sqrt{5}\right)\right) + 8$

$= 2\left(-3 + \sqrt{5}\right) + 8 = 2 + 2\sqrt{5}$

$2 : \begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

— OR —

$\int_{3}^{5} (2f'(x) + 4)\,dx = [2f(x) + 4x]_{x=3}^{x=5}$

$= (2f(5) + 20) - (2f(3) + 12)$

$= (2 \cdot 0 + 20) - \left(2\left(3 - \sqrt{5}\right) + 12\right)$

$= 2 + 2\sqrt{5}$

(c) $g'(x) = f(x) = 0 \Rightarrow x = -1,\ x = \frac{1}{2},\ x = 5$

x	$g(x)$
-2	0
-1	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{4}$
5	$11 - \frac{9\pi}{4}$

On the interval $-2 \le x \le 5$, the absolute maximum value of g is $g(5) = 11 - \frac{9\pi}{4}$.

$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -1 \text{ as a candidate} \\ 1 : \text{answer with justification} \end{cases}$

(d) $\lim_{x \to 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} = \frac{10^1 - 3f'(1)}{f(1) - \arctan 1}$

$= \frac{10 - 3 \cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}}$

1 : answer

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$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\dfrac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

3. Let f be the function defined above.

(a) Find the average rate of change of f on the interval $-3 \le x \le 4$.

(b) Write an equation for the line tangent to the graph of f at $x = 3$.

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Continue question 3 on page 15.

(c) Find the average value of f on the interval $-3 \le x \le 4$.

(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \le x \le 4$? Justify your answer.

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Question 3

$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\frac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

3. Let f be the function defined above.

(a) Find the average rate of change of f on the interval $-3 \le x \le 4$.
(b) Write an equation for the line tangent to the graph of f at $x = 3$.
(c) Find the average value of f on the interval $-3 \le x \le 4$.
(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \le x \le 4$? Justify your answer.

(a) Average rate of change $= \dfrac{f(4) - f(-3)}{4 - (-3)} = \dfrac{-1 - 0}{7} = -\dfrac{1}{7}$

1 : answer

(b) $f(3) = -3 + 3\cos\left(\dfrac{3\pi}{2}\right) = -3$

For $0 < x < 4$, $f'(x) = -1 + \left(-3\sin\left(\dfrac{\pi x}{2}\right)\right) \cdot \dfrac{\pi}{2}$

$f'(3) = -1 + \left(-3\sin\left(\dfrac{3\pi}{2}\right)\right) \cdot \dfrac{\pi}{2} = -1 + \dfrac{3\pi}{2}$

An equation for the tangent line is $y = -3 + \left(-1 + \dfrac{3\pi}{2}\right)(x - 3)$.

2 : { 1 : $f'(3)$; 1 : equation }

(c) The average value of f on the interval $-3 \le x \le 4$ is $\dfrac{1}{4 - (-3)} \displaystyle\int_{-3}^{4} f(x)\, dx$.

$\displaystyle\int_{-3}^{4} f(x)\, dx = \int_{-3}^{0} f(x)\, dx + \int_{0}^{4} f(x)\, dx$

$\displaystyle\int_{-3}^{0} f(x)\, dx = \int_{-3}^{0} \sqrt{9 - x^2}\, dx = \frac{9\pi}{4}$

$\displaystyle\int_{0}^{4} f(x)\, dx = \int_{0}^{4} \left(-x + 3\cos\left(\frac{\pi x}{2}\right)\right) dx = \left[-\frac{1}{2}x^2 + \frac{6}{\pi}\sin\left(\frac{\pi x}{2}\right)\right]_0^4 = -8$

$\dfrac{1}{4 - (-3)} \displaystyle\int_{-3}^{4} f(x)\, dx = \frac{1}{7}\left(\frac{9\pi}{4} - 8\right)$

4 : { 1 : integrals of f over $-3 \le x \le 0$ and $0 \le x \le 4$; 1 : value of $\displaystyle\int_{-3}^{0} \sqrt{9 - x^2}\, dx$; 1 : antiderivative of $-x + 3\cos\left(\dfrac{\pi x}{2}\right)$; 1 : answer }

(d) $\lim\limits_{x \to 0^-} f(x) = f(0) = 3$ and $\lim\limits_{x \to 0^+} f(x) = 3$, so f is continuous at $x = 0$.

Because f is continuous on $[-3, 4]$, the Extreme Value Theorem guarantees that f attains an absolute maximum on $[-3, 4]$.

2 : { 1 : continuity at $x = 0$; 1 : answer with justification }

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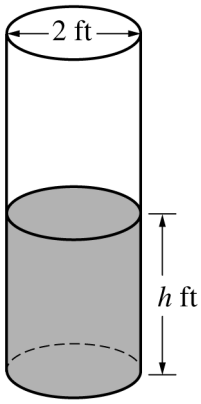

4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

   (b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

   (c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t.

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Question 4

(a) $V = \pi r^2 h = \pi (1)^2 h = \pi h$

$\left.\frac{dV}{dt}\right|_{h=4} = \pi \left.\frac{dh}{dt}\right|_{h=4} = \pi\left(-\frac{1}{10}\sqrt{4}\right) = -\frac{\pi}{5}$ cubic feet per second

$2 : \begin{cases} 1 : \frac{dV}{dt} = \pi \frac{dh}{dt} \\ 1 : \text{answer with units} \end{cases}$

(b) $\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left(-\frac{1}{10}\sqrt{h}\right) = \frac{1}{200}$

Because $\frac{d^2h}{dt^2} = \frac{1}{200} > 0$ for $h > 0$, the rate of change of the height is increasing when the height of the water is 3 feet.

$3 : \begin{cases} 1 : \frac{d}{dh}\left(-\frac{1}{10}\sqrt{h}\right) = -\frac{1}{20\sqrt{h}} \\ 1 : \frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} \\ 1 : \text{answer with explanation} \end{cases}$

(c) $\frac{dh}{\sqrt{h}} = -\frac{1}{10}\,dt$

$\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10}\,dt$

$2\sqrt{h} = -\frac{1}{10}t + C$

$2\sqrt{5} = -\frac{1}{10} \cdot 0 + C \Rightarrow C = 2\sqrt{5}$

$2\sqrt{h} = -\frac{1}{10}t + 2\sqrt{5}$

$h(t) = \left(-\frac{1}{20}t + \sqrt{5}\right)^2$

$4 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : h(t) \end{cases}$

Note: $0/4$ if no separation of variables

Note: max $2/4$ [1-1-0-0] if no constant of integration

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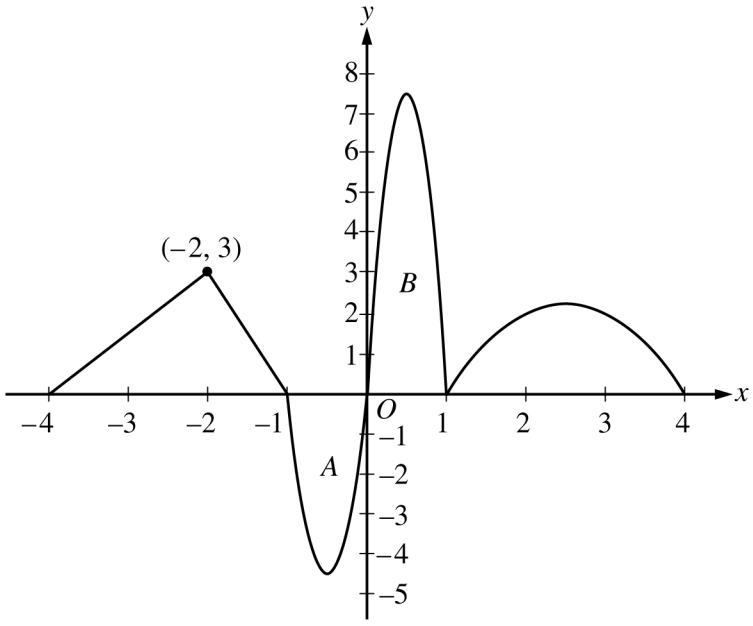

Graph of f

4. The continuous function f is defined for $-4 \le x \le 4$. The graph of f, shown above, consists of two line segments and portions of three parabolas. The graph has horizontal tangents at $x = -\frac{1}{2}$, $x = \frac{1}{2}$, and $x = \frac{5}{2}$. It is known that $f(x) = -x^2 + 5x - 4$ for $1 \le x \le 4$. The areas of regions A and B bounded by the graph of f and the x-axis are 3 and 5, respectively. Let g be the function defined by $g(x) = \int_{-4}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(4)$.

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Continue question 4 on page 17.

(b) Find the absolute minimum value of g on the closed interval $[-4, 4]$. Justify your answer.

(c) Find all intervals on which the graph of g is concave down. Give a reason for your answer.

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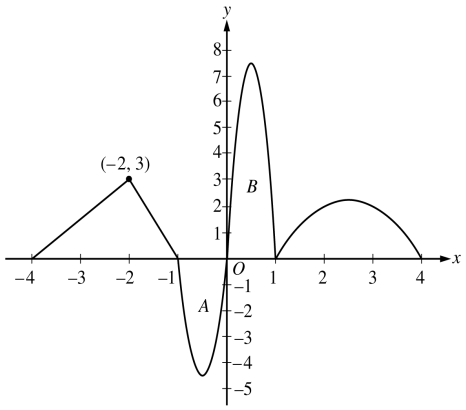
Question 4



Graph of f

4. The continuous function f is defined for $-4 \le x \le 4$. The graph of f, shown above, consists of two line segments and portions of three parabolas. The graph has horizontal tangents at $x = -\frac{1}{2}$, $x = \frac{1}{2}$, and $x = \frac{5}{2}$. It is known that $f(x) = -x^2 + 5x - 4$ for $1 \le x \le 4$. The areas of regions A and B bounded by the graph of f and the x-axis are 3 and 5, respectively. Let g be the function defined by $g(x) = \int_{-4}^{x} f(t)\,dt$.
   (a) Find $g(0)$ and $g(4)$.
   (b) Find the absolute minimum value of g on the closed interval $[-4, 4]$. Justify your answer.
   (c) Find all intervals on which the graph of g is concave down. Give a reason for your answer.

(a) $g(0) = \int_{-4}^{0} f(t)\,dt = \frac{9}{2} - 3 = \frac{3}{2}$

$g(4) = \int_{-4}^{4} f(t)\,dt$

$= \int_{-4}^{0} f(t)\,dt + \int_{0}^{1} f(t)\,dt + \int_{1}^{4} f(t)\,dt$

$= \frac{3}{2} + 5 + \int_{1}^{4} \left(-t^2 + 5t - 4\right) dt$

$= \frac{3}{2} + 5 + \left[-\frac{1}{3}t^3 + \frac{5}{2}t^2 - 4t\right]_{1}^{4}$

$= \frac{3}{2} + 5 + \left[\left(-\frac{1}{3}\cdot 4^3 + \frac{5}{2}\cdot 4^2 - 4\cdot 4\right) - \left(-\frac{1}{3}\cdot 1^3 + \frac{5}{2}\cdot 1^2 - 4\cdot 1\right)\right]$

$= \frac{3}{2} + 5 + \left(\frac{8}{3} - \left(-\frac{11}{6}\right)\right) = 11$

4 : { 1 : $g(0)$; 1 : integral of f over $1 \le t \le 4$; 1 : antiderivative; 1 : $g(4)$

(b) $g'(x) = f(x)$ is negative for $-1 < x < 0$, and nonnegative elsewhere. Thus, the absolute minimum value of g on $[-4, 4]$ can only occur at $x = -4$ or $x = 0$.

$g(-4) = 0 \qquad g(0) = \frac{3}{2}$

The absolute minimum value of g on $[-4, 4]$ is $g(-4) = 0$.

3 : { 1 : $g'(x) = f(x)$; 1 : identifies $x = -4$ and $x = 0$ as candidates; 1 : answer with justification

(c) The graph of g is concave down on the intervals $-2 < x < -\frac{1}{2}$, $\frac{1}{2} < x < 1$, and $\frac{5}{2} < x < 4$ because $g'(x) = f(x)$ is decreasing on these intervals.

2 : { 1 : intervals; 1 : reason

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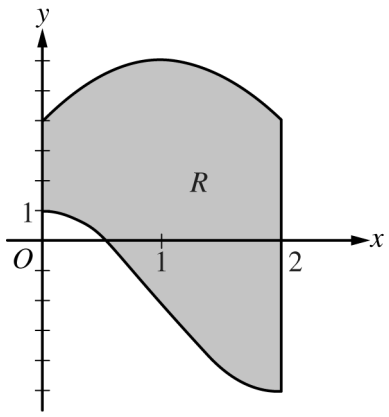

5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

   (a) Find the area of R.

   (b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

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Question 5

(a) $\int_0^2 (h(x) - g(x))\,dx = \int_0^2 \left(\left(6 - 2(x-1)^2\right) - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right)\right)\right) dx$

$= \left[\left(6x - \frac{2}{3}(x-1)^3\right) - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\right)\right]_{x=0}^{x=2}$

$= \left(\left(12 - \frac{2}{3}\right) - (-4 + 0)\right) - \left(\left(0 + \frac{2}{3}\right) - (0 + 0)\right)$

$= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}$

The area of R is $\frac{44}{3}$.

4 : { 1 : integrand; 1 : antiderivative of $3\cos\left(\frac{\pi}{2}x\right)$; 1 : antiderivative of remaining terms; 1 : answer }

(b) $\int_0^2 A(x)\,dx = \int_0^2 \frac{1}{x+3}\,dx$

$= \left[\ln(x+3)\right]_{x=0}^{x=2} = \ln 5 - \ln 3$

The volume of the solid is $\ln 5 - \ln 3$.

2 : { 1 : integral; 1 : answer }

(c) $\pi\int_0^2 \left((6 - g(x))^2 - (6 - h(x))^2\right) dx$

3 : { 1 : limits and constant; 1 : form of integrand; 1 : integrand }

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t (hours)	0	1	2	3	4
$B(t)$ (miles per hour)	1	8	1.5	–5	11

5. Brandon and Chloe ride their bikes for 4 hours along a flat, straight road. Brandon's velocity, in miles per hour, at time t hours is given by a differentiable function B for $0 \le t \le 4$. Values of $B(t)$ for selected times t are given in the table above. Chloe's velocity, in miles per hour, at time t hours is given by the piecewise function C defined by

$$C(t) = \begin{cases} te^{4-t^2} & \text{for } 0 \le t \le 2 \\ 12 - 3t - t^2 & \text{for } 2 < t \le 4. \end{cases}$$

(a) How many miles did Chloe travel from time $t = 0$ to time $t = 2$?

(b) At time $t = 3$, is Chloe's speed increasing or decreasing? Give a reason for your answer.

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Continue question 5 on page 19.

(c) Is there a time t, for $0 \le t \le 4$, at which Brandon's acceleration is equal to 2.5 miles per hour per hour? Justify your answer.

(d) Is there a time t, for $0 \le t \le 2$, at which Brandon's velocity is equal to Chloe's velocity? Justify your answer.

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Question 5

t (hours)	0	1	2	3	4
$B(t)$ (miles per hour)	1	8	1.5	–5	11

5. Brandon and Chloe ride their bikes for 4 hours along a flat, straight road. Brandon's velocity, in miles per hour, at time t hours is given by a differentiable function B for $0 \le t \le 4$. Values of $B(t)$ for selected times t are given in the table above. Chloe's velocity, in miles per hour, at time t hours is given by the piecewise function C defined by

$$C(t) = \begin{cases} te^{4-t^2} & \text{for } 0 \le t \le 2 \\ 12 - 3t - t^2 & \text{for } 2 < t \le 4. \end{cases}$$

(a) How many miles did Chloe travel from time $t = 0$ to time $t = 2$?

(b) At time $t = 3$, is Chloe's speed increasing or decreasing? Give a reason for your answer.

(c) Is there a time t, for $0 \le t \le 4$, at which Brandon's acceleration is equal to 2.5 miles per hour per hour? Justify your answer.

(d) Is there a time t, for $0 \le t \le 2$, at which Brandon's velocity is equal to Chloe's velocity? Justify your answer.

(a) $\displaystyle\int_0^2 te^{4-t^2}\,dt = -\frac{1}{2}e^{4-t^2}\Big|_{t=0}^{t=2} = -\frac{1}{2} + \frac{1}{2}e^4$

Chloe traveled $-\frac{1}{2} + \frac{1}{2}e^4$ miles from time $t = 0$ to time $t = 2$.

3 : { 1 : integral; 1 : antiderivative; 1 : answer }

(b) $C(3) = -6 < 0$
$C'(3) = -9 < 0$

Chloe's speed is increasing at time $t = 3$ because her velocity and acceleration have the same sign.

2 : { 1 : $C(3) < 0$ and $C'(3) < 0$; 1 : answer with reason }

(c) B is differentiable $\Rightarrow$ B is continuous on $[0, 4]$.

$\dfrac{B(4) - B(0)}{4 - 0} = \dfrac{11 - 1}{4 - 0} = 2.5$

By the Mean Value Theorem, there is a time t, for $0 < t < 4$, such that $B'(t) = 2.5$ miles per hour per hour.

2 : { 1 : $\dfrac{B(4) - B(0)}{4 - 0}$; 1 : answer with justification }

(d) B and C are continuous on $[0, 2]$, therefore $B - C$ is continuous on $[0, 2]$.

$B(0) - C(0) = 1 - 0 > 0$
$B(2) - C(2) = 1.5 - 2 < 0$

By the Intermediate Value Theorem, there is a time t, for $0 < t < 2$, such that $B(t) - C(t) = 0$, or $B(t) = C(t)$.

2 : { 1 : considers $B(t) - C(t)$; 1 : answer with justification }

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6. Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

   (a) Find $h'(2)$.

   (b) Let a be the function given by $a(x) = 3x^3h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

   (c) The function h satisfies $h(x) = \frac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim_{x \to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

   (d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

STOP

END OF EXAM

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Question 6

(a) $h'(2) = \frac{2}{3}$

1 : answer

(b) $a'(x) = 9x^2h(x) + 3x^3h'(x)$

$a'(2) = 9 \cdot 2^2 h(2) + 3 \cdot 2^3 h'(2) = 36 \cdot 4 + 24 \cdot \frac{2}{3} = 160$

3 : { 1 : form of product rule; 1 : $a'(x)$; 1 : $a'(2)$ }

(c) Because h is differentiable, h is continuous, so $\lim_{x \to 2} h(x) = h(2) = 4$.

Also, $\lim_{x \to 2} h(x) = \lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3}$, so $\lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4$.

Because $\lim_{x \to 2} \left(x^2 - 4\right) = 0$, we must also have $\lim_{x \to 2} \left(1 - (f(x))^3\right) = 0$.

Thus, $\lim_{x \to 2} f(x) = 1$.

Because f is differentiable, f is continuous, so $f(2) = \lim_{x \to 2} f(x) = 1$.

Also, because f is twice differentiable, f' is continuous, so $\lim_{x \to 2} f'(x) = f'(2)$ exists.

Using L'Hospital's Rule,

$\lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3} = \lim_{x \to 2} \frac{2x}{-3(f(x))^2 f'(x)} = \frac{4}{-3(1)^2 \cdot f'(2)} = 4.$

Thus, $f'(2) = -\frac{1}{3}$.

4 : { 1 : $\lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4$; 1 : $f(2)$; 1 : L'Hospital's Rule; 1 : $f'(2)$ }

(d) Because g and h are differentiable, g and h are continuous, so $\lim_{x \to 2} g(x) = g(2) = 4$ and $\lim_{x \to 2} h(x) = h(2) = 4$.

Because $g(x) \le k(x) \le h(x)$ for $1 < x < 3$, it follows from the squeeze theorem that $\lim_{x \to 2} k(x) = 4$.

Also, $4 = g(2) \le k(2) \le h(2) = 4$, so $k(2) = 4$.

Thus, k is continuous at $x = 2$.

1 : continuous with justification

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6. Consider the curve defined by $2x^2 + 3y^2 - 4xy = 36$.

(a) Show that $\frac{dy}{dx} = \frac{2y - 2x}{3y - 2x}$.

(b) Find the slope of the line tangent to the curve at each point on the curve where $x = 6$.

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Continue question 6 on page 21.

(c) Find the positive value of x at which the curve has a vertical tangent line. Show the work that leads to your answer.

(d) Let x and y be functions of time t that are related by the equation $2x^2 + 3y^2 - 4xy = 36$. At time $t = 1$, the value of x is 2, the value of y is -2, and the value of $\frac{dy}{dt}$ is 4. Find the value of $\frac{dx}{dt}$ at time $t = 1$.

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Question 6

6. Consider the curve defined by $2x^2 + 3y^2 - 4xy = 36$.

(a) Show that $\frac{dy}{dx} = \frac{2y - 2x}{3y - 2x}$.

(b) Find the slope of the line tangent to the curve at each point on the curve where $x = 6$.

(c) Find the positive value of x at which the curve has a vertical tangent line. Show the work that leads to your answer.

(d) Let x and y be functions of time t that are related by the equation $2x^2 + 3y^2 - 4xy = 36$. At time $t = 1$, the value of x is 2, the value of y is -2, and the value of $\frac{dy}{dt}$ is 4. Find the value of $\frac{dx}{dt}$ at time $t = 1$.

(a) $\frac{d}{dx}\left(2x^2 + 3y^2 - 4xy\right) = \frac{d}{dx}(36) \Rightarrow 4x + 6y\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} = 0$

$\Rightarrow (6y - 4x)\frac{dy}{dx} = 4y - 4x \Rightarrow \frac{dy}{dx} = \frac{4y - 4x}{6y - 4x} = \frac{2y - 2x}{3y - 2x}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$

(b) $x = 6 \Rightarrow 2 \cdot 6^2 + 3y^2 - 4 \cdot 6 \cdot y = 36$

$\Rightarrow y^2 - 8y + 12 = 0 \Rightarrow y = 2 \text{ or } y = 6$

$\left.\frac{dy}{dx}\right|_{(x, y)=(6, 2)} = \frac{4 - 12}{6 - 12} = \frac{4}{3}$

The slope of the line tangent to the curve at $(6, 2)$ is $\frac{4}{3}$.

$\left.\frac{dy}{dx}\right|_{(x, y)=(6, 6)} = \frac{12 - 12}{18 - 12} = 0$

The slope of the line tangent to the curve at $(6, 6)$ is 0.

$2 : \begin{cases} 1 : \text{slope at } (6, 2) \\ 1 : \text{slope at } (6, 6) \end{cases}$

(c) The curve has vertical tangent lines where $3y - 2x = 0$ and $2y - 2x \neq 0$.

$3y - 2x = 0 \Rightarrow y = \frac{2}{3}x \Rightarrow 2x^2 + 3\left(\frac{2}{3}x\right)^2 - 4x \cdot \frac{2}{3}x = 36$

$\Rightarrow \left(2 + \frac{4}{3} - \frac{8}{3}\right)x^2 = \frac{2}{3}x^2 = 36 \Rightarrow x^2 = 54$

Because x is positive, $x = 3\sqrt{6}$.

$2 : \begin{cases} 1 : \text{sets } 3y - 2x = 0 \\ 1 : \text{answer} \end{cases}$

(d) $\frac{d}{dt}\left(2x^2 + 3y^2 - 4xy\right) = \frac{d}{dt}(36)$

$\Rightarrow 4x\frac{dx}{dt} + 6y\frac{dy}{dt} - 4\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right) = 0$

$\Rightarrow (4x - 4y)\frac{dx}{dt} + (6y - 4x)\frac{dy}{dt} = 0$

$(4 \cdot 2 - 4 \cdot (-2))\left.\frac{dx}{dt}\right|_{t=1} + (6 \cdot (-2) - 4 \cdot 2) \cdot 4 = 0 \Rightarrow \left.\frac{dx}{dt}\right|_{t=1} = 5$

— OR —

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{2y - 2x}{3y - 2x} \cdot \frac{dx}{dt}$

$4 = \frac{2 \cdot (-2) - 2 \cdot 2}{3 \cdot (-2) - 2 \cdot 2} \cdot \left.\frac{dx}{dt}\right|_{t=1} = \frac{4}{5} \cdot \left.\frac{dx}{dt}\right|_{t=1} \Rightarrow \left.\frac{dx}{dt}\right|_{t=1} = 5$

$3 : \begin{cases} 1 : \text{chain rules or product rule} \\ 1 : \text{derivative with respect to } t \\ 1 : \text{answer} \end{cases}$

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r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

1. The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f, where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

   (a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

   (b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression $2\pi \int_0^4 r f(r)\, dr$. Approximate the value of $2\pi \int_0^4 r f(r)\, dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.

   (c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

   (d) The density of bacteria in the petri dish, for $1 \le r \le 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k, $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \le r \le 4$?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

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(c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

(d) The density of bacteria in the petri dish, for $1 \le r \le 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k, $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \le r \le 4$?

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Part A (AB or BC): Graphing calculator required
Question 1 — 9 points

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f, where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

	Model Solution	Scoring	
(a)	Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.		
	$f'(2.25) \approx \frac{f(2.5)-f(2)}{2.5-2} = \frac{10-6}{0.5} = 8$	Estimate	**1 point**
	At a distance of $r = 2.25$ centimeters from the center of the petri dish, the density of the bacteria population is increasing at a rate of 8 milligrams per square centimeter per centimeter.	Interpretation with units	**1 point**
(b)	The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression $2\pi\int_0^4 r f(r)\,dr$. Approximate the value of $2\pi\int_0^4 r f(r)\,dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.		
	$2\pi\int_0^4 r f(r)\,dr \approx 2\pi(1\cdot f(1)\cdot(1-0) + 2\cdot f(2)\cdot(2-1) + 2.5\cdot f(2.5)\cdot(2.5-2) + 4\cdot f(4)\cdot(4-2.5))$	Right Riemann sum setup	**1 point**
	$= 2\pi(1\cdot 2\cdot 1 + 2\cdot 6\cdot 1 + 2.5\cdot 10\cdot 0.5 + 4\cdot 18\cdot 1.5)$ $= 269\pi = 845.088$	Approximation	**1 point**
(c)	Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.		
	$\frac{d}{dr}(r f(r)) = f(r) + r f'(r)$	Product rule expression for $\frac{d}{dr}(r f(r))$	**1 point**
	Because f is nonnegative and increasing, $\frac{d}{dr}(r f(r)) > 0$ on the interval $0 \le r \le 4$. Thus, the integrand $r f(r)$ is strictly increasing. Therefore, the right Riemann sum approximation of $2\pi\int_0^4 r f(r)\,dr$ is an overestimate.	Answer with explanation	**1 point**

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(d) The density of bacteria in the petri dish, for $1 \le r \le 4$, is modeled by the function g defined by $g(r) = 2 - 16\left(\cos\left(1.57\sqrt{r}\right)\right)^3$. For what value of k, $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \le r \le 4$?

Average value $= g_{\text{avg}} = \frac{1}{4-1}\int_1^4 g(r)\,dr$	Definite integral	**1 point**
$\frac{1}{4-1}\int_1^4 g(r)\,dr = 9.875795$	Average value	**1 point**
$g(k) = g_{\text{avg}} \Rightarrow k = 2.497$	Answer	**1 point**

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Part A (AB or BC): Graphing calculator required
Question 1 9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f, where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

	Model Solution	Scoring	
(a)	Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.		
	$f'(2.25) \approx \frac{f(2.5)-f(2)}{2.5-2} = \frac{10-6}{0.5} = 8$	Estimate	**1 point**
	At a distance of $r = 2.25$ centimeters from the center of the petri dish, the density of the bacteria population is increasing at a rate of 8 milligrams per square centimeter per centimeter.	Interpretation with units	**1 point**

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Scoring notes:

- To earn the first point, the response must provide both a difference and a quotient and must explicitly use values of f from the table.
- Simplification of the numerical value is not required to earn the first point. If the numerical value is simplified, it must be correct.
- The interpretation requires all of the following: distance $r = 2.25,$ density of bacteria (population) is increasing or changing, at a rate of $8,$ and units of milligrams per square centimeter per centimeter.
- The second point (interpretation) cannot be earned without a nonzero presented value for $f'(2.25)$.
- To earn the second point, the interpretation must be consistent with the presented nonzero value for $f'(2.25)$. In particular, if the presented value for $f'(2.25)$ is negative, the interpretation must include "decreasing at a rate of $|f'(2.25)|$" or "changing at a rate of $f'(2.25)$." The second point cannot be earned for an incorrect statement such as "the bacteria density is decreasing at a rate of -8 …" even for a presented $f'(2.25) = -8$.
- The units ($\text{mg/cm}^2\text{/cm}$) may be attached to the estimate of $f'(2.25)$ and if so, do not need to be repeated in the interpretation.
- If units attached to the estimate do not agree with units in the interpretation, read the units in the interpretation.

Total for part (a) 2 points

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(b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression $2\pi\int_0^4 r\,f(r)\,dr$. Approximate the value of $2\pi\int_0^4 r\,f(r)\,dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.

$2\pi\int_0^4 r\,f(r)\,dr \approx 2\pi(1\cdot f(1)\cdot(1-0)+2\cdot f(2)\cdot(2-1)$ $+2.5\cdot f(2.5)\cdot(2.5-2)+4\cdot f(4)\cdot(4-2.5))$	Right Riemann sum setup	**1 point**
$=2\pi(1\cdot 2\cdot 1+2\cdot 6\cdot 1+2.5\cdot 10\cdot 0.5+4\cdot 18\cdot 1.5)$ $=269\pi=845.088$	Approximation	**1 point**

Scoring notes:

- The presence or absence of 2π has no bearing on earning the first point.
- The first point is earned for a sum of 4 products with at most one error in any single value among the four products. Multiplication by 1 in any term does not need to be shown, but all other products must be explicitly shown.
- A response of $1\cdot f(1)\cdot(1-0)+2\cdot f(2)\cdot(2-1)+2.5\cdot f(2.5)\cdot(2.5-2)+4\cdot f(4)\cdot(4-2.5)$ earns the first point but not the second.
- A response with any error in the Riemann sum is not eligible for the second point.
- A response that provides a completely correct left Riemann sum for $2\pi\int_0^4 r\,f(r)\,dr$ and approximation (91π) earns one of the two points. A response that has any error in a left Riemann sum or evaluation for $2\pi\int_0^4 r\,f(r)\,dr$ earns no points.
- A response that provides a completely correct right Riemann sum for $2\pi\int_0^4 f(r)\,dr$ and approximation (80π) earns one of the two points. A response that has any error in a right Riemann sum or evaluation for $2\pi\int_0^4 f(r)\,dr$ earns no points.
- Simplification of the numerical value is not required to earn the second point. If a numerical value is given, it must be correct to three decimal places.

Total for part (b) 2 points

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(c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

$\frac{d}{dr}(rf(r)) = f(r) + rf'(r)$	Product rule expression for $\frac{d}{dr}(rf(r))$	**1 point**
Because f is nonnegative and increasing, $\frac{d}{dr}(rf(r)) > 0$ on the interval $0 \le r \le 4$. Thus, the integrand $rf(r)$ is strictly increasing. Therefore, the right Riemann sum approximation of $2\pi\int_0^4 rf(r)\,dr$ is an overestimate.	Answer with explanation	**1 point**

Scoring notes:

- To earn the second point, a response must explain that $rf(r)$ is increasing, and therefore the right Riemann sum is an overestimate. The second point can be earned without having earned the first point.
- A response that attempts to explain based on a left Riemann sum for $2\pi\int_0^4 rf(r)\,dr$ from part (b) earns no points.
- A response that attempts to explain based on a right Riemann sum for $2\pi\int_0^4 f(r)\,dr$ from part (b) earns no points.

Total for part (c) 2 points

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(d) The density of bacteria in the petri dish, for $1 \le r \le 4$, is modeled by the function g defined by $g(r) = 2 - 16\left(\cos\left(1.57\sqrt{r}\right)\right)^3$. For what value of k, $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \le r \le 4$?

Average value $= g_{\text{avg}} = \frac{1}{4-1}\int_1^4 g(r)\,dr$	Definite integral	**1 point**
$\frac{1}{4-1}\int_1^4 g(r)\,dr = 9.875795$	Average value	**1 point**
$g(k) = g_{\text{avg}} \Rightarrow k = 2.497$	Answer	**1 point**

Scoring notes:

- The first point is earned for a definite integral, with or without $\frac{1}{4-1}$ or $\frac{1}{3}$.
- A response that presents a definite integral with incorrect limits but a correct integrand earns the first point.
- Presentation of the numerical value 9.875795 is not required to earn the second point. This point can be earned by the average value setup: $\frac{1}{3}\int_1^4 g(r)\,dr$.
- Once earned for the average value setup, the second point cannot be lost. Subsequent errors will result in not earning the third point.
- The third point is earned only for the value $k = 2.497$.
- The third point cannot be earned without the second.
- Special case: A response that does not provide the average value setup but presents an average value of -13.955 is using degree mode on their calculator. This response would not earn the second point, but could earn the third point for an answer of $k = 2.5$ (or 2.499).

Total for part (d) 3 points

Total for question 1 9 points

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2. A particle, P, is moving along the x-axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \le t \le \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q, also moves along the x-axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \le t \le \pi$. At time $t = 0$, particle Q is at position $x = 10$.

(a) Find the positions of particles P and Q at time $t = 1$.

(b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

(c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

(d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

2. A particle, P, is moving along the x-axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \le t \le \pi$. At time $t = 0$, particle P is at position $x = 5$.

   A second particle, Q, also moves along the x-axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \le t \le \pi$. At time $t = 0$, particle Q is at position $x = 10$.

   (a) Find the positions of particles P and Q at time $t = 1$.

   (b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

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A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

(c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

(d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

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Part A (AB): Graphing calculator required
Question 2 — 9 points

A particle, P, is moving along the x-axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \le t \le \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q, also moves along the x-axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \le t \le \pi$. At time $t = 0$, particle Q is at position $x = 10$.

	Model Solution	Scoring	
(a)	Find the positions of particles P and Q at time $t = 1$.		
	$x_P(1) = 5 + \int_0^1 v_P(t)\,dt = 5.370660$ At time $t = 1$, the position of particle P is $x = 5.371$ (or 5.370).	One definite integral	**1 point**
		One position	**1 point**
		The other position	**1 point**
	$x_Q(1) = 10 + \int_0^1 v_Q(t)\,dt = 8.564355$ At time $t = 1$, the position of particle Q is $x = 8.564$.		
(b)	Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.		
	$v_P(1) = \sin\left(1^{1.5}\right) = 0.841471 > 0$ At time $t = 1$, particle P is moving to the right.	Direction of motion for one particle	**1 point**
	$v_Q(1) = (1 - 1.8) \cdot 1.25^1 = -1 < 0$ At time $t = 1$, particle Q is moving to the left. At time $t = 1$, $x_P(1) < x_Q(1)$, so particle P is to the left of particle Q. Thus, at time $t = 1$, particles P and Q are moving toward each other.	Answer with explanation	**1 point**
(c)	Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.		
	$a_Q(1) = v_Q'(1) = 1.026856$ The acceleration of particle Q is 1.027 (or 1.026) at time $t = 1$.	Setup and acceleration	**1 point**
	$v_Q(1) = -1 < 0$ and $a_Q(1) > 0$ The speed of particle Q is decreasing at time $t = 1$ because the velocity and acceleration have opposite signs.	Speed decreasing with reason	**1 point**

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(d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

$\int_0^{\pi} \lvert v_P(t) \rvert \, dt = 1.93148$ Over the time interval $0 \le t \le \pi$, the total distance traveled by particle P is 1.931.	Definite integral	**1 point**
	Answer	**1 point**

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Part A (AB): Graphing calculator required
Question 2 — 9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

A particle, P, is moving along the x-axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \le t \le \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q, also moves along the x-axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \le t \le \pi$. At time $t = 0$, particle Q is at position $x = 10$.

	Model Solution	Scoring	
(a)	Find the positions of particles P and Q at time $t = 1$.		
	$x_P(1) = 5 + \int_0^1 v_P(t)\,dt = 5.370660$ At time $t = 1$, the position of particle P is $x = 5.371$ (or 5.370).	One definite integral	**1 point**
		One position	**1 point**
	$x_Q(1) = 10 + \int_0^1 v_Q(t)\,dt = 8.564355$ At time $t = 1$, the position of particle Q is $x = 8.564$.	The other position	**1 point**

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Scoring notes:

- The first point is earned for the explicit presentation of at least one definite integral, either $\int_0^1 v_P(t)\,dt$ or $\int_0^1 v_Q(t)\,dt$.
- The first point must be earned to be eligible for the second and third points.
- The second point is earned for adding the initial condition to at least one of the definite integrals and finding the correct position.
- Writing $\int_0^1 v_P(t) + 5 = 5.370660$ does not earn a position point, because the missing dt makes this statement unclear or false. However, $5 + \int_0^1 v_P(t) = 5.370660$ does earn the position point because it is not ambiguous. Similarly, for the position of Q.
- Read unlabeled answers presented left to right, or top to bottom, as $x_P(1)$ and $x_Q(1)$, respectively.
- Special case 1: A response of $x_P(1) = 5 + \int_0^a v_P(t)\,dt = 5.370660$ AND $x_Q(1) = 10 + \int_0^a v_Q(t)\,dt = 8.564355$ for $a \neq 1$ earns one point.
- Special case 2: A response of $x_P(1) = 5 + \int v_P(t)\,dt = 5.370660$ AND $x_Q(1) = 10 + \int v_Q(t)\,dt = 8.564355$ or the equivalent, never providing the definite integrals, earns one point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $x_P(1)$ is 5.007 (or 5.006).

Total for part (a) 3 points

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(b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

$v_P(1) = \sin\left(1^{1.5}\right) = 0.841471 > 0$ At time $t = 1$, particle P is moving to the right.	Direction of motion for one particle	**1 point**
$v_Q(1) = (1 - 1.8) \cdot 1.25^1 = -1 < 0$ At time $t = 1$, particle Q is moving to the left. At time $t = 1$, $x_P(1) < x_Q(1)$, so particle P is to the left of particle Q. Thus, at time $t = 1$, particles P and Q are moving toward each other.	Answer with explanation	**1 point**

Scoring notes:

- The first point is earned for using the sign of $v_P(1)$ or $v_Q(1)$ to determine the direction of motion for one of the particles. This point cannot be earned without reference to the sign of $v_P(1)$ or $v_Q(1)$.
- It is not necessary to present an explicit value for $v_P(1)$, or $v_Q(1)$, but if a value is presented, it must be correct as far as reported, up to three places after the decimal.
- Read with imported incorrect position values from part (a).
- If one or both position values were not found in part (a), but are found in part (b), the points for part (a) are not earned retroactively.
- To earn the second point, the explanation must be based on the signs of $v_P(1)$ and $v_Q(1)$ and the relative positions of P and Q at $t = 1$. References to other values of time, such as $t = 0$, are not sufficient.
- Degree mode: $v_P(1) = 0.017$ (See degree mode statement in part (a).)

Total for part (b) 2 points

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(c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

$a_Q(1) = v_Q'(1) = 1.026856$ The acceleration of particle Q is 1.027 (or 1.026) at time $t = 1$.	Setup and acceleration	**1 point**
$v_Q(1) = -1 < 0$ and $a_Q(1) > 0$ The speed of particle Q is decreasing at time $t = 1$ because the velocity and acceleration have opposite signs.	Speed decreasing with reason	**1 point**

Scoring notes:

- To earn the first point, the acceleration must be explicitly connected to v_Q' (e.g., $v_Q'(1) = 1.026856$).
- The first point is not earned for an unsupported value of 1.027 (or 1.026). The setup, $v_Q'(1)$, must be shown. Presenting only $a_Q(1) = 1.027$ (or 1.026) without indication that $v_Q' = a_Q$ is not enough to earn the first point.
- A response does not need to present a value for $v_Q(1)$; the sign is sufficient.
- To earn the second point, a response must compare the signs of a_Q and v_Q at $t = 1$. Considering only one sign is not sufficient.
- After the first point has been earned, a response declaring only "velocity and acceleration are of opposite signs at $t = 1$ so the particle is slowing down" (or equivalent) earns the second point.
- The second point may be earned without the first, as long as the response does not present an incorrect value or sign for $v_Q(1)$ and concludes the particle is slowing down because velocity and acceleration have opposite signs at $t = 1$.

Total for part (c) 2 points

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(d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

$\int_0^{\pi} \lvert v_P(t) \rvert \, dt = 1.93148$ Over the time interval $0 \le t \le \pi$, the total distance traveled by particle P is 1.931.	Definite integral	**1 point**
	Answer	**1 point**

Scoring notes:

- The first point is earned for $\int_0^{\pi} \lvert v_P(t) \rvert \, dt$.
- The first point can also be earned for a sum (or difference) of definite integrals, such as $\int_0^{2.145029} v_P(t) \, dt - \int_{2.145029}^{\pi} v_P(t) \, dt$, provided the response has indicated $v_P(2.145029) = 0$.
- The second point can only be earned for the correct answer.
- The unsupported value 1.931 earns no points.
- A response reporting the distance traveled by particle Q as $\int_0^{\pi} \lvert v_Q(t) \rvert \, dt = 3.506$ earns the first point and is not eligible for the second point.
- In degree mode, the total distance traveled is 0.122. (See degree mode statement in part (a).) In the degree mode case, the response must present $\int_0^{\pi} \lvert v_P(t) \rvert \, dt$ in order to earn the first point because $\int_0^{\pi} \lvert v_P(t) \rvert \, dt = \int_0^{\pi} v_P(t) \, dt$.

Total for part (d) 2 points

Total for question 2 9 points

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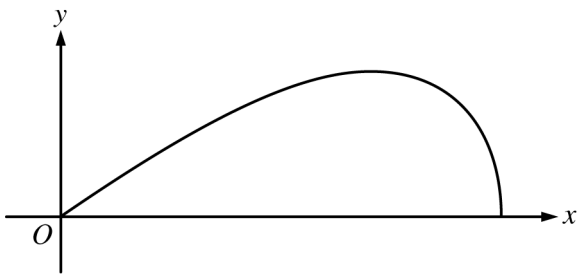

3. A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

   (a) Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

   (b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

   (c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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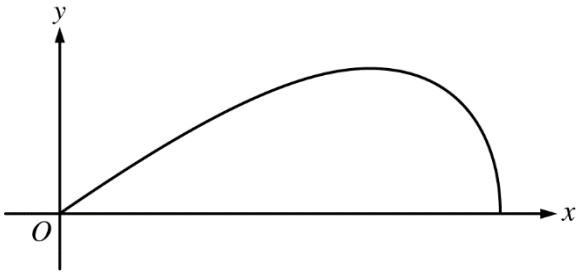
Part B (AB or BC): Graphing calculator not allowed
Question 3 — 9 points



A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

	Model Solution	Scoring	
(a)	Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.		
	$6x\sqrt{4 - x^2} = 0 \Rightarrow x = 0, x = 2$ Area $= \int_0^2 6x\sqrt{4 - x^2}\, dx$	Integrand	**1 point**
	Let $u = 4 - x^2$. $du = -2x\, dx \Rightarrow -\frac{1}{2}\, du = x\, dx$ $x = 0 \Rightarrow u = 4 - 0^2 = 4$ $x = 2 \Rightarrow u = 4 - 2^2 = 0$ $\int_0^2 6x\sqrt{4 - x^2}\, dx = \int_4^0 6\left(-\frac{1}{2}\right)\sqrt{u}\, du = -3\int_4^0 u^{1/2}\, du = 3\int_0^4 u^{1/2}\, du$ $= 2u^{3/2}\Big\|_{u=0}^{u=4} = 2 \cdot 8 = 16$	Antiderivative	**1 point**
	The area of the region is 16 square inches.	Answer	**1 point**

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(b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

The cross-sectional circular slice with the largest radius occurs where $cx\sqrt{4 - x^2}$ has its maximum on the interval $0 < x < 2$. $\frac{dy}{dx} = \frac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}} = 0 \Rightarrow x = \sqrt{2}$	Sets $\frac{dy}{dx} = 0$	**1 point**
$x = \sqrt{2} \Rightarrow y = c\sqrt{2}\sqrt{4 - \left(\sqrt{2}\right)^2} = 2c$ $2c = 1.2 \Rightarrow c = 0.6$	Answer	**1 point**

(c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Volume $= \int_0^2 \pi\left(cx\sqrt{4 - x^2}\right)^2 dx = \pi c^2 \int_0^2 x^2\left(4 - x^2\right) dx$	Form of the integrand	**1 point**
	Limits and constant	**1 point**
$= \pi c^2 \int_0^2 \left(4x^2 - x^4\right) dx = \pi c^2 \left(\frac{4}{3}x^3 - \frac{1}{5}x^5 \Big\vert_0^2\right)$	Antiderivative	**1 point**
$= \pi c^2 \left(\frac{32}{3} - \frac{32}{5}\right) = \frac{64\pi c^2}{15}$ $\frac{64\pi c^2}{15} = 2\pi \Rightarrow c^2 = \frac{15}{32} \Rightarrow c = \sqrt{\frac{15}{32}}$	Answer	**1 point**

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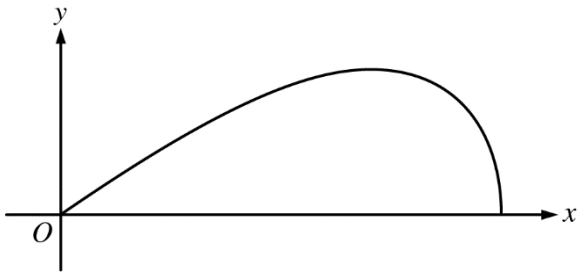
Part B (AB or BC): Graphing calculator not allowed
Question 3 9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.



A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

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Model Solution	Scoring

(a) Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

$6x\sqrt{4 - x^2} = 0 \Rightarrow x = 0, x = 2$ Area $= \int_0^2 6x\sqrt{4 - x^2}\, dx$	Integrand	**1 point**
Let $u = 4 - x^2$. $du = -2x\, dx \Rightarrow -\frac{1}{2}\, du = x\, dx$ $x = 0 \Rightarrow u = 4 - 0^2 = 4$ $x = 2 \Rightarrow u = 4 - 2^2 = 0$ $\int_0^2 6x\sqrt{4 - x^2}\, dx = \int_4^0 6\left(-\frac{1}{2}\right)\sqrt{u}\, du = -3\int_4^0 u^{1/2}\, du = 3\int_0^4 u^{1/2}\, du$ $= 2u^{3/2}\Big\|_{u=0}^{u=4} = 2 \cdot 8 = 16$	Antiderivative	**1 point**
The area of the region is 16 square inches.	Answer	**1 point**

Scoring notes:

- Units are not required for any points in this question and are not read if presented (correctly or incorrectly) in any part of the response.
- The first point is earned for presenting $cx\sqrt{4 - x^2}$ or $6x\sqrt{4 - x^2}$ as the integrand in a definite integral. Limits of integration (numeric or alphanumeric) must be presented (as part of the definite integral), but do not need to be correct in order to earn the first point.
- If an indefinite integral is presented with an integrand of the correct form, the first point can be earned if the antiderivative (correct or incorrect) is eventually evaluated using the correct limits of integration.
- The second point can be earned without the first point. The second point is earned for presentation of a correct antiderivative of a function of the form $Ax\sqrt{4 - x^2}$, for any nonzero constant A. If the response has subsequent errors in simplification of the antiderivative or sign errors, the response will earn the second point but will not earn the third point.
- Responses that use u-substitution and have incorrect limits of integration or do not change the limits of integration from x- to u-values are eligible for the second point.
- The response is eligible for the third point only if it has earned the second point.
- The third point is earned only for the answer 16 or equivalent. In the case where a response only presents an indefinite integral, the use of the correct limits of integration to evaluate the antiderivative must be shown to earn the third point.
- The response cannot correct -16 to $+16$ in order to earn the third point; there is no possible reversal here.

Total for part (a) 3 points

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(b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

The cross-sectional circular slice with the largest radius occurs where $cx\sqrt{4 - x^2}$ has its maximum on the interval $0 < x < 2$. $\frac{dy}{dx} = \frac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}} = 0 \Rightarrow x = \sqrt{2}$	Sets $\frac{dy}{dx} = 0$ **1 point**
$x = \sqrt{2} \Rightarrow y = c\sqrt{2}\sqrt{4 - \left(\sqrt{2}\right)^2} = 2c$ $2c = 1.2 \Rightarrow c = 0.6$	Answer **1 point**

Scoring notes:

- The first point is earned for setting $\frac{dy}{dx} = 0$, $\frac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}} = 0$, or $c\left(4 - 2x^2\right) = 0$.
- An unsupported $x = \sqrt{2}$ does not earn the first point.
- The second point can be earned without the first point but is earned only for the answer $c = 0.6$ with supporting work.

Total for part (b) 2 points

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(c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

$\text{Volume} = \int_0^2 \pi\left(cx\sqrt{4-x^2}\right)^2\,dx = \pi c^2\int_0^2 x^2\left(4-x^2\right)\,dx$	Form of the integrand	**1 point**
	Limits and constant	**1 point**
$= \pi c^2\int_0^2\left(4x^2-x^4\right)\,dx = \pi c^2\left(\frac{4}{3}x^3-\frac{1}{5}x^5\Big\|_0^2\right)$	Antiderivative	**1 point**
$= \pi c^2\left(\frac{32}{3}-\frac{32}{5}\right) = \frac{64\pi c^2}{15}$ $\frac{64\pi c^2}{15} = 2\pi \Rightarrow c^2 = \frac{15}{32} \Rightarrow c = \sqrt{\frac{15}{32}}$	Answer	**1 point**

Scoring notes:

- The first point is earned for presenting an integrand of the form $A\left(x\sqrt{4-x^2}\right)^2$ in a definite integral with any limits of integration (numeric or alphanumeric) and any nonzero constant A. Mishandling the c will result in the response being ineligible for the fourth point.
- The second point can be earned without the first point. The second point is earned for the limits of integration, $x = 0$ and $x = 2$, and the constant π (but not for 2π) as part of an integral with a correct or incorrect integrand.
- If an indefinite integral is presented with the correct constant π, the second point can be earned if the antiderivative (correct or incorrect) is evaluated using the correct limits of integration.
- A response that presents $2 = \int_0^2\left(cx\sqrt{4-x^2}\right)^2\,dx$ earns the first and second points.
- The third point is earned for presenting a correct antiderivative of the presented integrand of the form $A\left(x\sqrt{4-x^2}\right)^2$ for any nonzero A. If there are subsequent errors in simplification of the antiderivative, linkage errors, or sign errors, the response will not earn the fourth point.
- The fourth point cannot be earned without the third point. The fourth point is earned only for the correct answer. The expression does not need to be simplified to earn the fourth point.

Total for part (c) 4 points

Total for question 3 9 points

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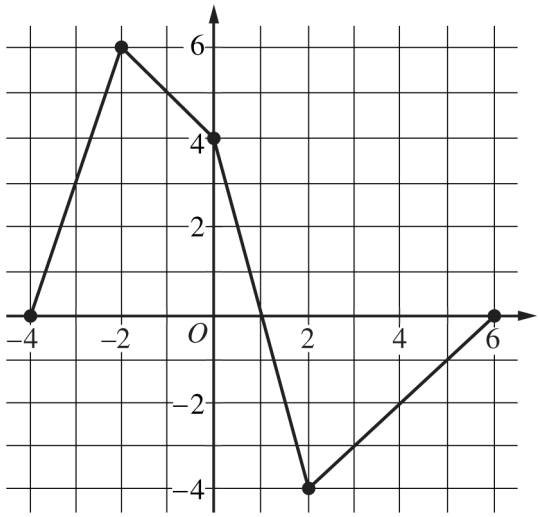

Graph of f

4. Let f be a continuous function defined on the closed interval $-4 \le x \le 6$. The graph of f, consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t)\,dt$.

(a) On what open intervals is the graph of G concave up? Give a reason for your answer.

(b) Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.

(c) Find $\lim\limits_{x \to 2} \dfrac{G(x)}{x^2 - 2x}$.

(d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c, $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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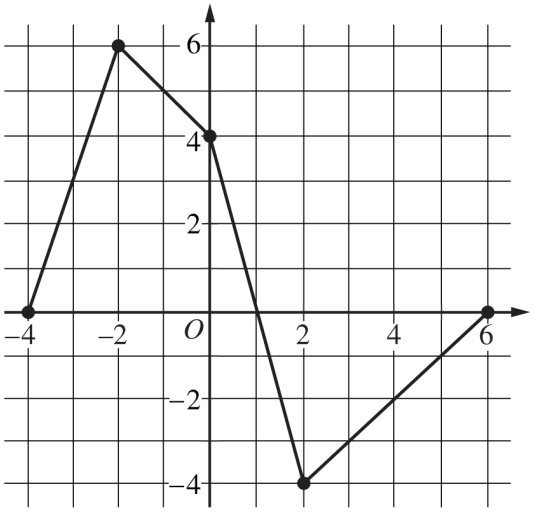

Graph of f

4. Let f be a continuous function defined on the closed interval $-4 \le x \le 6$. The graph of f, consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t)\,dt$.

(a) On what open intervals is the graph of G concave up? Give a reason for your answer.

(b) Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.

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(c) Find $\lim_{x \to 2} \dfrac{G(x)}{x^2 - 2x}$.

(d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c, $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

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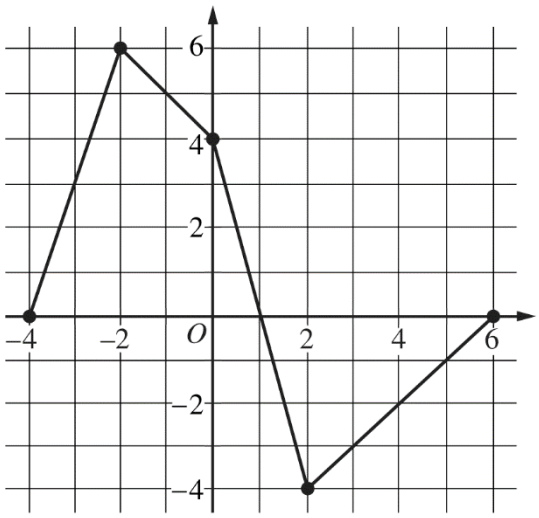
Part B (AB or BC): Graphing calculator not allowed

Question 4 — 9 points



Graph of f

Let f be a continuous function defined on the closed interval $-4 \le x \le 6$. The graph of f, consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t)\,dt$.

	Model Solution	Scoring	
	$G'(x) = f(x)$ in any part of the response	$G'(x) = f(x)$	**1 point**
(a)	On what open intervals is the graph of G concave up? Give a reason for your answer.		
	$G'(x) = f(x)$ The graph of G is concave up for $-4 < x < -2$ and $2 < x < 6$, because $G' = f$ is increasing on these intervals.	Answer with reason	**1 point**
(b)	Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.		
	$P'(x) = G(x) \cdot f'(x) + f(x) \cdot G'(x)$ $P'(3) = G(3) \cdot f'(3) + f(3) \cdot G'(3)$	Product rule	**1 point**
	Substituting $G(3) = \int_0^3 f(t)\,dt = -3.5$ and $G'(3) = f(3) = -3$ into the above expression for $P'(3)$ gives the following:	$G(3)$ or $G'(3)$	**1 point**
	$P'(3) = -3.5 \cdot 1 + (-3) \cdot (-3) = 5.5$	Answer	**1 point**

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(c) Find $\lim_{x \to 2} \frac{G(x)}{x^2 - 2x}$.

$\lim_{x \to 2} \left(x^2 - 2x\right) = 0$ Because G is continuous for $-4 \le x \le 6$, $\lim_{x \to 2} G(x) = \int_0^2 f(t)\, dt = 0$. Therefore, the limit $\lim_{x \to 2} \frac{G(x)}{x^2 - 2x}$ is an indeterminate form of type $\frac{0}{0}$.	Uses L'Hospital's Rule	**1 point**
Using L'Hospital's Rule, $\lim_{x \to 2} \frac{G(x)}{x^2 - 2x} = \lim_{x \to 2} \frac{G'(x)}{2x - 2}$ $= \lim_{x \to 2} \frac{f(x)}{2x - 2} = \frac{f(2)}{2} = \frac{-4}{2} = -2$	Answer with justification	**1 point**

(d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c, $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

$G(2) = \int_0^2 f(t)\, dt = 0$ and $G(-4) = \int_0^{-4} f(t)\, dt = -16$ Average rate of change $= \frac{G(2) - G(-4)}{2 - (-4)} = \frac{0 - (-16)}{6} = \frac{8}{3}$	Average rate of change	**1 point**
Yes, $G'(x) = f(x)$ so G is differentiable on $(-4, 2)$ and continuous on $[-4, 2]$. Therefore, the Mean Value Theorem applies and guarantees a value c, $-4 < c < 2$, such that $G'(c) = \frac{8}{3}$.	Answer with justification	**1 point**

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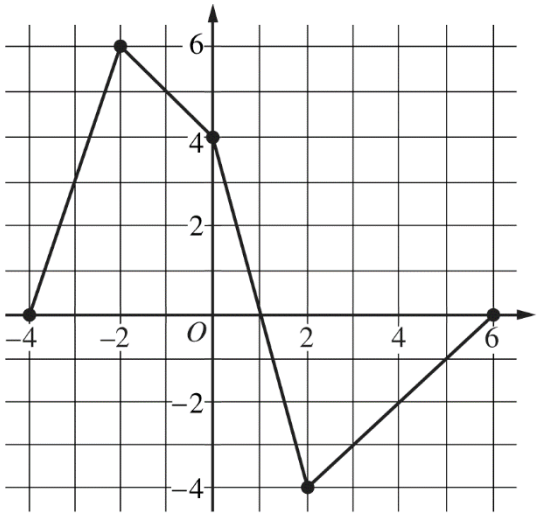
Part B (AB or BC): Graphing calculator not allowed
Question 4 — 9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.



Graph of f

Let f be a continuous function defined on the closed interval $-4 \le x \le 6$. The graph of f, consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t)\,dt$.

Model Solution	Scoring	
$G'(x) = f(x)$ in any part of the response	$G'(x) = f(x)$	**1 point**

Scoring notes:

- This "global point" can be earned in any one part. Expressions that show this connection and therefore earn this point include: $G' = f$, $G'(x) = f(x)$, $G''(x) = f'(x)$ in part (a), $G'(3) = f(3)$ in part (b), or $G'(2) = f(2)$ in part (c).

Total 1 point

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(a) On what open intervals is the graph of G concave up? Give a reason for your answer.

$G'(x) = f(x)$ The graph of G is concave up for $-4 < x < -2$ and $2 < x < 6$, because $G' = f$ is increasing on these intervals.	Answer with reason	**1 point**

Scoring notes:

- Intervals may also include one or both endpoints.

Total for part (a) 1 point

(b) Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.

$P'(x) = G(x) \cdot f'(x) + f(x) \cdot G'(x)$ $P'(3) = G(3) \cdot f'(3) + f(3) \cdot G'(3)$	Product rule	**1 point**
Substituting $G(3) = \int_0^3 f(t)\,dt = -3.5$ and $G'(3) = f(3) = -3$ into the above expression for $P'(3)$ gives the following:	$G(3)$ or $G'(3)$	**1 point**
$P'(3) = -3.5 \cdot 1 + (-3) \cdot (-3) = 5.5$	Answer	**1 point**

Scoring notes:

- The first point is earned for the correct application of the product rule in terms of x or in the evaluation of $P'(3)$. Once earned, this point cannot be lost.
- The second point is earned by correctly evaluating $G(3) = -3.5$, $G'(3) = -3$, or $f(3) = -3$.
- To be eligible to earn the third point, a response must have earned the first two points. Simplification of the numerical value is not required to earn the third point.

Total for part (b) 3 points

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(c) Find $\lim_{x\to 2}\dfrac{G(x)}{x^2-2x}$.

$\lim_{x\to 2}\left(x^2-2x\right)=0$ Because G is continuous for $-4\le x\le 6$, $\lim_{x\to 2}G(x)=\int_0^2 f(t)\,dt=0$. Therefore, the limit $\lim_{x\to 2}\dfrac{G(x)}{x^2-2x}$ is an indeterminate form of type $\dfrac{0}{0}$.	Uses L'Hospital's Rule	**1 point**
Using L'Hospital's Rule, $\lim_{x\to 2}\dfrac{G(x)}{x^2-2x}=\lim_{x\to 2}\dfrac{G'(x)}{2x-2}$ $=\lim_{x\to 2}\dfrac{f(x)}{2x-2}=\dfrac{f(2)}{2}=\dfrac{-4}{2}=-2$	Answer with justification	**1 point**

Scoring notes:

- To earn the first point, the response must show $\lim_{x\to 2}\left(x^2-2x\right)=0$ and $\lim_{x\to 2}G(x)=0$ and must show a ratio of the two derivatives, $G'(x)$ and $2x-2$. The ratio may be shown as evaluations of the derivatives at $x=2$, such as $\dfrac{G'(2)}{2}$.
- To earn the second point, the response must evaluate correctly with appropriate limit notation. In particular the response must include $\lim_{x\to 2}\dfrac{G'(x)}{2x-2}$ or $\lim_{x\to 2}\dfrac{f(x)}{2x-2}$.
- With any linkage errors (such as $\dfrac{G'(x)}{2x-2}=\dfrac{f(2)}{2}$), the response does not earn the second point.

Total for part (c) 2 points

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(d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c, $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

$G(2) = \int_0^2 f(t)\,dt = 0$ and $G(-4) = \int_0^{-4} f(t)\,dt = -16$ Average rate of change $= \dfrac{G(2) - G(-4)}{2 - (-4)} = \dfrac{0 - (-16)}{6} = \dfrac{8}{3}$	Average rate of change	**1 point**
Yes, $G'(x) = f(x)$ so G is differentiable on $(-4, 2)$ and continuous on $[-4, 2]$. Therefore, the Mean Value Theorem applies and guarantees a value c, $-4 < c < 2$, such that $G'(c) = \dfrac{8}{3}$.	Answer with justification	**1 point**

Scoring notes:

- To earn the first point, a response must present at least a difference and a quotient and a correct evaluation. For example, $\dfrac{0 + 16}{6}$ or $\dfrac{G(2) - G(-4)}{6} = \dfrac{16}{6}$.
- Simplification of the numerical value is not required to earn the first point, but any simplification must be correct.
- The second point can be earned without the first point if the response has the correct setup but an incorrect or no evaluation of the average rate of change. The Mean Value Theorem need not be explicitly stated provided the conditions and conclusion are stated.

Total for part (d) 2 points

Total for question 4 9 points

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5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

   (a) Show that $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$.

   (b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

   (c) For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

   (d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y \cos x}{4y - \sin x}$.

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

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(c) For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

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Part B (AB): Graphing calculator not allowed
Question 5 — 9 points

Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

	Model Solution	Scoring	
(a)	Show that $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$.		
	$\frac{d}{dx}\left(2y^2 - 6\right) = \frac{d}{dx}(y \sin x) \Rightarrow 4y\frac{dy}{dx} = \frac{dy}{dx}\sin x + y \cos x$	Implicit differentiation	**1 point**
	$\Rightarrow 4y\frac{dy}{dx} - \frac{dy}{dx}\sin x = y \cos x \Rightarrow \frac{dy}{dx}(4y - \sin x) = y \cos x$ $\Rightarrow \frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$	Verification	**1 point**
(b)	Write an equation for the line tangent to the curve at the point $\left(0, \sqrt{3}\right)$.		
	At the point $\left(0, \sqrt{3}\right)$, $\frac{dy}{dx} = \frac{\sqrt{3}\cos 0}{4\sqrt{3} - \sin 0} = \frac{1}{4}$. An equation for the tangent line is $y = \sqrt{3} + \frac{1}{4}x$.	Answer	**1 point**
(c)	For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.		
	$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x} = 0 \Rightarrow y \cos x = 0$ and $4y - \sin x \neq 0$	Sets $\frac{dy}{dx} = 0$	**1 point**
	$y \cos x = 0$ and $y > 0 \Rightarrow x = \frac{\pi}{2}$	$x = \frac{\pi}{2}$	**1 point**
	When $x = \frac{\pi}{2}$, $y \sin x = 2y^2 - 6 \Rightarrow y \sin\frac{\pi}{2} = 2y^2 - 6$ $\Rightarrow y = 2y^2 - 6 \Rightarrow 2y^2 - y - 6 = 0$ $\Rightarrow (2y + 3)(y - 2) = 0 \Rightarrow y = 2$ When $x = \frac{\pi}{2}$ and $y = 2$, $4y - \sin x = 8 - 1 \neq 0$. Therefore, the line tangent to the curve is horizontal at the point $\left(\frac{\pi}{2}, 2\right)$.	$y = 2$	**1 point**

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(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

$\frac{d^2y}{dx^2} = \frac{(4y - \sin x)\left(\frac{dy}{dx}\cos x - y\sin x\right) - (y\cos x)\left(4\frac{dy}{dx} - \cos x\right)}{(4y - \sin x)^2}$	Considers $\frac{d^2y}{dx^2}$	**1 point**
When $x = \frac{\pi}{2}$ and $y = 2$, $\frac{d^2y}{dx^2} = \frac{\left(4\cdot 2 - \sin\frac{\pi}{2}\right)\left(0\cdot\cos\frac{\pi}{2} - 2\cdot\sin\frac{\pi}{2}\right) - \left(2\cos\frac{\pi}{2}\right)\left(4\cdot 0 - \cos\frac{\pi}{2}\right)}{\left(4\cdot 2 - \sin\frac{\pi}{2}\right)^2}$ $= \frac{(7)(-2) - (0)(0)}{(7)^2} = \frac{-2}{7} < 0.$	$\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{2}, 2\right)$	**1 point**
f has a relative maximum at the point $\left(\frac{\pi}{2}, 2\right)$ because $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$.	Answer with justification	**1 point**

Alternate Solution for part (d)	Scoring for Alternate Solution	
For the function $y = f(x)$ near the point $\left(\frac{\pi}{2}, 2\right)$, $4y - \sin x > 0$ and $y > 0$.	Considers sign of $4y - \sin x$	**1 point**
Thus, $\frac{dy}{dx} = \frac{y\cos x}{4y - \sin x}$ changes from positive to negative at $x = \frac{\pi}{2}$.	$\frac{dy}{dx}$ changes from positive to negative at $x = \frac{\pi}{2}$	**1 point**
By the First Derivative Test, f has a relative maximum at the point $\left(\frac{\pi}{2}, 2\right)$.	Conclusion	**1 point**

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Part B (AB): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

(a) Show that $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$.

Model Solution	Scoring	
$\frac{d}{dx}\left(2y^2 - 6\right) = \frac{d}{dx}(y \sin x) \Rightarrow 4y\frac{dy}{dx} = \frac{dy}{dx}\sin x + y \cos x$	Implicit differentiation	**1 point**
$\Rightarrow 4y\frac{dy}{dx} - \frac{dy}{dx}\sin x = y \cos x \Rightarrow \frac{dy}{dx}(4y - \sin x) = y \cos x$ $\Rightarrow \frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$	Verification	**1 point**

Scoring notes:

- The first point is earned only for correctly implicitly differentiating $2y^2 - 6 = y \sin x$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y'.
- The second point may not be earned without the first point.
- It is sufficient to present $\frac{dy}{dx}(4y - \sin x) = y \cos x$ to earn the second point, provided that there are no subsequent errors.

Total for part (a) 2 points

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(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

At the point $(0, \sqrt{3})$, $\frac{dy}{dx} = \frac{\sqrt{3}\cos 0}{4\sqrt{3} - \sin 0} = \frac{1}{4}$. An equation for the tangent line is $y = \sqrt{3} + \frac{1}{4}x$.	Answer	**1 point**

Scoring notes:

- Any correct tangent line equation will earn the point. No supporting work is required. Simplification of the slope value is not required.

Total for part (b) 1 point

(c) For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

$\frac{dy}{dx} = \frac{y\cos x}{4y - \sin x} = 0 \Rightarrow y\cos x = 0$ and $4y - \sin x \ne 0$	Sets $\frac{dy}{dx} = 0$	**1 point**
$y\cos x = 0$ and $y > 0 \Rightarrow x = \frac{\pi}{2}$	$x = \frac{\pi}{2}$	**1 point**
When $x = \frac{\pi}{2}$, $y\sin x = 2y^2 - 6 \Rightarrow y\sin\frac{\pi}{2} = 2y^2 - 6$ $\Rightarrow y = 2y^2 - 6 \Rightarrow 2y^2 - y - 6 = 0$ $\Rightarrow (2y + 3)(y - 2) = 0 \Rightarrow y = 2$ When $x = \frac{\pi}{2}$ and $y = 2$, $4y - \sin x = 8 - 1 \ne 0$. Therefore, the line tangent to the curve is horizontal at the point $\left(\frac{\pi}{2}, 2\right)$.	$y = 2$	**1 point**

Scoring notes:

- The first point is earned by any of $\frac{dy}{dx} = 0$, $\frac{y\cos x}{4y - \sin x} = 0$, $y\cos x = 0$, or $\cos x = 0$.
- If additional "correct" x-values are considered outside of the given domain, the response must commit to only $x = \frac{\pi}{2}$ to earn the second point. Any presented y-values, correct or incorrect, are not considered for the second point.
- Entering with $x = \frac{\pi}{2}$ does not earn the first point, earns the second point, and is eligible for the third point. The third point is earned for finding $y = 2$. The coordinates do not have to be presented as an ordered pair.
- The third point is not earned with additional points present unless the response commits to the correct point.

Total for part (c) 3 points

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(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

$\frac{d^2y}{dx^2} = \frac{(4y - \sin x)\left(\frac{dy}{dx}\cos x - y\sin x\right) - (y\cos x)\left(4\frac{dy}{dx} - \cos x\right)}{(4y - \sin x)^2}$	Considers $\frac{d^2y}{dx^2}$	**1 point**
When $x = \frac{\pi}{2}$ and $y = 2$, $\frac{d^2y}{dx^2} = \frac{\left(4\cdot 2 - \sin\frac{\pi}{2}\right)\left(0\cdot\cos\frac{\pi}{2} - 2\cdot\sin\frac{\pi}{2}\right) - \left(2\cos\frac{\pi}{2}\right)\left(4\cdot 0 - \cos\frac{\pi}{2}\right)}{\left(4\cdot 2 - \sin\frac{\pi}{2}\right)^2}$ $= \frac{(7)(-2) - (0)(0)}{(7)^2} = \frac{-2}{7} < 0.$	$\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{2}, 2\right)$	**1 point**
f has a relative maximum at the point $\left(\frac{\pi}{2}, 2\right)$ because $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0.$	Answer with justification	**1 point**

Scoring notes:

- The first point is earned for an attempt to use the quotient rule (or product rule) to find $\frac{d^2y}{dx^2}$.
- The second point is earned for correctly finding $\frac{d^2y}{dx^2}$ and evaluating to find that $\frac{d^2y}{dx^2} < 0$ at $\left(\frac{\pi}{2}, 2\right)$. The explicit value of $-\frac{2}{7}$ or the equivalent does not need to be reported, but any reported values must be correct in order to earn this point.
- The third point can be earned without the second point by reaching a consistent conclusion based on the reported sign of a nonzero value of $\frac{d^2y}{dx^2}$ obtained utilizing $\frac{dy}{dx} = 0$.
- Imports: A response is eligible to earn all 3 points in part (d) with a point of the form $\left(\frac{\pi}{2}, k\right)$ with $k > 0,$ imported from part (c).

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Alternate Solution for part (d)	Scoring for Alternate Solution	
For the function $y = f(x)$ near the point $\left(\frac{\pi}{2}, 2\right)$, $4y - \sin x > 0$ and $y > 0$.	Considers sign of $4y - \sin x$	**1 point**
Thus, $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$ changes from positive to negative at $x = \frac{\pi}{2}$.	$\frac{dy}{dx}$ changes from positive to negative at $x = \frac{\pi}{2}$	**1 point**
By the First Derivative Test, f has a relative maximum at the point $\left(\frac{\pi}{2}, 2\right)$.	Conclusion	**1 point**

Scoring notes:

- The first point for considering the sign of $4y - \sin x$ may also be earned by stating that $4y - \sin x$ is not equal to zero.
- The second and third points can be earned without the first point.
- To earn the second point, a response must state that $\frac{dy}{dx}$ (or $\cos x$) changes from positive to negative at $x = \frac{\pi}{2}$.
- The third point cannot be earned without the second point.
- A response that concludes there is a minimum at this point does not earn the third point.

Total for part (d) 3 points

Total for question 5 9 points

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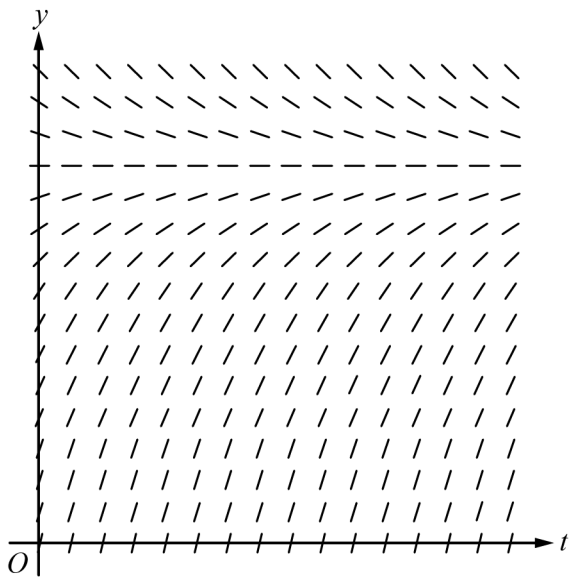
6. A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

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(d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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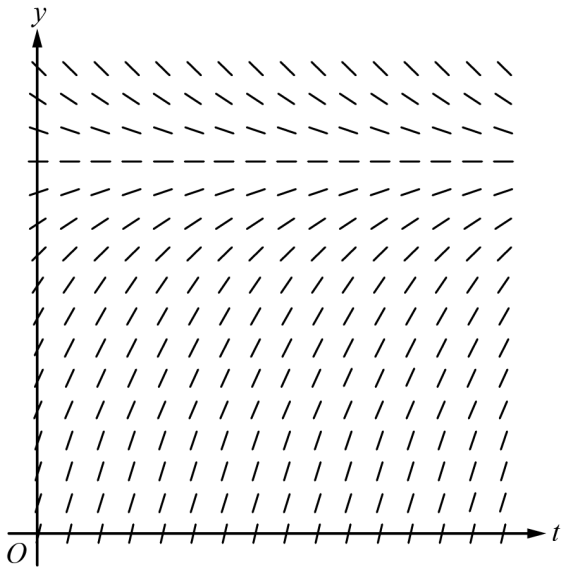
NO CALCULATOR ALLOWED

6. A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

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Do not write beyond this border.

NO CALCULATOR ALLOWED

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

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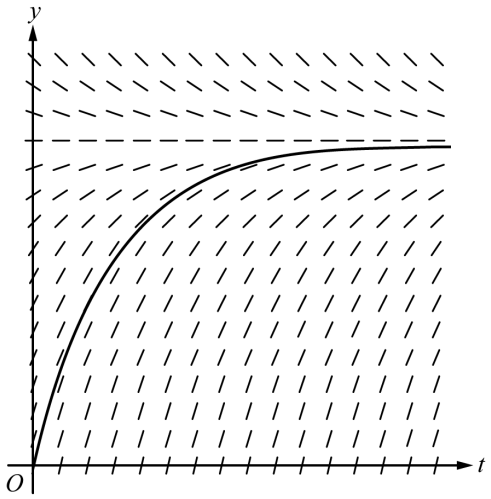
Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

Model Solution	Scoring

(a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



Solution curve		**1 point**

(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

Over time the amount of medication in the patient approaches 12 milligrams.	Interpretation	**1 point**

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

$\frac{dy}{dt} = \frac{12 - y}{3} \Rightarrow \frac{dy}{12 - y} = \frac{dt}{3}$	Separation of variables	**1 point**
$\int \frac{dy}{12 - y} = \int \frac{dt}{3} \Rightarrow -\ln\lvert 12 - y\rvert = \frac{t}{3} + C$	Antiderivatives	**1 point**
$\ln\lvert 12 - y\rvert = -\frac{t}{3} - C \Rightarrow \lvert 12 - y\rvert = e^{-t/3 - C}$ $\Rightarrow y = 12 + Ke^{-t/3}$ $0 = 12 + K \Rightarrow K = -12$	Constant of integration and uses initial condition	**1 point**
$y = A(t) = 12 - 12e^{-t/3}$	Solves for y	**1 point**

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(d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

$\frac{dy}{dt} = 3 - \frac{y}{t+2} \Rightarrow \frac{d^2y}{dt^2} = (-1)\frac{\frac{dy}{dt}(t+2) - y}{(t+2)^2}$	Quotient rule	**1 point**
$B'(1) = 3 - \frac{B(1)}{3} = 3 - \frac{2.5}{3} = \frac{6.5}{3}$ $B''(1) = -\frac{B'(1)\cdot 3 - B(1)}{3^2} = -\frac{6.5 - 2.5}{9} = -\frac{4}{9} < 0$	$B''(1) < 0$	**1 point**
The rate of change of the amount of medication is decreasing at time $t = 1$ because $B''(1) < 0$ and $\frac{d^2y}{dt^2}$ is continuous in an interval containing $t = 1$.	Answer with reason	**1 point**

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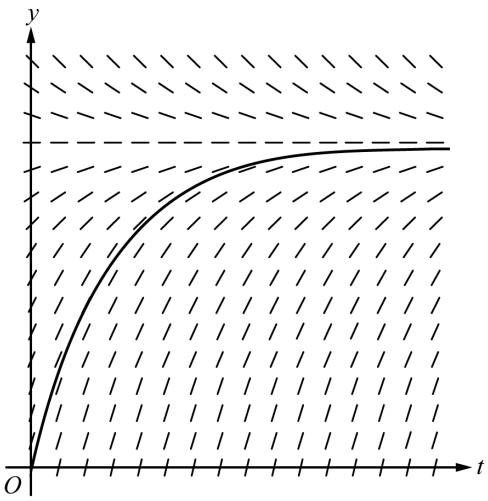
Part B (AB): Graphing calculator not allowed
Question 6 9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

	Model Solution	Scoring
(a)	A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.	
		Solution curve **1 point**

Scoring notes:

- To earn the point the solution curve must pass through the point $(0, 0)$, be generally increasing and concave down, and approach the horizontal asymptote from below as t increases. The point is not earned if two or more solution curves are presented.

Total for part (a) 1 point

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(b) Using correct units, interpret the statement $\lim_{t\to\infty} A(t) = 12$ in the context of this problem.

Over time the amount of medication in the patient approaches 12 milligrams.	Interpretation	**1 point**

Scoring notes:

- To earn the point the interpretation must include "medication in the patient," "approaches 12," and units (milligrams), or their equivalents.

Total for part (b) 1 point

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

$\frac{dy}{dt} = \frac{12 - y}{3} \Rightarrow \frac{dy}{12 - y} = \frac{dt}{3}$	Separation of variables	**1 point**
$\int \frac{dy}{12 - y} = \int \frac{dt}{3} \Rightarrow -\ln\lvert 12 - y\rvert = \frac{t}{3} + C$	Antiderivatives	**1 point**
$\ln\lvert 12 - y\rvert = -\frac{t}{3} - C \Rightarrow \lvert 12 - y\rvert = e^{-t/3 - C}$ $\Rightarrow y = 12 + Ke^{-t/3}$ $0 = 12 + K \Rightarrow K = -12$	Constant of integration and uses initial condition	**1 point**
$y = A(t) = 12 - 12e^{-t/3}$	Solves for y	**1 point**

Scoring notes:

- A response of $\frac{dy}{12 - y} = 3\,dt$ is a bad separation and does not earn the first point. However, this response is eligible for the second and third points. It cannot earn the fourth point.
- Absolute value bars are not required in this part.
- A response that correctly separates to $\frac{3\,dy}{12 - y} = dt,$ but then incorrectly simplifies to $\frac{dy}{4 - y} = dt$ earns the first point (for the initial correct separation), is eligible for the second point (for $-\ln\lvert 4 - y\rvert = t,$ with or without $+C$), but is not eligible for the third or fourth points.
- $+\ln\lvert 12 - y\rvert = \frac{t}{3}$ (with or without $+C$) does not earn the second point and is not eligible for the fourth point; $+\ln\lvert 12 - y\rvert = \frac{t}{3} + C$ is eligible for the third point.
- In all other cases, the points are earned consecutively – the second point cannot be earned without the first, the third without the second, etc.

Total for part (c) 4 points

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(d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

$\frac{dy}{dt} = 3 - \frac{y}{t+2} \Rightarrow \frac{d^2y}{dt^2} = (-1)\frac{\frac{dy}{dt}(t+2) - y}{(t+2)^2}$	Quotient rule	**1 point**
$B'(1) = 3 - \frac{B(1)}{3} = 3 - \frac{2.5}{3} = \frac{6.5}{3}$ $B''(1) = -\frac{B'(1)\cdot 3 - B(1)}{3^2} = -\frac{6.5 - 2.5}{9} = -\frac{4}{9} < 0$	$B''(1) < 0$	**1 point**
The rate of change of the amount of medication is decreasing at time $t = 1$ because $B''(1) < 0$ and $\frac{d^2y}{dt^2}$ is continuous in an interval containing $t = 1$.	Answer with reason	**1 point**

Scoring notes:

- The first point is for correctly applying the quotient rule to $\frac{y}{t+2}$ or applying the product rule to $y(t+2)^{-1}$. Errors in differentiating the constant, 3, or handling the sign of the second term of $\frac{dy}{dt}$ will result in not earning the second point.
- The second point cannot be earned unless the second derivative $\frac{d^2y}{dt^2}$ is correct.
- For the second point, it is sufficient to state the sign of $B''(1)$ is negative with supporting work. If a value is declared for $B''(1)$, it must be correct in order to earn the second point.
- Eligibility for the third point: An attempt at using the quotient rule (or product rule) to find $B''(1)$. In this case, the third point will be earned for a consistent conclusion based on the declared value (or sign) of $B''(1)$.

Total for part (d) 3 points

Total for question 6 9 points

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1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

   (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

   (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

   (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

   (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t, for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400)\, dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

   (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

   (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

   (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

   (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t, for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400)\,dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

Model Solution	Scoring
(a) The total number of vehicles that arrive at the toll plaza from 6 A.M. to 10 A.M. is given by $\int_1^5 A(t)\,dt$.	1: Answer
(b) Average $= \frac{1}{5-1}\int_1^5 A(t)\,dt = 375.536966$	2: {1: Uses average value formula; 1: Answer}
(c) $A'(1) = 148.947272$ Because $A'(1) > 0$, the rate at which the vehicles arrive at the toll plaza is increasing.	2: {1: Considers $A'(1)$; 1: Answer with reason}
(d) $N'(t) = A(t) - 400 = 0$ $\Rightarrow A(t) = 400 \Rightarrow t = 1.469372,\ t = 3.597713$ $a = 1.469372$ $b = 3.597713$ t | $N(t) = \int_a^t (A(x) - 400)\,dx$ a | 0 b | 71.254129 4 | 62.338346 The greatest number of vehicles in line is 71.	4: {1: Considers $N'(1) = 0$; 1: $t = a$ and $t = b$; 1: Answer; 1: Justification}

Part A (AB or BC): Graphing calculator required
Question 1 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

	Model Solution	Scoring
(a)	Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).	
	The total number of vehicles that arrive at the toll plaza from 6 A.M. to 10 A.M. is given by $\int_1^5 A(t)\,dt$.	Answer **1 point**

Scoring notes:

- The response must be a definite integral with correct lower and upper limits to earn this point.
- Because $|A(t)| = A(t)$ for $1 \le t \le 5$, a response of $\int_1^5 \left|450\sqrt{\sin(0.62t)}\right|\,dt$ or $\int_1^5 |A(t)|\,dt$ earns the point.
- A response missing dt or using dx is eligible to earn the point.
- A response with a copy error in the expression for $A(t)$ will earn the point only in the presence of $\int_1^5 A(t)\,dt$.

Total for part (a) 1 point

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(b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

$\text{Average} = \frac{1}{5-1}\int_1^5 A(t)\,dt = 375.536966$	Uses average value formula: $\frac{1}{b-a}\int_a^b A(t)\,dt$ **1 point**
The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is 375.537 (or 375.536) vehicles per hour.	Answer **1 point**

Scoring notes:

- The use of the average value formula, indicating that $a = 1$ and $b = 5$, can be presented in single or multiple steps to earn the first point. For example, the following response earns both points: $\int_1^5 A(t)\,dt = 1502.147865$, so the average value is 375.536966.
- A response that presents a correct integral along with the correct average value, but provides incorrect or incomplete communication, earns 1 out of 2 points. For example, the following response earns 1 out of 2 points: $\int_1^5 A(t)\,dt = 1502.147865 = 375.536966$.
- The answer must be correct to three decimal places. For example, $\frac{1}{5-1}\int_1^5 A(t)\,dt = 375.536966 \approx 376$ earns only the first point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{4}\int_1^5 A(t)\,dt = 79.416068$.
- Special case: $\frac{1}{5}\int_1^5 A(t)\,dt = 300.429573$ earns 1 out of 2 points.

Total for part (b) 2 points

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(c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

$A'(1) = 148.947272$	Considers $A'(1)$	**1 point**
Because $A'(1) > 0$, the rate at which the vehicles arrive at the toll plaza is increasing.	Answer with reason	**1 point**

Scoring notes:

- The response need not present the value of $A'(1)$. The second line of the model solution earns both points.
- An incorrect value assigned to $A'(1)$ earns the first point (but will not earn the second point).
- Without a reference to $t = 1$, the first point is earned by any of the following:
  - 148.947 accurate to the number of decimals presented, with zero up to three decimal places (i.e., 149, 148, 148.9, 148.95, or 148.94)
  - $A'(t) = 148.947$ by itself
- To be eligible for the second point, the first point must be earned.
- To earn the second point, there must be a reference to $t = 1$.
- Degree mode: $A'(1) = 23.404311$

Total for part (c) 2 points

(d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t, for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400)\,dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

$N'(t) = A(t) - 400 = 0$ $\Rightarrow A(t) = 400 \Rightarrow t = 1.469372,\ t = 3.597713$ $a = 1.469372$ $b = 3.597713$	Considers $N'(t) = 0$	**1 point**
	$t = a$ and $t = b$	**1 point**
t | $N(t) = \int_a^t (A(x) - 400)\,dx$ a | 0 b | 71.254129 4 | 62.338346 The greatest number of vehicles in line is 71.	Answer	**1 point**
	Justification	**1 point**

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Scoring notes:

- It is not necessary to indicate that $A(t) = 400$ to earn the first point, although this statement alone would earn the first point.
- A response of "$A(t) \geq 400$ when $1.469372 \leq t \leq 3.597713$" will earn the first 2 points. A response of "$A(t) \geq 400$" along with the presence of exactly one of the two numbers above will earn the first point, but not the second. A response of "$A(t) \geq 400$" by itself will not earn either of the first 2 points.
- To earn the second point the values for a and b must be accurate to the number of decimals presented, with at least one and up to three decimal places. These may appear only in a candidates table, as limits of integration, or on a number line.
- A response with incorrect notation involving t or x is eligible to earn all 4 points.
- A response that does not earn the first point is still eligible for the remaining 3 points.
- To earn the third point, a response must present the greatest number of vehicles. This point is earned for answers of either 71 or $71.254***$ only.
- A correct justification earns the fourth point, even if the third point is not earned because of a decimal presentation error.
- When using a Candidates Test, the response must include the values for $N(a)$, $N(b)$, and $N(4)$ to earn the fourth point. These values must be correct to the number of decimals presented, with up to three decimal places. (Correctly rounded integer values are acceptable.)
- Alternate solution for the third and fourth points:

  For $a \leq t \leq b$, $A(t) \geq 400$. For $b \leq t \leq 4$, $A(t) \leq 400$.

  Thus, $N(t) = \int_a^t (A(x) - 400)\,dx$ is greatest at $t = b$.

  $N(b) = 71.254129$, and the greatest number of vehicles in line is 71.
- Degree mode: The response is only eligible to earn the first point because in degree mode $A(t) < 400$.

Total for part (d) 4 points

Total for question 1 9 points

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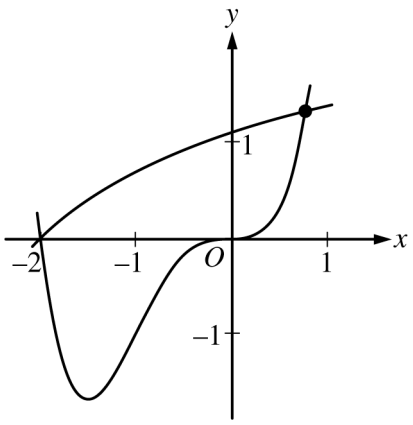

2. Let f and g be the functions defined by $f(x) = \ln(x + 3)$ and $g(x) = x^4 + 2x^3$. The graphs of f and g, shown in the figure above, intersect at $x = -2$ and $x = B$, where $B > 0$.

   (a) Find the area of the region enclosed by the graphs of f and g.

   (b) For $-2 \le x \le B$, let $h(x)$ be the vertical distance between the graphs of f and g. Is h increasing or decreasing at $x = -0.5$? Give a reason for your answer.

   (c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x-axis are squares. Find the volume of the solid.

   (d) A vertical line in the xy-plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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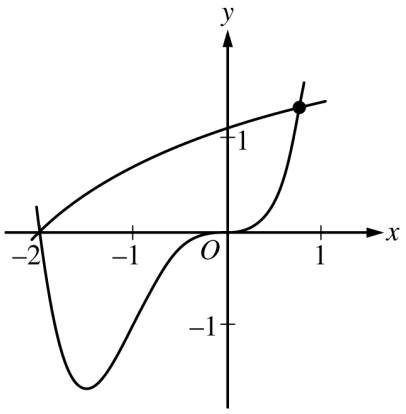
A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.



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A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

(c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x-axis are squares. Find the volume of the solid.

(d) A vertical line in the xy-plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

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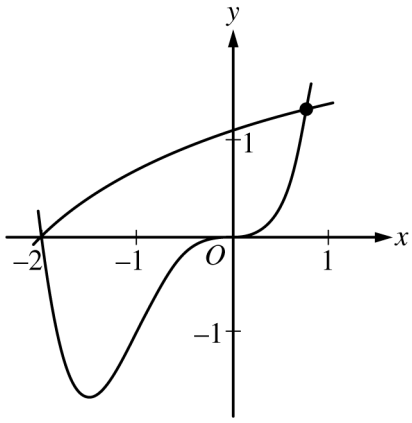

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(a) Find the area of the region enclosed by the graphs of f and g.

(b) For $-2 \le x \le B$, let $h(x)$ be the vertical distance between the graphs of f and g. Is h increasing or decreasing at $x = -0.5$? Give a reason for your answer.

(c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x-axis are squares. Find the volume of the solid.

(d) A vertical line in the xy-plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

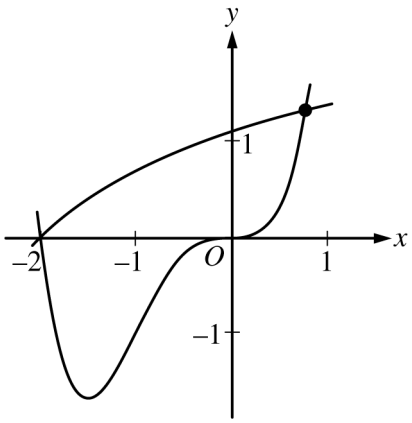
Model Solution	Scoring
(a) $\ln(x+3) = x^4 + 2x^3 \Rightarrow x = -2,\ x = B = 0.781975$ $\int_{-2}^{B} (f(x) - g(x))\,dx = 3.603548$ The area of the region is 3.604 (or 3.603).	3: {1: Integrand; 1: Limits of integration; 1: Answer}
(b) $h(x) = f(x) - g(x)$ $h'(x) = f'(x) - g'(x)$ $h'(-0.5) = f'(-0.5) - g'(-0.5) = -0.6$ (or -0.599) Since $h'(-0.5) < 0$, h is decreasing at $x = -0.5$.	2: {1: Considers $h'(-0.5)$; 1: Answer with reason}
(c) $\int_{-2}^{B} (f(x) - g(x))^2\,dx = 5.340102$	2: {1: Integrand; 1: Answer}
(d) The cross section has area $A(x) = (f(x) - g(x))^2$. $\frac{d}{dt}[A(x)] = \frac{dA}{dx} \cdot \frac{dx}{dt}$ $\left.\frac{d}{dt}[A(x)]\right\|_{x=-0.5} = A'(-0.5) \cdot 7 = -9.271842$	2: {1: $\frac{dA}{dx} \cdot \frac{dx}{dt}$; 1: Answer}

Part A (AB): Graphing calculator required
Question 2 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Let f and g be the functions defined by $f(x) = \ln(x+3)$ and $g(x) = x^4 + 2x^3$. The graphs of f and g, shown in the figure above, intersect at $x = -2$ and $x = B$, where $B > 0$.

	Model Solution	Scoring	
(a)	Find the area of the region enclosed by the graphs of f and g.		
	$\ln(x+3) = x^4 + 2x^3 \Rightarrow x = -2,\ x = B = 0.781975$ $\int_{-2}^{B} (f(x) - g(x))\,dx = 3.603548$ The area of the region is 3.604 (or 3.603).	Integrand	**1 point**
		Limits of integration	**1 point**
		Answer	**1 point**

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Scoring notes:

- Other forms of the integrand in a definite integral, e.g., $|f(x) - g(x)|$, $|g(x) - f(x)|$, or $g(x) - f(x)$, earn the first point.
- To earn the second point, the response must have a lower limit of -2 and an upper limit expressed as either the letter B with no value attached, or a number that is correct to the number of digits presented, with at least one and up to three decimal places.
  - Case 1: If the response did not earn the second point because of an incorrect value of B, $0 < B < 1$, but used a lower limit of -2, the response earns the third point only for a consistent answer.
  - Case 2: If the response did not earn the second point because the lower limit used was $x = 0$, but the response used a correct upper limit of B, the response earns the third point for a consistent answer of 0.708 (or 0.707).
  - Case 3: If a response uses any other incorrect limits it does not earn the second or third points.
- A response containing the integrand $g(x) - f(x)$ must interpret the value of the resulting integral correctly to earn the third point. For example, the following response earns all 3 points: $\int_{-2}^{B}(g(x) - f(x))\,dx = -3.604$ so the area is 3.604. However, the response "Area $= \int_{-2}^{B}(g(x) - f(x))\,dx = 3.604$" presents an untrue statement and earns the first and second points, but not the third point.
- A response must earn the first point in order to be eligible for the third point. If the response has earned the second point, then only the correct answer will earn the third point.
- Instructions for scoring a response that presents an integrand of $\ln(x + 3) - x^4 + 2x^3$ and the correct answer are shown in the "Global Special case" after part (d).

Total for part (a) 3 points

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(b) For $-2 \le x \le B$, let $h(x)$ be the vertical distance between the graphs of f and g. Is h increasing or decreasing at $x = -0.5$? Give a reason for your answer.

$h(x) = f(x) - g(x)$ $h'(x) = f'(x) - g'(x)$ $h'(-0.5) = f'(-0.5) - g'(-0.5) = -0.6$ (or -0.599) Since $h'(-0.5) < 0$, h is decreasing at $x = -0.5$.	Considers $h'(-0.5)$ **1 point** – OR – $f'(x) - g'(x)$ Answer with reason **1 point**

Scoring notes:

- The response need not present the value of $h'(-0.5)$. The last line earns both points. However, if a value is presented it must be correct for the digits reported up to three decimal places.
- A response that reports an incorrect value of $h'(-0.5)$ earns only the first point.
- A response that presents only $h'(x)$ does not earn either point.
- The only response that earns the second point for concluding "h is increasing" is described in the Global Special case provided after part (d).
- A response that compares the values of $f'(x)$ and $g'(x)$ at $x = -0.5$ earns the first point and is eligible for the second point. This comparison can be made symbolically or verbally; for example, the response "the rate of change of $f(x)$ is less than the rate of change of $g(x)$ at $x = -0.5$" earns the first point.

Total for part (b) 2 points

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(c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x-axis are squares. Find the volume of the solid.

$\int_{-2}^{B}(f(x)-g(x))^2\,dx = 5.340102$	Integrand	**1 point**
The volume of the solid is 5.340.	Answer	**1 point**

Scoring notes:

- The first point is earned for an integrand of $k(f(x)-g(x))^2$ or its equivalent with $k \neq 0$ in any definite integral. If $k \neq 1$, then the response is not eligible for the second point.
- A response that does not earn the first point is ineligible to earn the second point, with the following exceptions:
  - A response which has a presentation error in the integrand (for example, mismatched or missing parentheses, misplaced exponent) does not earn the first point, but would earn the second point for the correct answer. A response which has a presentation error in the integrand and which reports an incorrect answer earns no points.
  - A response that presents an integrand of $\left(\ln(x+3)-x^4+2x^3\right)^2$. Scoring instructions for this case are provided in the Global Special case after part (d).
- A response that uses incorrect limits is only eligible for the second point provided the limits are imported from part (a) in case 1 or case 2. In both of these situations, the second point is earned only for answers consistent with the imported limits.

Total for part (c) 2 points

(d) A vertical line in the xy-plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

The cross section has area $A(x) = (f(x)-g(x))^2$. $\frac{d}{dt}[A(x)] = \frac{dA}{dx}\cdot\frac{dx}{dt}$	$\frac{dA}{dx}\cdot\frac{dx}{dt}$	**1 point**
$\left.\frac{d}{dt}[A(x)]\right\|_{x=-0.5} = A'(-0.5)\cdot 7 = -9.271842$ At $x = -0.5$, the area of the cross section above the line is changing at a rate of -9.272 (or -9.271) square units per second.	Answer	**1 point**

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Scoring notes:

- The first point may be earned by presenting $\frac{dA}{dx} \cdot \frac{dx}{dt}$, $A' \cdot \frac{dx}{dt}$, $A'(x) \cdot \frac{dx}{dt}$, $A'(x) \cdot x'$, $A'(-0.5) \cdot 7$, or $k \cdot 7$, where k is a declared value of $A'(-0.5)$, or any equivalent expression, including $2(f(x) - g(x))(f'(x) - g'(x))\frac{dx}{dt}$.
- If a response defines $f(x) - g(x)$ as a function in parts (b) or (c) (for example, $h(x) = f(x) - g(x)$), then a correct expression for $\frac{dA}{dt}$ (for example, $2h\frac{dh}{dt}$) earns the first point.
- A response that imports a function $A(x)$ declared in part (c) is eligible for both points (the answer must be consistent with the imported function $A(x)$).
- A response that presents an incorrect function for $A(x)$ that is not imported from part (c) is eligible only for the first point.
- Except when $A(x)$ is imported from part (c), the second point is earned only for the correct answer.
- A response that does not earn the first point is ineligible to earn the second point except in the special case noted below.

Total for part (d) 2 points

Total for question 2 9 points

Global Special case: A response may incorrectly simplify $f(x) - g(x)$ to $j(x) = \ln(x + 3) - x^4 + 2x^3$ instead of $\ln(x + 3) - x^4 - 2x^3$. Because this question is calculator active, a response with this incorrect simplification may nevertheless present correct answers.

- In any part of the question, a response that starts correctly by using $f(x) - g(x)$, then presents $j(x)$, is eligible for all points in that part.
- The first time a response implicitly presents $f(x) - g(x)$ as $j(x) = \ln(x + 3) - x^4 + 2x^3$ (with no explicit connection) in any part of this question, the response loses a point. The response is then eligible for all remaining points for a correct or consistent answer.
- In part (a), the consistent answer using $j(x)$ is negative and will not earn the third point.
- In part (b), the consistent answer using $j(x)$ is that $j'(-0.5) = 2.4 > 0$, so h is increasing at $x = -0.5$.
- In part (c), the consistent answer using $j(x)$ is 252.187 (or 252.188).
- In part (d), the consistent answer using $j(x)$ is 20.287.

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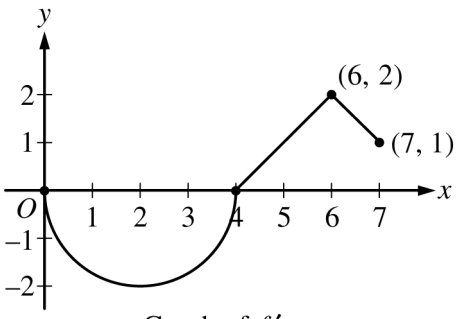

Graph of f'

3. Let f be a differentiable function with $f(4) = 3$. On the interval $0 \le x \le 7$, the graph of f', the derivative of f, consists of a semicircle and two line segments, as shown in the figure above.

   (a) Find $f(0)$ and $f(5)$.

   (b) Find the x-coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

   (c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \le x \le 7$? Show the analysis that leads to your answer.

   (d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \le x \le 7$. Justify your answer.

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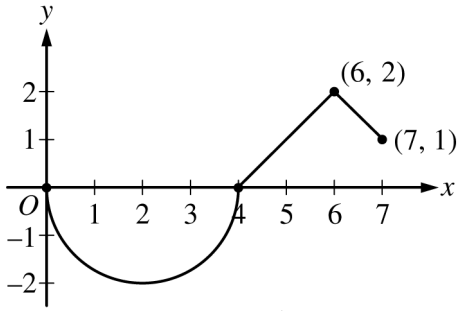


Graph of f'

3. Let f be a differentiable function with $f(4) = 3$. On the interval $0 \le x \le 7$, the graph of f', the derivative of f, consists of a semicircle and two line segments, as shown in the figure above.

(a) Find $f(0)$ and $f(5)$.

(b) Find the x-coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

(c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \le x \le 7$? Show the analysis that leads to your answer.

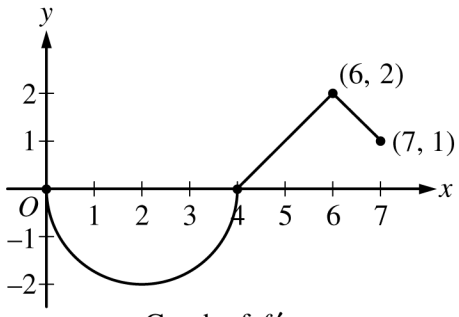
(d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \le x \le 7$. Justify your answer.

Model Solution	Scoring
(a) $f(0) = f(4) + \int_4^0 f'(x)\,dx = 3 - \int_0^4 f'(x)\,dx = 3 + 2\pi$ $f(5) = f(4) + \int_4^5 f'(x)\,dx = 3 + \frac{1}{2} = \frac{7}{2}$	3: {1: Area of either region; 1: $f(0)$; 1: $f(5)$}
(b) The graph of f has a point of inflection at each of $x = 2$ and $x = 6$, because $f'(x)$ changes from decreasing to increasing at $x = 2$ and from increasing to decreasing at $x = 6$.	2: {1: Answer; 1: Justification}
(c) $g'(x) = f'(x) - 1$ $f'(x) - 1 \le 0 \Rightarrow f'(x) \le 1$ The graph of g is decreasing on the interval $0 \le x \le 5$ because $g'(x) \le 0$ on this interval.	2: {1: $g'(x) = f'(x) - 1$; 1: Interval with reason}
(d) g is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$. Therefore, the absolute minimum occurs at $x = 5$, and $g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$ is the absolute minimum value of g.	2: {1: Considers $g'(x) = 0$; 1: Answer with justification}

Part B (AB or BC): Graphing calculator not allowed
Question 3 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Graph of f'

Let f be a differentiable function with $f(4) = 3$. On the interval $0 \le x \le 7$, the graph of f', the derivative of f, consists of a semicircle and two line segments, as shown in the figure above.

(a) Find $f(0)$ and $f(5)$.

Model Solution	Scoring	
$f(0) = f(4) + \int_4^0 f'(x)\,dx = 3 - \int_0^4 f'(x)\,dx = 3 + 2\pi$ $f(5) = f(4) + \int_4^5 f'(x)\,dx = 3 + \frac{1}{2} = \frac{7}{2}$	Area of either region – OR – $\int_0^4 f'(x)\,dx$ – OR – $\int_4^5 f'(x)\,dx$	**1 point**
	$f(0)$	**1 point**
	$f(5)$	**1 point**

Scoring notes:

- A response with answers of only $f(0) = \pm 2\pi$, or only $f(5) = \frac{1}{2}$, or both earns 1 of the 3 points.
- A response displaying $f(5) = \frac{7}{2}$ and a missing or incorrect value for $f(0)$ earns 2 of the 3 points.
- The second and third points can be earned in either order.
- Read unlabeled values from left to right and from top to bottom as $f(0)$ and $f(5)$. A single value must be labeled as either $f(0)$ or $f(5)$ in order to earn any points.

Total for part (a) 3 points

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(b) Find the x-coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

The graph of f has a point of inflection at each of $x = 2$ and $x = 6$, because $f'(x)$ changes from decreasing to increasing at $x = 2$ and from increasing to decreasing at $x = 6$.	Answer	**1 point**
	Justification	**1 point**

Scoring notes:

- A response that gives only one of $x = 2$ or $x = 6$, along with a correct justification, earns 1 of the 2 points.
- A response that claims that there is a point of inflection at any value other than $x = 2$ or $x = 6$ earns neither point.
- To earn the second point a response must use correct reasoning based on the graph of f'. Examples of correct reasoning include:
  - Correctly discussing the signs of the slopes of the graph of f'
  - Citing $x = 2$ and $x = 6$ as the locations of local extrema on the graph of f'
- Examples of reasoning not (sufficiently) connected to the graph of f' include:
  - Reasoning based on sign changes in f'' unless the connection is made between the sign of f'' and the slopes of the graph of f'
  - Reasoning based only on the concavity of the graph of f
- The second point cannot be earned by use of vague or undefined terms such as "it" or "the function" or "the derivative."
- Responses that report inflection points as ordered pairs must report the points $(2, 3 + \pi)$ and $(6, 5)$ in order to earn the first point. If the y-coordinates are reported incorrectly, the response remains eligible for the second point.

Total for part (b) 2 points

(c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \le x \le 7$? Show the analysis that leads to your answer.

$g'(x) = f'(x) - 1$	$g'(x) = f'(x) - 1$	**1 point**
$f'(x) - 1 \le 0 \Rightarrow f'(x) \le 1$ The graph of g is decreasing on the interval $0 \le x \le 5$ because $g'(x) \le 0$ on this interval.	Interval with reason	**1 point**

Scoring notes:

- The first point can be earned for $f'(x) \le 1$ or the equivalent, in words or symbols.
- Endpoints do not need to be included in the interval to be eligible for the second point.

Total for part (c) 2 points

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(d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \le x \le 7$. Justify your answer.

g is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$. Therefore, the absolute minimum occurs at $x = 5$, and $g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$ is the absolute minimum value of g.	Considers $g'(x) = 0$	**1 point**
	Answer with justification	**1 point**

Scoring notes:

- A justification that uses a local argument, such as "g' changes from negative to positive (or g changes from decreasing to increasing) at $x = 5$" must also state that $x = 5$ is the only critical point.
- If $g'(x) = 0$ (or equivalent) is not declared explicitly, a response that isolates $x = 5$ as the only critical number belonging to $(0, 7)$ earns the first point.
- A response that imports $g'(x) = f'(x)$ from part (c) is eligible for the first point but not the second.
  - In this case, consideration of $x = 4$ as the only critical number on $(0, 7)$ earns the first point.
- Solution using Candidates Test:

  $g'(x) = f'(x) - 1 = 0 \Rightarrow x = 5,\ x = 7$

x	$g(x)$
0	$3 + 2\pi$
5	$-\frac{3}{2}$
7	$-\frac{1}{2}$

  The absolute minimum value of g on the interval $0 \le x \le 7$ is $-\frac{3}{2}$.

- When using a Candidates Test, a response may import an incorrect value of $f(0) = g(0) > -\frac{3}{2}$ from part (a). The second point can only be earned for an answer of $-\frac{3}{2}$.

Total for part (d) 2 points

Total for question 3 9 points

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t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

4. An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r, where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \le t \le 12$.

   (a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \le t \le 10$. Show the computations that lead to your answer, and indicate units of measure.

   (b) Is there a time t, $0 \le t \le 3$, for which $r'(t) = -6$? Justify your answer.

   (c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of $\int_0^{12} r'(t)\,dt$.

   (d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

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NO CALCULATOR ALLOWED

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

4. An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r, where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \le t \le 12$.

   (a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \le t \le 10$. Show the computations that lead to your answer, and indicate units of measure.

   (b) Is there a time t, $0 \le t \le 3$, for which $r'(t) = -6$? Justify your answer.

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NO CALCULATOR ALLOWED

(c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of $\int_0^{12} r'(t)\,dt$.

(d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

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t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

4. An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r, where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \le t \le 12$.

   (a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \le t \le 10$. Show the computations that lead to your answer, and indicate units of measure.

   (b) Is there a time t, $0 \le t \le 3$, for which $r'(t) = -6$? Justify your answer.

   (c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of $\int_0^{12} r'(t)\,dt$.

   (d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

Model Solution	Scoring
(a) $r''(8.5) \approx \frac{r'(10) - r'(7)}{10 - 7} = \frac{-3.8 - (-4.4)}{10 - 7}$ $= \frac{0.6}{3} = 0.2$ centimeter per day per day	2: {1: Answer with work; 1: Units}
(b) $r'(0) = -6.1 < -6 < -5.0 = r'(3)$ $r(t)$ is twice-differentiable. $\Rightarrow r'(t)$ is differentiable. $\Rightarrow r'(t)$ is continuous. Therefore, by the Intermediate Value Theorem, there is a time t, $0 < t < 3$, such that $r'(t) = -6$.	2: {1: $r'(0) < -6 < r'(3)$; 1: Conclusion using IVT}
(c) $\int_0^{12} r'(t)\,dt \approx 3r'(3) + 4r'(7) + 3r'(10) + 2r'(12)$ $= 3(-5.0) + 4(-4.4) + 3(-3.8) + 2(-3.5) = -51$	2: {1: Form of right Riemann sum; 1: Answer}
(d) $\frac{dV}{dt} = \frac{2}{3}\pi rh\frac{dr}{dt} + \frac{1}{3}\pi r^2\frac{dh}{dt}$ $\left.\frac{dV}{dt}\right\|_{t=3} = \frac{2}{3}\pi(100)(50)(-5) + \frac{1}{3}\pi(100)^2(-2) = -\frac{70{,}000\pi}{3}$	3: {1: Product rule; 1: Chain rule; 1: Answer}

Part B (AB or BC): Graphing calculator not allowed
Question 4 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	−6.1	−5.0	−4.4	−3.8	−3.5

An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r, where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \le t \le 12$.

	Model Solution	Scoring
(a)	Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \le t \le 10$. Show the computations that lead to your answer, and indicate units of measure.	
	$r''(8.5) \approx \dfrac{r'(10) - r'(7)}{10 - 7} = \dfrac{-3.8 - (-4.4)}{10 - 7}$	$r''(8.5)$ with supporting work **1 point**
	$= \dfrac{0.6}{3} = 0.2$ centimeter per day per day	Units **1 point**

Scoring notes:

- To earn the first point the supporting work must include at least a difference and a quotient.
- Simplification is not required to earn the first point. If the numerical value is simplified, it must be correct.
- The second point can be earned with an incorrect approximation for $r''(8.5)$ but cannot be earned without some value for $r''(8.5)$ presented.
- Units may be written in any equivalent form (such as cm/day^2).

Total for part (a) 2 points

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(b) Is there a time t, $0 \le t \le 3$, for which $r'(t) = -6$? Justify your answer.

$r(t)$ is twice-differentiable. $\Rightarrow$ $r'(t)$ is differentiable. $\Rightarrow$ $r'(t)$ is continuous. $r'(0) = -6.1 < -6 < -5.0 = r'(3)$ Therefore, by the Intermediate Value Theorem, there is a time t, $0 < t < 3$, such that $r'(t) = -6$.	$r'(0) < -6 < r'(3)$	**1 point**
	Conclusion using Intermediate Value Theorem	**1 point**

Scoring notes:

- To earn the first point, the response must establish that -6 is between $r'(0)$ and $r'(3)$ (or -6.1 and -5). This statement may be represented symbolically (with or without including one or both endpoints in an inequality) or verbally. A response of "$r'(t) = -6$ because $r'(0) = -6.1$ and $r'(3) = -5$" does not state that -6 is between -6.1 and -5. Thus this response does not earn the first point.
- To earn the second point:
  - The response must state that $r'(t)$ is continuous because $r'(t)$ is differentiable (or because $r(t)$ is twice differentiable).
  - The response must have earned the first point.
    - Exception: A response of "$r'(t) = -6$ because $r'(0) = -6.1$ and $r'(3) = -5$" does not earn the first point because of imprecise communication but may nonetheless earn the second point if all other criteria for the second point are met.
  - The response must conclude that there is a time t such that $r'(t) = -6$. (A statement of "yes" would be sufficient.)
- To earn the second point, the response need not explicitly name the IVT, but if a theorem is named, it must be correct.

Total for part (b) 2 points

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(c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of $\int_0^{12} r'(t)\,dt$.

$\int_0^{12} r'(t)\,dt \approx 3r'(3) + 4r'(7) + 3r'(10) + 2r'(12)$	Form of right Riemann sum	**1 point**
$= 3(-5.0) + 4(-4.4) + 3(-3.8) + 2(-3.5)$ $= -51$	Answer	**1 point**

Scoring notes:

- To earn the first point, at least seven of the eight factors in the Riemann sum must be correct. If there is any error in the Riemann sum, the response does not earn the second point.
- A response of $3(-5.0) + 4(-4.4) + 3(-3.8) + 2(-3.5)$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A response that presents the correct answer, with accompanying work that shows the four products in the Riemann sum (without explicitly showing all of the factors and/or the sum process) does not earn the first point but earns the second point. For example, $-15 + 4(-4.4) + 3(-3.8) + -7$ does not earn the first point but earns the second point. Similarly, $-15,\ -17.6,\ -11.4,\ -7 \rightarrow -51$ does not earn the first point but earns the second point.
- A response that presents the correct answer (-51) with no supporting work earns no points.
- A response that provides a completely correct left Riemann sum and approximation $\int_0^{12} r'(t)\,dt$ (i.e., $3r'(0) + 4r'(3) + 3r'(7) + 2r'(10) = 3(-6.1) + 4(-5.0) + 3(-4.4) + 2(-3.8) = -59.1$) earns 1 of the 2 points. A response that has any error in a left Riemann sum or evaluation for $\int_0^{12} r'(t)\,dt$ earns no points.
- Units are not required or read in this part.

Total for part (c) 2 points

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(d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

$\frac{dV}{dt} = \frac{2}{3}\pi rh\frac{dr}{dt} + \frac{1}{3}\pi r^2\frac{dh}{dt}$	Product rule	**1 point**
	Chain rule	**1 point**
$\left.\frac{dV}{dt}\right\|_{t=3} = \frac{2}{3}\pi(100)(50)(-5) + \frac{1}{3}\pi(100)^2(-2) = -\frac{70{,}000\pi}{3}$ The rate of change of the volume of the sculpture at $t = 3$ is approximately $-\frac{70{,}000\pi}{3}$ cubic centimeters per day.	Answer	**1 point**

Scoring notes:

- The first 2 points could be earned in either order.
- A response with a completely correct product rule, missing one or both of the correct differentials, earns the product rule point, but not the chain rule point. For example, $\frac{dV}{dt} = \frac{2}{3}\pi rh + \frac{1}{3}\pi r^2$ earns the first point, but not the second.
- A response that treats r or h (but not both) as a constant is eligible for the chain rule point but not the product rule point. For example, $\frac{dV}{dt} = \frac{2}{3}\pi rh\frac{dr}{dt}$ is correct if h is constant, and thus earns the chain rule point.
- Note: Neither $\frac{dV}{dt} = \frac{2}{3}\pi r\frac{dh}{dt}$ nor $\frac{dV}{dt} = \frac{2}{3}\pi rh\frac{dr}{dt}\frac{dh}{dt}$ earn any points.
- A response that assumes a functional relationship between r and h (such as $r = 2h$), and uses this relationship to create a function for volume in terms of one variable, is eligible for at most the chain rule point. For example, $r = 2h \rightarrow V = \frac{1}{3}\pi(2h)^2 h = \frac{4}{3}\pi h^3 \rightarrow \frac{dV}{dt} = 4\pi h^2\frac{dh}{dt}$ earns only the chain rule point.
- A response that mishandles the constant $\frac{1}{3}\pi$ cannot earn the third point, but is eligible for the first 2 points.
- The third point cannot be earned without both of the first 2 points.
- $\frac{dV}{dt} = \frac{2}{3}\pi(100)(50)(-5) + \frac{1}{3}\pi(100)^2(-2)$ earns all 3 points.
- Units are not required or read in this part.

Total for part (d) 3 points

Total for question 4 9 points

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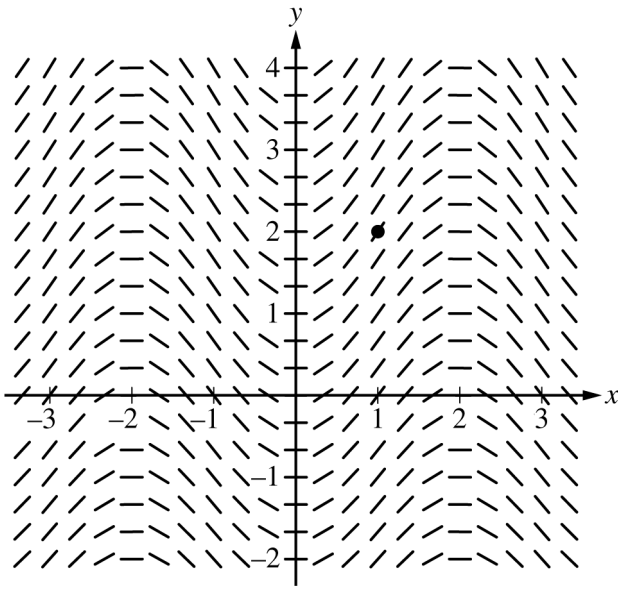
5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

(a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

(c) It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

(d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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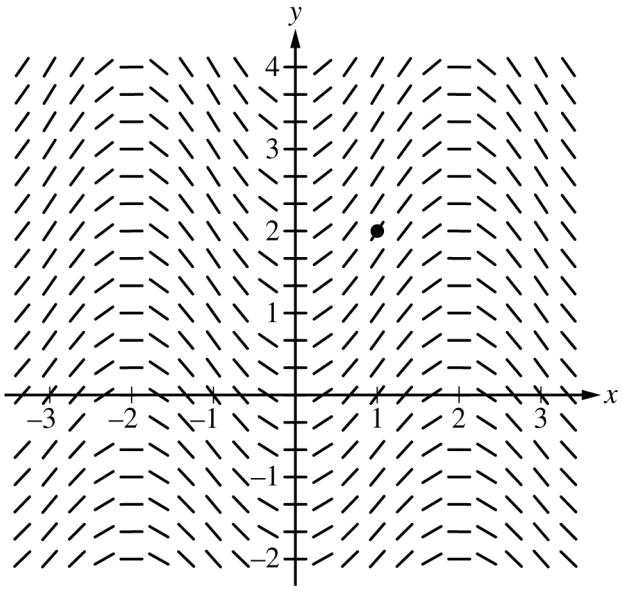
NO CALCULATOR ALLOWED

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

(a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

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(c) It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

(d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.

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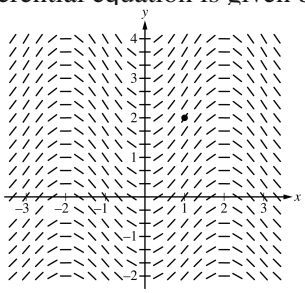
5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

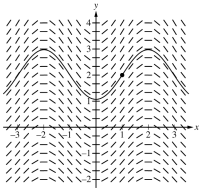
(a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

(c) It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

(d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.

Model Solution	Scoring
(a) 	1: Solution curve
(b) $\left.\frac{dy}{dx}\right\|_{(x,\,y)=(1,\,2)} = \frac{1}{2}\cdot 3\cdot\sin\left(\frac{\pi}{2}\right) = \frac{3}{2}$ An equation for the tangent line is $y = 2 + \frac{3}{2}(x-1)$. $f(0.8) \approx 2 + \frac{3}{2}(0.8-1) = 1.7$	2: {1: Tangent line equation; 1: Approximation}
(c) Because $f''(x) > 0$, f is concave up on $-1 \le x \le 1$, the tangent line lies below the graph of $y = f(x)$ at $x = 0.8$, and the approximation for $f(0.8)$ is an underestimate.	1: Answer with reason
(d) $\int \frac{dy}{\sqrt{y+7}} = \int \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)dx$ $2\sqrt{y+7} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right) + C$ $f(1) = 2 \Rightarrow 2\sqrt{2+7} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}\cdot 1\right) + C$ $\Rightarrow 6 = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}\right) + C \Rightarrow C = 6$ $\sqrt{y+7} = 3 - \frac{1}{2\pi}\cos\left(\frac{\pi}{2}x\right)$ $\quad y = \left(3 - \frac{1}{2\pi}\cos\left(\frac{\pi}{2}x\right)\right)^2 - 7$	5: {1: separation; 2: Antiderivatives; 1: Constant of integration and uses intial condition; 1: Solves for y}

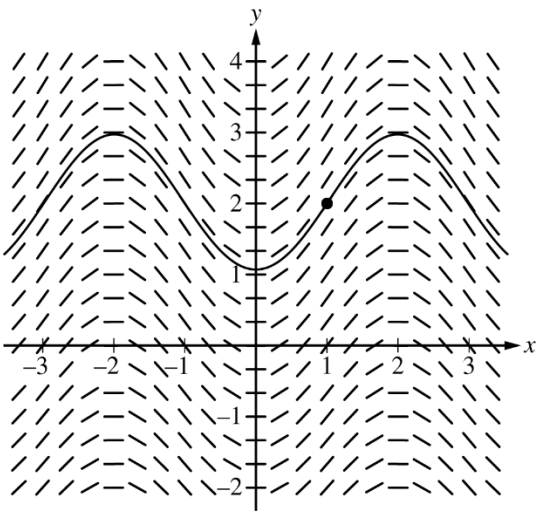
Part B (AB): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

	Model Solution	Scoring
(a)	A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.	
		Solution curve **1 point**

Scoring notes:

- The solution curve must pass through the point $(1, 2)$, extend reasonably close to the left and right edges of the square, and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.
- The solution curve must indicate $f(x) > 0$ for all points on the curve.
- All local maximum/minimum points on the solution curve must occur at horizontal line segments in the slope field.

Total for part (a) 1 point

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(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

$\left.\frac{dy}{dx}\right\|_{(x, y)=(1, 2)} = \frac{1}{2} \cdot 3 \cdot \sin\left(\frac{\pi}{2}\right) = \frac{3}{2}$ An equation for the tangent line is $y = 2 + \frac{3}{2}(x - 1)$.	Tangent line equation	**1 point**
$f(0.8) \approx 2 + \frac{3}{2}(0.8 - 1) = 1.7$	Approximation	**1 point**

Scoring notes:

- The tangent line equation can be presented in any equivalent form.
- An incorrect tangent line equation with a slope of $\frac{3}{2}$ is eligible to earn the second point for a consistent answer.
- A response of only $2 + \frac{3}{2}(0.8 - 1)$ earns the second point but not the first.

Total for part (b) 2 points

(c) It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

Because $f''(x) > 0$, f is concave up on $-1 \le x \le 1$, the tangent line lies below the graph of $y = f(x)$ at $x = 0.8$, and the approximation for $f(0.8)$ is an underestimate.	Answer with reason	**1 point**

Scoring notes:

- The reason must include $f''(x) > 0$, $f'(x)$ is increasing, or $f(x)$ is concave up.

Total for part (c) 1 point

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(d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.

$\int \frac{dy}{\sqrt{y+7}} = \int \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)dx$	Separation of variables	**1 point**
$2\sqrt{y+7} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right) + C$	One correct antiderivative	**1 point**
	The other correct antiderivative	**1 point**
$f(1) = 2 \Rightarrow 2\sqrt{2+7} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}\cdot 1\right) + C$ $\Rightarrow 6 = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}\right) + C \Rightarrow C = 6$ $\sqrt{y+7} = 3 - \frac{1}{2\pi}\cos\left(\frac{\pi}{2}x\right)$	Constant of integration and uses initial condition	**1 point**
$y = \left(3 - \frac{1}{2\pi}\cos\left(\frac{\pi}{2}x\right)\right)^2 - 7$	Solves for y	**1 point**

Scoring notes:

- A response with no separation of variables earns 0 out of 5 points.
- A response with no constant of integration can earn at most the first 3 points.
- A response is eligible for the fourth point only if it has earned the first point and at least 1 of the 2 antiderivative points.
  - Special case: The incorrect separation of $\sqrt{y+7}\ dy = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)dx$ does not earn the first point, is only eligible for the antiderivative point for $-\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right)$, and is eligible for the fourth point.
- An eligible response earns the fourth point by correctly including the constant of integration in an equation and substituting 1 for x and 2 for y.
- A response is eligible for the fifth point only if it has earned the first 4 points.

Total for part (d) 5 points

Total for question 5 9 points

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6. Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle Q moves along the y-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

(a) Find $v_P(t)$, the velocity of particle P at time t.

(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

(c) Find $y_Q(t)$, the position of particle Q at time t.

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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NO CALCULATOR ALLOWED

6. Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle Q moves along the y-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

(a) Find $v_P(t)$, the velocity of particle P at time t.

(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

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NO CALCULATOR ALLOWED

(c) Find $y_Q(t)$, the position of particle Q at time t.

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

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6. Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle Q moves along the y-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

(a) Find $v_P(t)$, the velocity of particle P at time t.

(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

(c) Find $y_Q(t)$, the position of particle Q at time t.

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

Model Solution	Scoring
(a) $v_P(t) = x_P'(t) = 4e^{-t}$	1: Answer
(b) $a_Q(t) = v_Q'(t) = \frac{-2}{t^3}$ For $t > 0$, $a_Q(t) < 0$ and $v_Q(t) > 0$. Because the velocity and acceleration have opposite signs, the speed of particle Q is decreasing for all $t > 0$.	3: 1: $a_Q(t)$ 1: Considers signs of $a_Q(t)$ and $v_Q(t)$ 1: Answer with justification
(c) $y_Q(t) = y_Q(1) + \int_1^t \frac{1}{s^2}\,ds$ $= 2 - \left(\frac{1}{s}\Big\vert_1^t\right) = 2 - \frac{1}{t} + 1 = 3 - \frac{1}{t}$	3: 1: Integral 1: Uses initial condition 1: Answer
(d) For particle P, $\lim_{t\to\infty}\left(6 - 4e^{-t}\right) = 6$. For particle Q, $\lim_{t\to\infty}\left(3 - \frac{1}{t}\right) = 3$. Because $6 > 3$, particle P will eventually be farther from the origin.	2: 1: One limit 1: Answer with reason

Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle Q moves along the y-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

	Model Solution	Scoring	
(a)	Find $v_P(t)$, the velocity of particle P at time t.		
	$v_P(t) = x_P'(t) = 4e^{-t}$	Answer	**1 point**

Scoring notes:

- A response that equates $x_P(t)$ with $v_P(t)$ does not earn the point.
- An unlabeled response earns the point.

Total for part (a) 1 point

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(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

$a_Q(t) = v_Q'(t) = \frac{-2}{t^3}$	$a_Q(t)$	**1 point**
For $t > 0$, $a_Q(t) < 0$ and $v_Q(t) > 0$.	Considers signs of $a_Q(t)$ and $v_Q(t)$	**1 point**
Because the velocity and acceleration have opposite signs, the speed of particle Q is decreasing for all $t > 0$.	Answer with justification	**1 point**

Scoring notes:

- Earning the first point is not necessary for a response to be eligible to earn the second or third points; however, the response must present an expression for $a_Q(t)$ to be eligible for third point.
- A response earns the second point with either of the following statements: "$v_Q(t)$ and $a_Q(t)$ have opposite signs" or "$v_Q(t)$ and $a_Q(t)$ have the same sign." This statement, however, must be consistent with $v_Q(t)$ and the presented expression for $a_Q(t)$.
- A response must earn the second point to be eligible for the third point. The answer must be consistent with the presented justification. Furthermore, responses for which $a_Q(t) > 0$ for $t > 0$ must conclude that there is no time at which the speed of the particle is decreasing.
- A response that indicates $v_Q(t) < 0$ does not earn the third point, even if the answer and justification are consistent with a reported sign of $a_Q(t)$.

Total for part (b) 3 points

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(c) Find $y_Q(t)$, the position of particle Q at time t.

$y_Q(t) = y_Q(1) + \int_1^t \frac{1}{s^2}\,ds$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 2 - \left(\frac{1}{s}\Big\vert_1^t\right) = 2 - \frac{1}{t} + 1 = 3 - \frac{1}{t}$	Answer	**1 point**

Scoring notes:

- A response that presents $\int_1^t \frac{1}{t^2}\,dt$ (using the same variable as a limit and integrand function) does not earn the first point unless it is followed by an attempt at integration.
- A response that presents either $\int \frac{1}{t^2}\,dt$ or $-\frac{1}{t}$ (with no integral) earns the first point. If the response continues and presents $2 = -1 + C$, then the response earns the second point.
- A response that presents only $y_Q(t) = -\frac{1}{t} + 3$ will earn all 3 points. Note that the right side of this equation suffices to earn all points. A response of $y_Q(t) = -\frac{1}{t} + C$, where $C \neq 3$, (with no additional supporting work) earns only the first point.

Total for part (c) 3 points

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

For particle P, $\lim_{t\to\infty}\left(6 - 4e^{-t}\right) = 6$. For particle Q, $\lim_{t\to\infty}\left(3 - \frac{1}{t}\right) = 3$.	One correct limit	**1 point**
Because $6 > 3$, particle P will eventually be farther from the origin.	Answer with reason	**1 point**

Scoring notes:

- A response with an incorrect $y_Q(t)$ from part (c) is eligible for both points in part (d) provided $y_Q(t)$ is a non-constant function. The second point is earned for a consistent answer with reason, and limits correct for particle P and the presented $y_Q(t)$.
- Responses that present statements such as "$6 - 4e^{-t}$ approaches 6" or "Q goes to 3" earn the first point and are eligible for the second point.
- A response that treats ∞ as an input for $x_P(t)$ or $y_Q(t)$, such as "$6 - 4e^{-\infty}$" or "$3 - \frac{1}{\infty}$" is not eligible for the second point.

Total for part (d) 2 points

Total for question 6 9 points

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t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

   (a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\,dt$ in the context of the problem. Use a right Riemann sum with the three subintervals [60, 90], [90, 120], and [120, 135] to approximate the value of $\int_{60}^{135} f(t)\,dt$.

   (b) Must there exist a value of c, for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

   (c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right)\cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \le t \le 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \le t \le 150$.

   Show the setup for your calculations.

   (d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

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A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

   (a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\,dt$ in the context of the problem. Use a right Riemann sum with the three subintervals [60, 90], [90, 120], and [120, 135] to approximate the value of $\int_{60}^{135} f(t)\,dt$.

   (b) Must there exist a value of c, for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

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A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

(c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right)\cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \le t \le 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \le t \le 150$.

Show the setup for your calculations.

(d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

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t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

   (a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\,dt$ in the context of the problem. Use a right Riemann sum with the three subintervals [60, 90], [90, 120], and [120, 135] to approximate the value of $\int_{60}^{135} f(t)\,dt$.

   (b) Must there exist a value of c, for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

   (c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right)\cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \le t \le 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \le t \le 150$. Show the setup for your calculations.

   (d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

(a)

$\int_{60}^{135} f(t)dt$ is the number of gallons pumped from $t = 60$ to $t = 135$ seconds.

$\int_{60}^{135} f(t)dt \approx (0.15)(30) + (0.1)(30) + (0.05)(15) = 8.25$ gal

3: {1: interpretation with units; 1: Form of Right Riemann sum; 1: Answer}

(b)

Yes, because f is differentiable and therefore continuous and the average rate of change on $[60, 120]$ is $\frac{f(120) - f(60)}{120 - 60} = \frac{0.1 - 0.1}{60} = 0$

The Mean Value Theorem guarantees there exists a value of c, $60 < c < 120$, such that $f'(c) = \frac{f(120) - f(60)}{120 - 60} = 0$.

2: {1: $f(120) - f(60) = 0$; 1: answer with justification}

(c)

average rate of flow $= \frac{1}{150}\int_0^{150} g(t)dt = 0.09599\ldots$ gal per sec

2: {1: average value formula; 1: answer}

(d) $g'(140) = -0.004908\ldots$ The rate of flow of gasoline is decreasing at a rate of 0.004908 ... gallons per second per second at $t = 140$ seconds.

2: {1: $g'(140)$; 1: interpretation}

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Part A (AB or BC): Graphing calculator required
Question 1 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

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Model Solution	Scoring

(a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\,dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t)\,dt$.

$\int_{60}^{135} f(t)\,dt$ represents the total number of gallons of gasoline pumped into the gas tank from time $t = 60$ seconds to time $t = 135$ seconds.	Interpretation with units	**1 point**
$\int_{60}^{135} f(t)\,dt$ $\approx f(90)(90-60) + f(120)(120-90) + f(135)(135-120)$ $= (0.15)(30) + (0.1)(30) + (0.05)(15) = 8.25$	Form of Riemann sum	**1 point**
	Answer	**1 point**

Scoring notes:

- To earn the first point, the response must reference gallons of gasoline added/pumped and the time interval $t = 60$ to $t = 135$.
- To earn the second point, at least five of the six factors in the Riemann sum must be correct.
- If there is any error in the Riemann sum, the response does not earn the third point.
- A response of $(0.15)(30) + (0.1)(30) + (0.05)(15)$ earns both the second and third points, unless there is a subsequent error in simplification, in which case the response would earn only the second point.
- A response that presents a correct value with accompanying work that shows the three products in the Riemann sum but does not show all six of the factors and/or the sum process, does not earn the second point but does earn the third point. For example, responses of either $4.5 + 3.0 + 0.75$ or $(0.15)(30), 0.1(30), 0.05(15) \rightarrow 8.25$ earn the third point but not the second.
- A response of $f(90)(90-60) + f(120)(120-90) + f(135)(135-120) = 8.25$ earns both the second and the third points.
- A response that presents an answer of only "8.25" does not earn either the second or third point.
- A response that provides a completely correct left Riemann sum with accompanying work, $f(60)(30) + f(90)(30) + f(120)(15) = 9$, or $(0.1)(30) + (0.15)(30) + (0.1)(15)$ earns 1 of the last 2 points. A response with any errors or missing factors in a left Riemann sum earns neither of the last 2 points.

Total for part (a) 3 points

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(b) Must there exist a value of c, for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

f is differentiable. $\Rightarrow$ f is continuous on $[60, 120]$. $\frac{f(120) - f(60)}{120 - 60} = \frac{0.1 - 0.1}{60} = 0$ By the Mean Value Theorem, there must exist a c, for $60 < c < 120$, such that $f'(c) = 0$.	$f(120) - f(60) = 0$	**1 point**
	Answer with justification	**1 point**

Scoring notes:

- To earn the first point, a response must present either $f(120) - f(60) = 0$, $0.1 - 0.1 = 0$ (perhaps as the numerator of a quotient), or $f(60) = f(120)$.
- To earn the second point a response must
  - have earned the first point,
  - state that f is continuous because f is differentiable (or equivalent),
  - answer "yes" in some way.
- A response may reference either the Mean Value Theorem or Rolle's Theorem.
- A response that references the Intermediate Value Theorem cannot earn the second point.

Total for part (b) 2 points

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(c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right)\cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \le t \le 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \le t \le 150$. Show the setup for your calculations.

$\frac{1}{150-0}\int_0^{150} g(t)\,dt$	Average value formula	**1 point**
$= 0.0959967$ The average rate of flow of gasoline, in gallons per second, is 0.096 (or 0.095).	Answer	**1 point**

Scoring notes:

- The exact value of $\frac{1}{150}\int_0^{150} g(t)\,dt$ is $\frac{12}{125}\sin\left(\frac{25}{16}\right)$.
- A response may present the average value formula in single or multiple steps. For example, the following response earns both points: $\int_0^{150} g(t)\,dt = 14.399504$ so the average rate is 0.0959967.
- A response that presents the average value formula in multiple steps but provides incorrect or incomplete communication, (e.g. $\int_0^{150} g(t)\,dt = \frac{14.399504}{150} = 0.0959967$), earns 1 out of 2 points.
- A response of $\int_0^{150} g(t)\,dt = 0.0959967$ does not earn either point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{150}\int_0^{150} g(t)\,dt = 0.149981$ or 0.002618.

Total for part (c) 2 points

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(d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

$g'(140) \approx -0.004908$ $g'(140) = -0.005$ (or -0.004)	$g'(140)$	**1 point**
The rate at which gasoline is flowing into the tank is decreasing at a rate of 0.005 (or 0.004) gallon per second per second at time $t = 140$ seconds.	Interpretation	**1 point**

Scoring notes:

- The exact value of $g'(140)$ is $\frac{1}{500}\cos\left(\frac{49}{36}\right) - \frac{49}{9000}\sin\left(\frac{49}{36}\right)$.
- The value of $g'(140)$ may appear only in the interpretation.
- To be eligible for the second point, a response must present some numerical value for $g'(140)$.
- To earn the second point, the interpretation must include "the rate of flow of gasoline is changing at a rate of [the declared value of $g'(140)$]" and "at $t = 140$" (or equivalent).
- An interpretation of "decreasing at a rate of -0.005" or "increasing at a rate of 0.005" does not earn the second point.
- Degree mode: In degree mode, $g'(140) = 0.001997$ or 0.00187.

Total for part (d) 2 points

Total for question 1 9 points

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2. Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

   (a) Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.

   (b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.

   (c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.

   (d) Find the total distance Stephen swims over the time interval $0 \le t \le 90$ seconds. Show the setup for your calculations.

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A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

2. Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

(a) Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.

(b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.

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2. Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

   (a) Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.

   (b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.

   (c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.

   (d) Find the total distance Stephen swims over the time interval $0 \le t \le 90$ seconds. Show the setup for your calculations.

(a) $v(t) = 0 \Rightarrow t = 56$ Stephen changes directions at $t = 56$ because $v(t)$ changes from positive to negative.

2: {1: Considers sign of $v(t)$; 1: Answer with reason}

(b) $a(6) = v'(6) = -0.3601\ldots$ meters per second per second
$v(6) = -0.1595\ldots$ Stephen is speeding up at $t = 60$ because $a(6)$ and $v(6)$ have the same sign.

3: {1: $a(60) = v'(60)$; 1: acceleration units; 1: speeding up with reason}

(c) $\int_{20}^{80} v(t)dt = 23.3839\ldots$ meters

2: {1: integrand; 1: answer}

(d) $\int_{0}^{90} |v(t)|dt = 62.1642\ldots$ meters

2: {1: integral; 1: answer}

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Part A (AB): Graphing calculator required
Question 2 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

	Model Solution	Scoring	
(a)	Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.		
	For $0 < t < 90$, $v(t) = 0 \Rightarrow t = 56$.	Considers sign of $v(t)$	**1 point**
	Stephen changes direction when his velocity changes sign. This occurs at $t = 56$ seconds.	Answer with reason	**1 point**

Scoring notes:

- A response that considers $v(t) = 0$ earns the first point.
- A response of "Stephen changes direction when his velocity changes sign" earns the first point for considering the sign of $v(t)$ but must include the answer of $t = 56$ in order to earn the second point.
- A response of $t = 56$ with no supporting work does not earn either point.
- Any presented values of t outside the interval $0 < t < 90$ will not affect scoring.

Total for part (a) 2 points

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(b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.

$v'(60) = a(60) = -0.0360162$ Stephen's acceleration at time $t = 60$ seconds is -0.036 meter per second per second.	$a(60)$ with setup	**1 point**
	Acceleration units	**1 point**
$v(60) = -0.1595124 < 0$ Stephen is speeding up at time $t = 60$ seconds because Stephen's velocity and acceleration are both negative at that time.	Speeding up with reason	**1 point**

Scoring notes:

- The minimum work needed to earn the first point is $v'(60) = -0.036$.
  - $a(60) = -0.0360162$ is not sufficient to earn the first point. The connection $v'(t) = a(t)$ or $v'(60) = a(60)$ must be explicitly shown.
- A response must declare a value for $a(60)$ to be eligible for the second point.
- In order to earn the third point, the presented conclusion must be consistent with a negative velocity at time $t = 60$ and the presented value of $a(60)$.
- A response does not need to find the value of $v(60)$; an implied sign is sufficient.
  - The statement "Stephen is speeding up because $a(60)$ and $v(60)$ have the same sign" (or equivalent) earns the third point, provided a negative value is presented for $a(60)$.
- A response that reports an incorrect sign or value of $v(60)$ does not earn the third point. Any presented value of $v(60)$ must be correct for the number of digits presented, from one up to three decimal places in order to earn the third point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode).
  - In degree mode, there are two possible values for $v'(60)$. A response that declares $v'(60) = -0.000141$ does not earn the first point, but would earn the third point in the presence of $v(60) = 0.042089$ or $v(60) > 0$ and the conclusion that Stephen is slowing down.
  - Similarly, a response that declares $v'(60) = 0.039304$ does not earn the first point, but would earn the third point in the presence of $v(60) = 0.042089$ or $v(60) > 0$ and the conclusion that Stephen is speeding up.

Total for part (b) 3 points

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(c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.

$\int_{20}^{80} v(t)\,dt$	Integral	**1 point**
$= 23.383997$ The distance between Stephen's positions at $t = 20$ seconds and $t = 80$ seconds is 23.384 (or 23.383) meters.	Answer	**1 point**

Scoring notes:

- The first point is earned only for $\int_{20}^{80} v(t)\,dt$ (or the equivalent) with or without the differential.
- The second point is earned only for an answer of 23.384 (or 23.383) regardless of whether the first point was earned.
- Degree mode: In degree mode, $\int_{20}^{80} v(t)\,dt = 2.407982$.

Total for part (c) 2 points

(d) Find the total distance Stephen swims over the time interval $0 \le t \le 90$ seconds. Show the setup for your calculations.

$\int_{0}^{90} \lvert v(t) \rvert\,dt$	Integral	**1 point**
$= 62.164216$ The total distance Stephen swims over the time interval $0 \le t \le 90$ seconds is 62.164 meters.	Answer	**1 point**

Scoring notes:

- The first point is earned only for $\int_{0}^{90} \lvert v(t) \rvert\,dt$ or $\int_{0}^{56} v(t)\,dt - \int_{56}^{90} v(t)\,dt$ (or the equivalent), with or without the differential(s).
- The second point is earned only for an answer of 62.164 regardless of whether the first point was earned.
- Degree mode: In degree mode, $\int_{0}^{90} \lvert v(t) \rvert\,dt = 3.127892$.

Total for part (d) 2 points

Total for question 2 9 points

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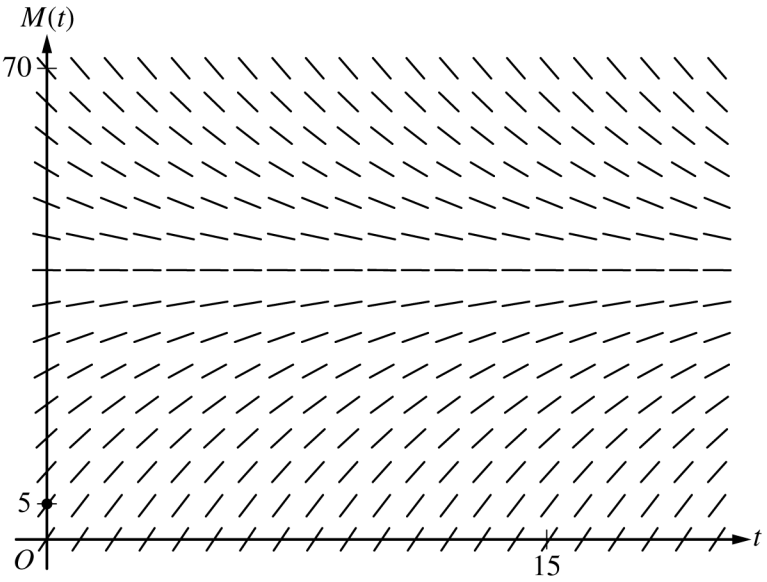
3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.



(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

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NO CALCULATOR ALLOWED

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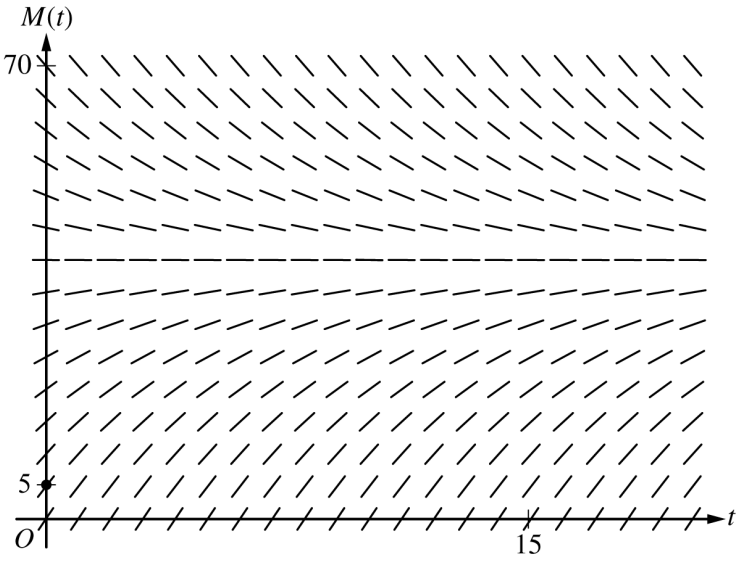

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NO CALCULATOR ALLOWED

(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

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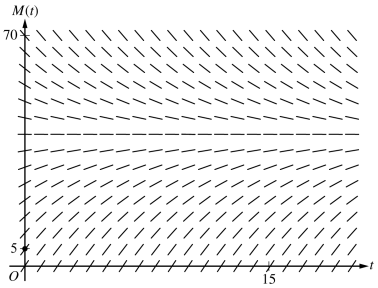

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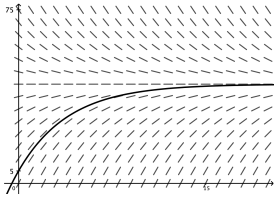
(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

(a) 

(b) $M(t) \approx 5 + \left.\frac{dM}{dt}\right|_{(0,5)} t =$
$5 + \frac{1}{4}(40 - 5)t = 5 + \frac{35}{4}t$
$M(2) \approx 5 + \frac{35}{4}(2) = 5 + \frac{35}{2} = \frac{45}{2}\ °C$

(c) $\frac{d^2M}{dt^2} = \frac{d}{dt}\left(\frac{1}{4}(40 - M)\right) = \frac{1}{4}\left(-\frac{dm}{dt}\right) = -\frac{1}{4}\left(\frac{1}{4}(40 - M)\right)$
$\left.\frac{d^2M}{dt^2}\right|_{(0,5)} = -\frac{1}{4}\left(\frac{1}{4}(40 - 5)\right) = -\frac{35}{16} < 0$ The approximation from part (b) is an overestimate because $M(t)$ is concave down near $t = 0$.

(d) $\int \frac{1}{40-M} dM = \int \frac{1}{4} dt \Rightarrow -\ln|40 - M| = \frac{1}{4}t + C$
$(0,5) \Rightarrow -\ln|40 - 5| = \frac{1}{4}(0) + C \Rightarrow C = -\ln 35$
$\ln|40 - M| = -\frac{1}{4}t + \ln 35 \Rightarrow |40 - M| = 35e^{-\frac{1}{4}t}$
$M = 40 \pm 35e^{-\frac{1}{4}t} \quad M(0) = 5 \Rightarrow M = 40 - 35e^{-\frac{1}{4}t}$

1: solution curve

2: 1: $\left.\frac{dM}{dt}\right|_{t=0}$
1: approximation

2: 1: $\frac{d^2M}{dt^2}$
1: overestimate with reason

4: 1: separates variables
1: Find antiderivatives
1: Constant of integration and uses initial condition
1: solves for $M(t)$

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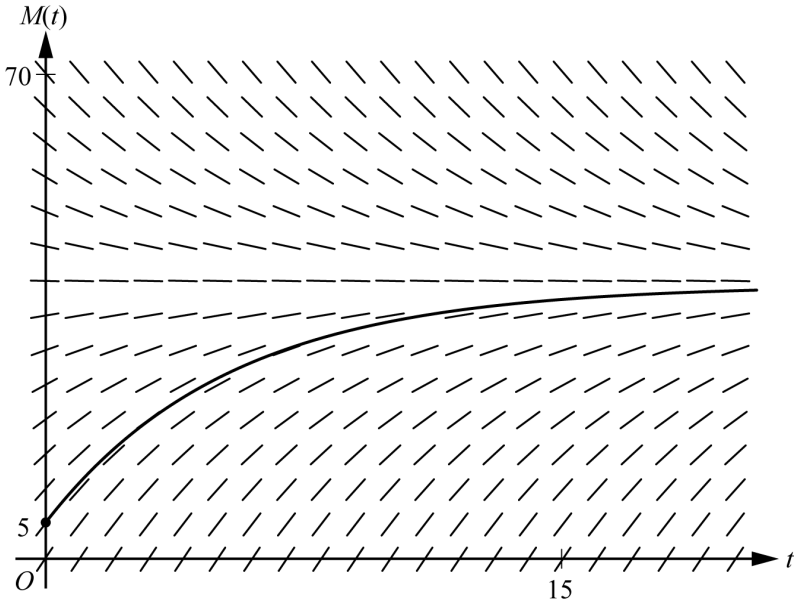
Part A (AB or BC): Graphing calculator not allowed
Question 3 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is $5°\text{C}$. It can be shown that $M(t) < 40$ for all values of t.

	Model Solution	Scoring
(a)	A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.	
		Solution curve **1 point**

Scoring notes:

- The solution curve must pass through the point $(0, 5)$, extend reasonably close to the left and right edges of the rectangle, and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered
- The solution curve must lie entirely below the horizontal line segments at $M = 40$.

Total for part (a) 1 point

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(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

$\left.\frac{dM}{dt}\right\|_{t=0} = \frac{1}{4}(40 - 5) = \frac{35}{4}$	$\left.\frac{dM}{dt}\right\|_{t=0}$	**1 point**
The tangent line equation is $y = 5 + \frac{35}{4}(t - 0)$. $M(2) \approx 5 + \frac{35}{4} \cdot 2 = 22.5$ The temperature of the milk at time $t = 2$ minutes is approximately 22.5° Celsius.	Approximation	**1 point**

Scoring notes:

- The value of the slope may appear in a tangent line equation or approximation.
- A response of $5 + \frac{35}{4} \cdot 2$ is the minimal response to earn both points.
- A response of $\frac{1}{4}(40 - 5)$ earns the first point. If there are any subsequent errors in simplification, the response does not earn the second point.
- In order to earn the second point the response must present an approximation found by using a tangent line that:
  - passes through the point $(0, 5)$, and
  - has slope $\frac{35}{4}$ or a nonzero slope that is declared to be the value of $\frac{dM}{dt}$.
- An unsupported approximation does not earn the second point.
- The approximation need not be simplified, but the response does not earn the second point if the approximation is simplified incorrectly.

Total for part (b) 2 points

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(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

$\frac{d^2M}{dt^2} = -\frac{1}{4}\frac{dM}{dt} = -\frac{1}{4}\left(\frac{1}{4}(40 - M)\right) = -\frac{1}{16}(40 - M)$	$\frac{d^2M}{dt^2}$	**1 point**
Because $M(t) < 40$, $\frac{d^2M}{dt^2} < 0$, so the graph of M is concave down. Therefore, the tangent line approximation of $M(2)$ is an overestimate.	Overestimate with reason	**1 point**

Scoring notes:

- The first point is earned for either $\frac{d^2M}{dt^2} = -\frac{1}{4}\left(\frac{1}{4}(40 - M)\right)$ or $\frac{d^2M}{dt^2} = -\frac{1}{16}(40 - M)$ (or equivalent). A response that presents any subsequent simplification error does not earn the second point.
- A response that presents an expression for $\frac{d^2M}{dt^2}$ in terms of $\frac{dM}{dt}$ but fails to continue to an expression in terms of M (i.e., $\frac{d^2M}{dt^2} = -\frac{1}{4}\frac{dM}{dt}$) does not earn the first point but is eligible for the second point.
- If the response presents an expression for $\frac{d^2M}{dt^2}$ that is incorrect, the response is eligible for the second point only if the expression is a nonconstant linear function that is negative for $5 < M < 40$.
    - Special case: A response that presents $\frac{d^2M}{dt^2} = \frac{1}{16}(40 - M)$ does not earn the first point, but is eligible to earn the second point for a consistent answer and reason
- To earn the second point a response must include one of $\frac{d^2M}{dt^2} < 0$, or $\frac{dM}{dt}$ is decreasing, or the graph of M is concave down, as well as the conclusion that the approximation is an overestimate.
- A response that presents an argument based on $\frac{d^2M}{dt^2}$ or concavity at a single point does not earn the second point.

Total for part (c) 2 points

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(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

$\frac{dM}{40 - M} = \frac{1}{4}dt$ $\int \frac{dM}{40 - M} = \int \frac{1}{4}\, dt$	Separates variables	**1 point**
$-\ln\lvert 40 - M \rvert = \frac{1}{4}t + C$	Finds antiderivatives	**1 point**
$-\ln\lvert 40 - 5 \rvert = 0 + C \Rightarrow C = -\ln 35$ $M(t) < 40 \Rightarrow 40 - M > 0 \Rightarrow \lvert 40 - M \rvert = 40 - M$ $-\ln(40 - M) = \frac{1}{4}t - \ln 35$ $\ln(40 - M) = -\frac{1}{4}t + \ln 35$	Constant of integration and uses initial condition	**1 point**
$40 - M = 35e^{-t/4}$ $M = 40 - 35e^{-t/4}$	Solves for M	**1 point**

Scoring notes:

- A response with no separation of variables earns 0 out of 4 points.
- A response that presents an antiderivative of $-\ln(40 - M)$ without absolute value symbols is sufficient.
- A response with no constant of integration can earn at most the first 2 points.
- A response is eligible for the third point only if it has earned the first 2 points.
    - Special Case: A response that presents $+\ln(40 - M) = \frac{t}{4} + C$ (or equivalent) does not earn the second point, is eligible for the third point, but not eligible for the fourth.
- An eligible response earns the third point by correctly including the constant of integration in an equation and substituting 0 for t and 5 for M.
- A response is eligible for the fourth point only if it has earned the first 3 points.
- A response earns the fourth point only for an answer of $M = 40 - 35e^{-t/4}$ or equivalent.

Total for part (d) 4 points

Total for question 3 9 points

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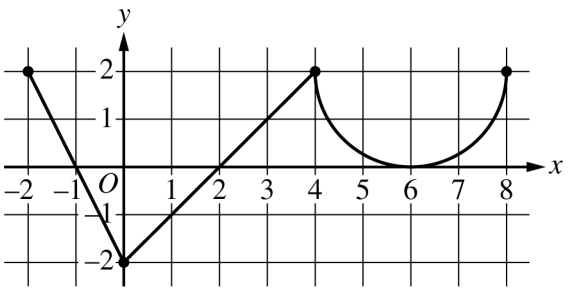

Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

   (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

   (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

   (c) Find the value of $\lim_{x \to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

   (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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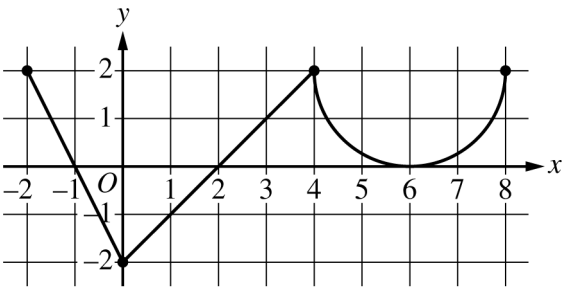

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   (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

   (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

   (c) Find the value of $\lim_{x\to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

   (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

(a) Neither because f' does not change signs at $x = 6$.

1: answer with reason

(b) f is concave down on $(-2,0)$ and $(4,6)$ because f' is decreasing.

2: $\begin{cases} 1: \text{intervals} \\ 1: \text{Reason} \end{cases}$

(c) $\lim_{x\to 2}[6f(x) - 3x] = 6f(2) - 6 = 6(1) - 6 = 0$

f is differentiable at $x = 2$, so f is continuous at $x = 2$,

so $\lim_{x\to 2} f(x) = f(2) = 1$

$\lim_{x\to 2}[x^2 - 5x + 6] = 2^2 - 10 + 6 = 0$

$\lim_{x\to 2} \frac{6f(x)-3x}{x^2-5x+6} = \underbrace{\lim_{x\to 2} \frac{6f'(x)-3}{2x-5}}_{\substack{\text{l'hospital's} \\ \text{rule}}} = \frac{6f'(2)-3}{4-5} = \frac{6(0)-3}{-1} = \frac{-3}{-1} = 3$

3: $\begin{cases} 1: \text{Limits of numerator and denomonator} \\ 1: \text{Uses L'Hospital's Rule} \\ 1: \text{Answer} \end{cases}$

(d) The absolute minimum value of f occurs at an endpoint of the interval or a critical point. $f'(x) = 0 \Rightarrow x = -1, 2$ and 6

x	$f(x)$
-2	$f(2) - f(-2) = \int_{-2}^{2} f'(x)dx \Rightarrow f(-2) = f(2) - \int_{-2}^{2} f'(x)dx$ $f(-2) = 1 - \left[\frac{1}{2}(2)(1) + \frac{1}{2}(3)(-2)\right] = 1 - [1 - 3] = 3$
-1	$f(-1) = f(2) - \int_{-1}^{2} f'(x)dx = 1 - \left[\frac{1}{2}(3)(-2)\right] = 4$
2	1 Absolute minimum by Candidate's Test
6	$f(6) - f(2) = \int_{2}^{6} f'(x)dx \Rightarrow f(6) = f(2) + \int_{2}^{6} f'(x)dx$ $f(6) = 1 + \left[\frac{1}{2}(2)(2) + \left[(2)(2) - \frac{1}{4}\pi(2^2)\right]\right] = 7 - \pi \approx 4$
8	$f(8) = f(2) + \int_{2}^{8} f'(x)dx$ $f(8) = 1 + \left[\frac{1}{2}(2)(2) + \left[(4)(2) - \frac{1}{2}\pi(2^2)\right]\right] = 11 - 2\pi \approx 5$

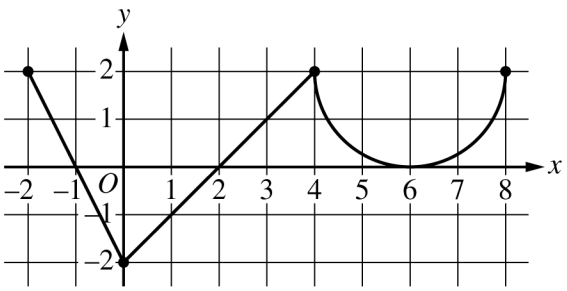
3: $\begin{cases} 1: \text{Considers } f'(x) = 0 \\ 1: \text{Justification} \\ 1: \text{answer} \end{cases}$

Part A (AB or BC): Graphing calculator not allowed
Question 4 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Graph of f'

The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

	Model Solution	Scoring
(a)	Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.	
	$f'(x) > 0$ on $(2, 6)$ and $f'(x) > 0$ on $(6, 8)$ $f'(x)$ does not change sign at $x = 6$, so there is neither a relative maximum nor a relative minimum at this location.	Answer with reason **1 point**

Scoring notes:

- A response that declares $f'(x)$ does not change sign at $x = 6$, so neither, is sufficient to earn the point.
- A response does not have to present intervals on which $f'(x)$ is positive or negative, but if any are given, they must be correct. Any presented intervals may include none, one, or both endpoints.
- A response that declares $f'(x) > 0$ before and after $x = 6$ does not earn the point.

Total for part (a) 1 point

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(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

The graph of f is concave down on $(-2, 0)$ and $(4, 6)$ because f' is decreasing on these intervals $(-2, 0)$ $(4, 6)$.	Intervals	**1 point**
	Reason	**1 point**

Scoring notes:

- The first point is earned only by an answer of $(-2, 0)$ and $(4, 6)$, or these intervals including one or both endpoints.
- A response must earn the first point to be eligible for second point.
- To earn the second point a response must correctly discuss the behavior of f', the slopes of f', or the sign of f''.
- Special case: A response that presents exactly one of the two correct intervals with a correct reason earns 1 out of 2 points.

Total for part (b) 2 points

(c) Find the value of $\lim_{x \to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

Because f is differentiable at $x = 2$, f is continuous at $x = 2$, so $\lim_{x \to 2} f(x) = f(2) = 1$. $\lim_{x \to 2} (6f(x) - 3x) = 6 \cdot 1 - 3 \cdot 2 = 0$ $\lim_{x \to 2} \left(x^2 - 5x + 6\right) = 0$	Limits of numerator and denominator	**1 point**
Because $\lim_{x \to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6}$ is of indeterminate form $\dfrac{0}{0}$, L'Hospital's Rule can be applied. Using L'Hospital's Rule, $\lim_{x \to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6} = \lim_{x \to 2} \dfrac{6f'(x) - 3}{2x - 5} = \dfrac{6 \cdot 0 - 3}{2 \cdot 2 - 5} = 3$	Uses L'Hospital's Rule	**1 point**
	Answer	**1 point**

Scoring notes:

- The first point is earned by the presentation of two separate limits for the numerator and denominator.
- A response that presents a limit explicitly equal to $\dfrac{0}{0}$ does not earn the first point.
- The second point is earned by applying L'Hospital's Rule, that is, by presenting at least one correct derivative in the limit of a ratio of derivatives.
- The third point is earned for the correct answer with supporting work.

Total for part (c) 3 points

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(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

$f'(x) = 0 \Rightarrow x = -1,\ x = 2,\ x = 6$	Considers $f'(x) = 0$	**1 point**
The function f is continuous on $[-2, 8]$, so the candidates for the location of an absolute minimum for f are $x = -2$, $x = -1$, $x = 2$, $x = 6$, and $x = 8$. $\begin{array}{c\|c} x & f(x) \\ \hline -2 & 3 \\ -1 & 4 \\ 2 & 1 \\ 6 & 7 - \pi \\ 8 & 11 - 2\pi \end{array}$ The absolute minimum value of f is $f(2) = 1$.	Justification	**1 point**
	Answer	**1 point**

Scoring notes:

- To earn the first point, a response must state $f' = 0$ or equivalent. Listing the zeros of f' is not sufficient.
- A response that presents any error in evaluating f at any critical point or endpoint will not earn the justification point.
- A response need not present the value of $f(-1)$ provided $x = -1$ is eliminated because it is the location of a local maximum. A response need not present the value of $f(6)$ provided $x = 6$ is eliminated by reference to part (a), or eliminated through analysis.
- A response need not present the value of $f(8)$ provided there is a presentation that argues $f'(x) \geq 0$ for $x > 2$ and therefore $f(8) > f(2)$.
- A response that does not consider both endpoints does not earn the justification point.
- The answer point is earned only for indicating that the minimum value is 1. It is not earned for noting that the minimum occurs at $x = 2$.

Total for part (d) 3 points

Total for question 4 9 points

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x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

   (a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

   (b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

   (c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\,dt$. Find $m(2)$. Show the work that leads to your answer.

   (d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
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   (a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

   (b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

   (c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\,dt$. Find $m(2)$. Show the work that leads to your answer.

   (d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

(a) $h'(x) = f'\big(g(x)\big) \cdot g'(x)$

$h'(7) = f'\big(g(7)\big) \cdot g'(7) = f'(0) \cdot (8) = \frac{3}{2}(8) = 12$

2: {1: chain rule; 1: answer}

(b) $k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + \big(f(x)\big)^2 \cdot g'(x)$

$k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + \big(f(4)\big)^2 \cdot g'(4)$

$k''(4) = 2(4) \cdot (3) \cdot (-3) + (4)^2 \cdot (2) = -72 + 32 < 0$

The graph of k is concave down at $x = 4$ because $k''(4) < 0$.

3: {1: product rule; 1: $k''(4)$; 1: answer with reason}

(c)

$m(2) = 5(2)^3 + \int_0^2 f'(x)dx = 40 + [f(2) - f(0)]$

$= 40 + [7 - 10] = 37$

1: answer with supporting work

(d) $m'(x) = 15x^2 + f'(x) \Rightarrow m'(2) = 15(2)^2 + f'(2) = 60 + (-8) > 0$

m is increasing at $x = 2$ because $m'(2) > 0$.

3: {1: Considers $m'(x)$; 1: $m'(2)$ with supporting work; 1: answer with reason.}

Part A (AB): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

	Model Solution	Scoring	
(a)	Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.		
	$h'(x) = f'(g(x)) \cdot g'(x)$ $h'(7) = f'(g(7)) \cdot g'(7)$	Chain rule	**1 point**
	$= f'(0) \cdot 8 = \frac{3}{2} \cdot 8 = 12$	Answer	**1 point**

Scoring notes:

- The first point is earned for either $h'(x) = f'(g(x)) \cdot g'(x)$ or $h'(7) = f'(g(7)) \cdot g'(7)$.
- If the first point is earned, the second point is earned only for an answer of 12 (or equivalent).
- If the first point is not earned, the second point can be earned only for a response of either $f'(0) \cdot 8 = 12$ or $\frac{3}{2} \cdot 8$.
- A response of 12 with no supporting work does not earn either point.

Total for part (a) 2 points

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(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

$k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$	Product or chain rule	**1 point**
$k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$ $= 2 \cdot 4 \cdot 3 \cdot (-3) + 4^2 \cdot 2 = -72 + 32 = -40$	$k''(4)$	**1 point**
The graph of k is concave down at the point where $x = 4$ because $k''(4) < 0$ and k'' is continuous.	Answer with reason	**1 point**

Scoring notes:

- The first point is earned for either $k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$.
- The first point is also earned by any of the following incorrect expressions, each of which has a single error in the application of the product rule or the chain rule:
    - $2f(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $2f(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
    - $2f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $2f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
    - $f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
    - $2f(x) \cdot f'(x) \cdot g'(x)$ or $2f(4) \cdot f'(4) \cdot g'(4)$
    - Note: A response that presents one of these expressions cannot earn the second point.
- To earn the second point a response must correctly find $k''(4) = -40$ (or equivalent) with supporting work.
- The third point is earned for an answer and reason that are consistent with any declared nonzero value of $k''(4)$.

Total for part (b) 3 points

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(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\,dt$. Find $m(2)$. Show the work that leads to your answer.

$m(2) = 5 \cdot 8 + \int_0^2 f'(t)\,dt = 40 + (f(2) - f(0))$ $= 40 + (7 - 10) = 37$	Answer with supporting work	**1 point**

Scoring notes:

- The point is earned only for an answer of 37 (or equivalent) with supporting work equivalent to $5 \cdot 8 + (f(2) - f(0))$, $40 + (f(2) - f(0))$, $5 \cdot 8 + (7 - 10)$, or $40 + (7 - 10)$.
- An answer of 37 with no supporting work does not earn the point.

Total for part (c) **1 point**

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

$m'(x) = 15x^2 + f'(x)$	Considers $m'(x)$	**1 point**
$m'(2) = 15 \cdot 4 + f'(2) = 60 + (-8) = 52$	$m'(2)$ with supporting work	**1 point**
The graph of m is increasing at $x = 2$ because $m'(2) > 0$.	Answer with justification	**1 point**

Scoring notes:

- The first point is earned for considering $m'(x)$, $m'(2)$, or m'. This consideration may appear in a justification statement.
- The second point is earned for $m'(2) = 15 \cdot 2^2 + f'(2)$, $m'(2) = 60 + f'(2)$, or $m'(2) = 60 - 8$ but is not earned for an unsupported response of $m'(2) = 52$.
- The third point is earned for an answer and justification consistent with any declared value of $m'(2)$.

Total for part (d) **3 points**

Total for question 5 **9 points**

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6. Consider the curve given by the equation $6xy = 2 + y^3$.

   (a) Show that $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$.

   (b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

   (c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

   (d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\frac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\frac{dx}{dt} = \frac{2}{3}$ unit per second. What is the value of $\frac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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6. Consider the curve given by the equation $6xy = 2 + y^3$.

(a) Show that $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$.

(b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

(c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

(d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\frac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\frac{dx}{dt} = \frac{2}{3}$ unit per second. What is the value of $\frac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

(a) $6(y + xy') = 3y^2y' \Rightarrow 6y + 6xy' = 3y^2y'$

$6xy' - 3y^2y' = -6y \Rightarrow y' = \frac{-6y}{6x - 3y^2} = \frac{-2y}{2x - y^2} = \frac{2y}{y^2 - 2x}$

2: {1: implicit differentiation; 1: Verification}

(b) Horizontal tangent line $\Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{2y}{y^2 - 2x} = 0$

$2y = 0, y^2 - 2x \neq 0 \Rightarrow y = 0 \Rightarrow 6x(0) \overset{?}{=} 2 + 0^3 \Rightarrow 0 \neq 2$

Therefore, no such point exists because there are no points on the curve where $y = 0$.

2: {1: Sets $dy = 0$; 1: answer with reason}

(c) Vertical tangent line $\Rightarrow \frac{dy}{dx}$ is undefined $\Rightarrow y^2 - 2x = 0, 2y \neq 0$

$y^2 - 2x = 0 \Rightarrow y^2 = 2x \Rightarrow x = \frac{y^2}{2}$

$6\left(\frac{y^2}{2}\right)y = 2 + y^3 \Rightarrow 3y^3 = 2 + y^3 \Rightarrow 2y^3 = 2 \Rightarrow \boxed{y = 1, x = \frac{1}{2}}$

3: {1: sets $y^2 - 2x = 0$; 1: Substitutes $x = \frac{y^2}{2}$ into $6xy = 2$_y; 1: answer}

(d)

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{2y}{y^2 - 2x}\left(\frac{dx}{dt}\right)$

$\left.\frac{dy}{dt}\right|_{\left(\frac{1}{2}, -2\right)} = \frac{2(-2)}{(-2)^2 - 2\left(\frac{1}{2}\right)}\left(\frac{2}{3}\right) = \frac{-4}{3}\left(\frac{2}{3}\right) = -\frac{8}{9}$

2: {1: Uses implicit differentiation with respect to t.; 1: answer}

Part A (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve given by the equation $6xy = 2 + y^3$.

	Model Solution	Scoring	
(a)	Show that $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$.		
	$\frac{d}{dx}(6xy) = \frac{d}{dx}\left(2 + y^3\right) \Rightarrow 6y + 6x\frac{dy}{dx} = 3y^2\frac{dy}{dx}$	Implicit differentiation	**1 point**
	$\Rightarrow 2y = \frac{dy}{dx}\left(y^2 - 2x\right) \Rightarrow \frac{dy}{dx} = \frac{2y}{y^2 - 2x}$	Verification	**1 point**

Scoring notes:

- The first point is earned only for the correct implicit differentiation of $6xy = 2 + y^3$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y'.
- The second point may not be earned without the first point.
- It is sufficient to present $2y = \frac{dy}{dx}\left(y^2 - 2x\right)$ to earn the second point, provided there are no subsequent errors.

Total for part (a) — 2 points

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(b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

For the line tangent to the curve to be horizontal, it is necessary that $2y = 0$ (so $y = 0$) and that $y^2 - 2x \neq 0$.	Sets $2y = 0$	**1 point**
Substituting $y = 0$ into $6xy = 2 + y^3$ yields the equation $6x \cdot 0 = 2$, which has no solution. Therefore, there is no point on the curve at which the line tangent to the curve is horizontal.	Answer with reason	**1 point**

Scoring notes:

- The first point is earned with any of $2y = 0$, $y = 0$, $\frac{dy}{dx} = 0$, $dy = 0$, $y' = 0$, or $\frac{2y}{y^2 - 2x} = 0$.
- A response need not state that at a horizontal tangent, $y^2 - 2x \neq 0$.

Total for part (b) **2 points**

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(c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

For the line tangent to this curve to be vertical, it is necessary that $2y \neq 0$ and that $y^2 - 2x = 0$ (so $x = \frac{y^2}{2}$).	Sets $y^2 - 2x = 0$	**1 point**
Substituting $x = \frac{y^2}{2}$ into $6xy = 2 + y^3$ yields the equation $3y^2 \cdot y = 2 + y^3$ $\Rightarrow 2y^3 = 2 \Rightarrow y = 1.$	Substitutes $x = \frac{y^2}{2}$ into $6xy = 2 + y^3$	**1 point**
Substituting $y = 1$ in $6xy = 2 + y^3$ yields $6x = 2 + 1$, or $x = \frac{1}{2}$. The tangent line to the curve is vertical at the point $\left(\frac{1}{2}, 1\right)$.	Answer	**1 point**

Scoring notes:

- The first point can be earned by presenting $y^2 = 2x$ or $y = \sqrt{2x}$.
- The second point can be earned for the substitution of $y = \sqrt{2x}$ into $6xy = 2 + y^3$, or for substituting $x = \frac{2 + y^3}{6y}$ into $y^2 - 2x = 0$.
- A response earns all three points by setting $y^2 - 2x = 0$, declaring the point $\left(\frac{1}{2}, 1\right)$, and verifying that this point is on the curve $6xy = 2 + y^3$.
- A response that identifies the point $\left(\frac{1}{2}, 1\right)$ but does not verify that the point is on the curve, does not earn the second or the third point.
- To earn the third point the response must present both coordinates of the point $\left(\frac{1}{2}, 1\right)$. The coordinates need not appear as an ordered pair as long as they are labeled.

Total for part (c) **3 point**

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(d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\frac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\frac{dx}{dt} = \frac{2}{3}$ unit per second. What is the value of $\frac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

$6y\frac{dx}{dt} + 6x\frac{dy}{dt} = 0 + 3y^2\frac{dy}{dt}$	Uses implicit differentiation with respect to t	**1 point**
At the point $(x, y) = \left(\frac{1}{2}, -2\right)$, $6(-2)\left(\frac{2}{3}\right) + 6\left(\frac{1}{2}\right)\frac{dy}{dt} = 3(-2)^2\frac{dy}{dt}$ $\Rightarrow -8 + 3\frac{dy}{dt} = 12\frac{dy}{dt}$ $\Rightarrow \frac{dy}{dt} = -\frac{8}{9}$ unit per second	Answer	**1 point**

Scoring notes:

- The first point is earned by presenting one or more of the terms $6y\frac{dx}{dt}$, $6x\frac{dy}{dt}$, or $3y^2\frac{dy}{dt}$.
- Units will not affect scoring in this part.
- An unsupported response of $-\frac{8}{9}$ earns no points.
- Alternate solution:

  $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

  $\left.\frac{dy}{dx}\right|_{(x, y)=(1/2, -2)} = \frac{2(-2)}{(-2)^2 - 2(1/2)} = -\frac{4}{3}$

  $\left.\frac{dy}{dt}\right|_{(x, y)=(1/2, -2)} = \left.\frac{dy}{dx} \cdot \frac{dx}{dt}\right|_{(x, y)=(1/2, -2)} = -\frac{4}{3} \cdot \frac{2}{3} = -\frac{8}{9}$ unit per second

  - The first point is earned for the statement $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$, or equivalent.
  - A numerical expression, such as $-\frac{4}{3} \cdot \frac{2}{3}$ or $\frac{2(-2)}{(-2)^2 - 2\left(\frac{1}{2}\right)} \cdot \frac{2}{3}$ earns both points.

Total for part (d) **2 points**

Total for question 6 **9 points**

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t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

1. The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

   (a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \le t \le 7$. Show the work that leads to your answer and include units of measure.

   (b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.

   (c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

   (d) For the model defined in part (c), it can be shown that $C''(t) = \frac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

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   (b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.

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A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

(c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \dfrac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

(d) For the model defined in part (c), it can be shown that $C''(t) = \dfrac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

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t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

1. The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.
   (a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \le t \le 7$. Show the work that leads to your answer and include units of measure.
   (b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.
   (c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.
   (d) For the model defined in part (c), it can be shown that $C''(t) = \frac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

(a) $C'(5) \approx \frac{C(7) - C(3)}{7 - 3} = \frac{69 - 85}{4} = -\frac{16}{4} = -4$ ° per minute

2: {1: Estimate with supporting work; 1: Units}

(b) $\int_0^{12} C(t)dt \approx (100)(3) + (85)(4) + (69)(5) = 985$

$\frac{1}{12}\int_0^{12} C(t)dt$ is the average temperature of the coffee on the interval $0 \le t \le 12$ minutes.

3: {1: Form of left Riemann sum; 1: estimate; 1: interpretation}

(c) $C(20) = 55 + \int_{12}^{20} C'(t)dt = 40.3291\cdots$

3: {1: integral; 1: Uses initial condition; 1: Answer}

(d) $C(t)$ is changing at an increasing rate for $12 < t < 20$ because $C''(t) > 0$ on the interval $12 < t < 20$.

1: Answer with reason

Part A (AB or BC): Graphing calculator required
Question 1 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

	Model Solution	Scoring	
(a)	Approximate $C'(5)$ using the average rate of change of C over the interval $3 \le t \le 7$. Show the work that leads to your answer and include units of measure.		
	$C'(5) \approx \frac{C(7)-C(3)}{7-3} = \frac{69-85}{4} = -4$ degrees Celsius per minute	Estimate with supporting work	**1 point**
		Units	**1 point**

Scoring notes:

- To earn the first point a response must include a difference and a quotient as the supporting work.
- $\frac{-16}{7-3}$, $\frac{69-85}{7-3}$, or $\frac{69-85}{4}$ is sufficient to earn the first point.
- A response that presents only units without a numerical approximation for $C'(5)$ does not earn the second point.
- The second point is also earned for "degrees per minute" attached to a numerical value.

Total for part (a) 2 points

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(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.

$\int_0^{12} C(t)\,dt \approx (3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ $= 3\cdot 100 + 4\cdot 85 + 5\cdot 69 = 985$	Form of left Riemann sum	**1 point**
	Estimate	**1 point**
$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of the coffee (in degrees Celsius) over the interval from $t = 0$ to $t = 12$.	Interpretation	**1 point**

Scoring notes:

- Read "$=$" as "$\approx$" for the first point.
- To earn the first point at least five of the six factors in the Riemann sum must be correct. If any of the six factors is incorrect, the response does not earn the second point.
- A response of $(3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ earns the first point. Values must be pulled from the table to earn the second point.
- A response of $3\cdot 100 + 4\cdot 85 + 5\cdot 69$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A completely correct right Riemann sum (e.g., $3\cdot 85 + 4\cdot 69 + 5\cdot 55$) earns 1 of the first 2 points. An unsupported answer of 806 does not earn either of the first 2 points.
- Units will not affect scoring for the second point.
- To earn the third point the interpretation must include both "average temperature" and the time interval. The response need not include a reference to units. However, if incorrect units are given in the interpretation, the response does not earn the third point.

Total for part (b) 3 points

(c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

$C(20) = C(12) + \int_{12}^{20} C'(t)\,dt$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 55 - 14.670812 = 40.329188$ The temperature of the coffee at time $t = 20$ is 40.329 degrees Celsius.	Answer	**1 point**

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Scoring notes:

- The first point is earned for a definite integral with integrand $C'(t)$. If the limits of integration are incorrect, the response does not earn the third point.
- A linkage error such as $C(20) = \int_{12}^{20} C'(t)\,dt = 55 - 14.670812$ or $\int_{12}^{20} C'(t)\,dt = -14.670812 = 40.329188$ earns the first 2 points but does not earn the third point.
- Missing differential (dt):
  - Unambiguous responses of $C(20) = C(12) + \int_{12}^{20} C'(t)$ or $C(20) = 55 + \int_{12}^{20} C'(t)$ earn the first 2 points and are eligible for the third point.
  - Ambiguous responses of $C(20) = \int_{12}^{20} C'(t) + C(12)$ or $C(20) = \int_{12}^{20} C'(t) + 55$ do not earn the first point, earn the second point, and earn the third point if the given numeric answer is correct. If there is no numeric answer given, these responses do not earn the third point.
- The second point is earned for adding $C(12)$ or 55 to a definite integral with a lower limit of 12, either symbolically or numerically.
- The third point is earned for an answer of $55 - 14.671$ or $-14.671 + 55$ with no additional simplification, provided there is some supporting work for these values.
- An answer of just 40.329 with no supporting work does not earn any points.

Total for part (c) 3 points

(d) For the model defined in part (c), it can be shown that $C''(t) = \dfrac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

Because $C''(t) > 0$ on the interval $12 < t < 20$, the rate of change in the temperature of the coffee, $C'(t)$, is increasing on this interval. That is, on the interval $12 < t < 20$, the temperature of the coffee is changing at an increasing rate.	Answer with reason	**1 point**

Scoring notes:

- This point is earned only for a correct answer with a correct reason that references the sign of the second derivative of C.
- A response that provides a reason based on the evaluation of $C''(t)$ at a single point does not earn this point.
- A response that uses ambiguous pronouns (such as "It is positive, so increasing") does not earn this point.
- A response does not need to reference the interval $12 < t < 20$ to earn the point.

Total for part (d) 1 point

Total for question 1 9 points

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2. A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln\left(t^2 - 4t + 5\right) - 0.2t$.

   (a) There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R. For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.

   (b) Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.

   (c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.

   (d) Find the total distance traveled by the particle over the interval $1 \leq t \leq 4$. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

2. A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln\left(t^2 - 4t + 5\right) - 0.2t$.

(a) There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R. For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.

(b) Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.

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A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

(c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.

(d) Find the total distance traveled by the particle over the interval $1 \le t \le 4$. Show the setup for your calculations.

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2. A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln\left(t^2 - 4t + 5\right) - 0.2t$.

(a) There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R. For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.

(b) Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.

(c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.

(d) Find the total distance traveled by the particle over the interval $1 \leq t \leq 4$. Show the setup for your calculations.

(a) $v(t_R) = 0 \quad t_R = 1.425$ (or 1.426)
The particle is moving right because $v(t) > 0$ for $0 < t < t_R$.

2: {1: t_R; 1: Direction with explanation}

(b) $a(1.5) = v'(1.5) = -1$
The speed is increasing at time $t = 1.5$ because $v(1.5) = -0.076 < 0$ and $a(1.5) < 0$.

2: {1: Acceleration with setup; 1: Answer with explanation}

(c) $x(4) = -3 + \int_1^4 v(t)dt = -2.802$ (or -2.803)

3: {1: integral; 1: Uses initial condition; 1: Answer}

(d) total distance $= \int_1^4 |v(t)|dt = 0.958$

2: {1: Integral; 1: answer}

Part A (AB): Graphing calculator required
Question 2 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln\left(t^2 - 4t + 5\right) - 0.2t$.

	Model Solution	Scoring	
(a)	There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R. For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.		
	$v(t) = 0 \Rightarrow t = 1.425610$ Therefore, the particle is at rest (not moving) at $t_R = 1.426$ (or 1.425).	t_R	**1 point**
	For $0 < t < t_R$, $v(t) > 0$. Therefore, the particle is moving to the right on that interval.	Direction with explanation	**1 point**

Scoring notes:

- The first point is earned for considering $v(t) = 0$ and reporting the value $t = 1.426$ (or $t = 1.425$).
- A response that finds no value or an incorrect value of t_R is not eligible to earn the second point.
- A response need not demonstrate an evaluation of $v(t)$ at any value of t, $0 < t < 1.426$. The evaluations do not need to be presented with three correct decimal places, but they must be correct for the number of digits shown for one up to three digits after the decimal.

Total for part (a) 2 points

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(b) Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.

$a(1.5) = v'(1.5) = -1$ The acceleration of the particle at time $t = 1.5$ is -1 (or -0.999).	Acceleration with setup	**1 point**
$v(1.5) = -0.076856 < 0$ Because $a(1.5)$ and $v(1.5)$ have the same sign, the speed is increasing at time $t = 1.5$.	Answer with explanation	**1 point**

Scoring notes:

- A response must demonstrate the relationship $v' = a$ in order to earn the first point. This relationship could be shown by "$v'(1.5) = -1$ (or -0.999)" or "$v' = a$ and $a(1.5) = -1$ (or -0.999)." A response of just "$a(1.5) = -1$ (or -0.999)" is not sufficient to earn the first point.
- A response must declare a value for $a(1.5)$ to be eligible for the second point.
- The second point can only be earned for a response that is consistent with a negative velocity at time $t = 1.5$ and the presented value of $a(1.5)$.
- Any presented value of $v(1.5)$ must be correct for the number of digits presented, from one up to three decimal places in order to earn the second point.
- A response does not need to report a value for $v(1.5)$; an implied sign is sufficient:
  - Any statement equivalent to "The speed of the particle is increasing because $a(1.5)$ and $v(1.5)$ have the same sign" will earn the second point, provided the presented value of $a(1.5)$ is negative.
- A response that presents or references an incorrect value or sign of $v(1.5)$ does not earn the second point.
- Alternate solution for the second point:

  Speed $= |v(t)|$ and its derivative is positive when $t = 1.5$, therefore the speed of the particle is increasing.

Total for part (b) 2 points

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(c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.

$x(4) = x(1) + \int_1^4 v(t)\,dt$	Integral	**1 point**
	Uses initial condition	**1 point**
$= -3 + 0.197117 = -2.802883$ The position of the particle at time $t = 4$ is -2.803 (or -2.802).	Answer	**1 point**

Scoring notes:

- The first point is earned for a definite integral with integrand $v(t)$. If the limits of integration are incorrect, the response does not earn the third point.
- A response with a linkage error such as $x(4) = \int_1^4 v(t)\,dt = -3 + 0.197$ or $\int_1^4 v(t)\,dt = 0.197 = -2.803$ earns the first 2 points but does not earn the third point.
- Missing differential (dt):
  - Unambiguous responses of $x(1) + \int_1^4 v(t)$ and $-3 + \int_1^4 v(t)$ both earn the first 2 points and are eligible for the third point.
  - Ambiguous responses of $\int_1^4 v(t) + x(1)$ and $\int_1^4 v(t) - 3$ do not earn the first point, earn the second point, and earn the third point if the given numeric answer is correct. If there is no numeric answer given, neither of these responses earn the third point.
- The second point is earned for either symbolically or numerically adding $x(1)$ or -3 to a definite integral with a lower limit of 1.
- The third point is earned for an answer of $-3 + 0.197$ or $0.197 - 3$ with no additional simplification, provided there is some supporting work for these values.
- An answer of just -2.803 (or -2.802) with no supporting work does not earn any points.

Total for part (c) 3 points

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(d) Find the total distance traveled by the particle over the interval $1 \le t \le 4$. Show the setup for your calculations.

$\int_1^4 \lvert v(t) \rvert \, dt$	Integral	**1 point**
$= 0.9581$ The total distance traveled by the particle over the interval $1 \le t \le 4$ is 0.958.	Answer	**1 point**

Scoring notes:

- The first point is earned for any one of the following:
  - $\int_1^4 \lvert v(t) \rvert \, dt$
  - $\int_1^{1.425} v(t) \, dt - \int_{1.425}^{2.883} v(t) \, dt + \int_{2.883}^4 v(t) \, dt$
  - $\int_1^{1.426} v(t) \, dt - \int_{1.426}^{2.883} v(t) \, dt + \int_{2.883}^4 v(t) \, dt$
- Due to variations in numerical integration techniques on some calculators, responses of 0.958, 0.959, or 0.96 earn the second point.

Total for part (d) 2 points

Total for question 2 9 points

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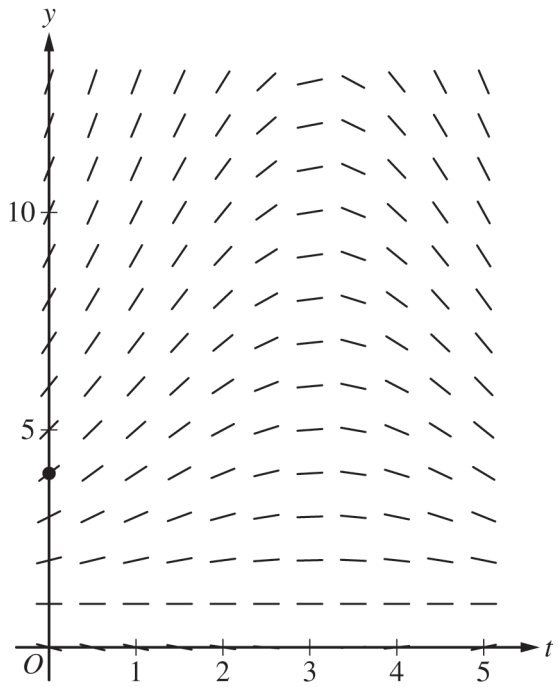
3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

NO CALCULATOR ALLOWED

3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

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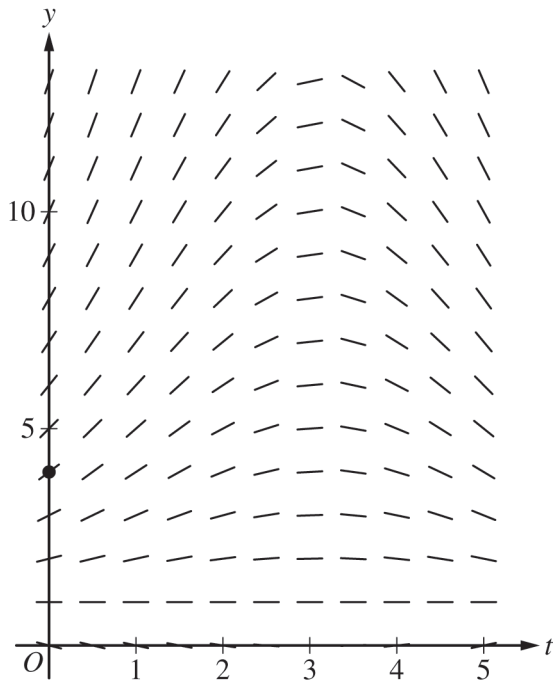

(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

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NO CALCULATOR ALLOWED

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

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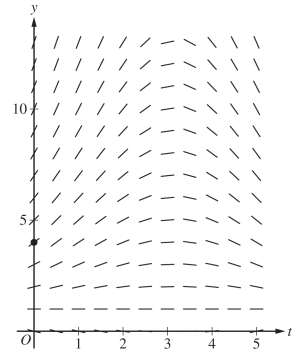
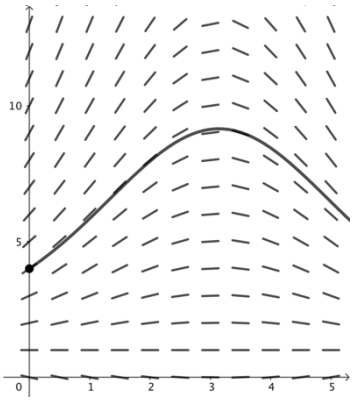


(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0,4)$.

(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

(a)



1: solution curve

(b) $\frac{dH}{dt} = 0 \rightarrow H - 1 = 0 \rightarrow H = 1$ but $H > 1$

and $\frac{1}{2}(H-1) > 0$ for $0 < t < 5$.

$\cos\left(\frac{t}{2}\right) = 0 \rightarrow \frac{t}{2} = \frac{\pi}{2}\cdots \rightarrow t = \pi, 3\pi, \cdots 0 < t = \pi < 5.$

$\cos\left(\frac{t}{2}\right)$ changes from positive to negative at $t = \pi$

so there is a relative maximum at $t = \pi$.

3:
- 1: Considers sign of $\frac{dH}{Dt}$
- 1: $t = \pi$
- 1: answer with justification

(c) $\frac{1}{H-1}dH = \frac{1}{2}\cos\left(\frac{t}{2}\right) \rightarrow \int \frac{1}{H-1}dH = \int \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$

$\ln(H-1) = \sin\left(\frac{t}{2}\right) + C$

$H(0) = 4 \rightarrow \ln 3 = \sin 0 + C \rightarrow C = \ln 3$

$\ln(H-1) = \sin\left(\frac{t}{2}\right) + \ln 3 \rightarrow H - 1 = e^{\sin\left(\frac{t}{2}\right)+\ln 3} = 3e^{\sin\left(\frac{t}{2}\right)}$

$H = 1 + 3e^{\sin\left(\frac{t}{2}\right)}$

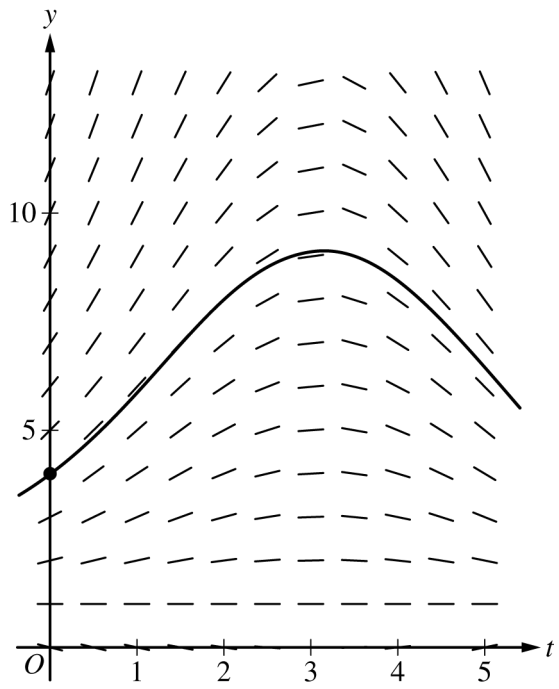
5:
- 1: Separation
- 1: One antiderivative.
- 1: Second antiderivative
- 1: Constant of integration uses initial condition
- 1: Solves for H

Part B (AB or BC): Graphing calculator not allowed
Question 3 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

	Model Solution	Scoring
(a)	A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.	
		Solution curve **1 point**

Scoring notes:

- The solution curve must pass through the point $(0, 4)$, extend to at least $t = 4.5$, and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.

Total for part (a) 1 point

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(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

Because $H(t) > 1$, then $\frac{dH}{dt} = 0$ implies $\cos\left(\frac{t}{2}\right) = 0$.	Considers sign of $\frac{dH}{dt}$	**1 point**
This implies that $t = \pi$ is a critical point.	Identifies $t = \pi$	**1 point**
For $0 < t < \pi$, $\frac{dH}{dt} > 0$ and for $\pi < t < 5$, $\frac{dH}{dt} < 0$. Therefore, $t = \pi$ is the location of a relative maximum value of H.	Answer with justification	**1 point**

Scoring notes:

- The first point is earned for considering $\frac{dH}{dt} = 0$, $\frac{dH}{dt} > 0$, $\frac{dH}{dt} < 0$, $\cos\left(\frac{t}{2}\right) = 0$, $\cos\left(\frac{t}{2}\right) > 0$, or $\cos\left(\frac{t}{2}\right) < 0$.
- The second point is earned for identifying $t = \pi$, with or without supporting work. A response may consider $H = 1$ or $t = 1$ as potential critical points without penalty.
- The third point cannot be earned without the first point. The third point is earned only for a correct justification and a correct answer of "relative maximum."
- The justification can be shown by determining the sign of $\frac{dH}{dt}$ (or $\cos\left(\frac{t}{2}\right)$) at a single value in $0 < t < \pi$ and at a single value in $\pi < t < 5$. It is not necessary to state that $\frac{dH}{dt}$ does not change sign on these intervals.
- The third point can also be earned by using the Second Derivative Test. For example:

  $$\frac{d^2H}{dt^2} = \frac{1}{2}(H-1)\left(-\frac{1}{2}\sin\left(\frac{t}{2}\right)\right) + \cos\left(\frac{t}{2}\right)\cdot\frac{1}{2}\cdot\frac{dH}{dt}$$

  $$\left.\frac{d^2H}{dt^2}\right|_{t=\pi} < 0$$

  Therefore, $t = \pi$ is the location of a relative maximum value of H.

Total for part (b) 3 points

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(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

$\frac{dH}{H-1} = \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$	Separation of variables	**1 point**
$\int \frac{dH}{H-1} = \int \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$ $\Rightarrow \ln\lvert H-1\rvert = \sin\left(\frac{t}{2}\right) + C$	One antiderivative Second antiderivative	**1 point** **1 point**
$\ln\lvert 4-1\rvert = \sin\left(\frac{0}{2}\right) + C \Rightarrow C = \ln 3$ Because $H(0) = 4$, $H > 1$, so $\lvert H-1\rvert = H-1$. $\ln(H-1) = \sin\left(\frac{t}{2}\right) + \ln 3$	Constant of integration and uses initial condition	**1 point**
$H-1 = e^{\sin(t/2)+\ln 3} = 3e^{\sin(t/2)}$ $H(t) = 1 + 3e^{\sin(t/2)}$	Solves for H	**1 point**

Scoring notes:

- A response with no separation of variables earns 0 out of 5 points.
- A response that presents $\int \frac{dH}{H-1} = \ln(H-1)$ without absolute value symbols earns that antiderivative point.
- A response with no constant of integration can earn at most the first 3 points.
- A response is eligible for the fourth point only if it has earned the first point and at least 1 of the 2 antiderivative points.
- An eligible response earns the fourth point by correctly including the constant of integration in an equation and substituting 0 for t and 4 for H.
- A response is eligible for the fifth point only if it has earned the first 4 points.
- A response earns the fifth point only for an answer of $H(t) = 1 + 3e^{\sin(t/2)}$ or a mathematically equivalent expression for $H(t)$ such as $H(t) = 1 + e^{\sin(t/2)+\ln 3}$.
- A response does not need to argue that $\lvert H-1\rvert = H-1$ in order to earn the fifth point.
- Special case: A response that presents an incorrect separation of variables of $\frac{1}{2}\cdot\frac{dH}{H-1} = \cos\left(\frac{t}{2}\right)dt$ does not earn the first point or the fifth point, but is eligible for the 2 antiderivative points. If the response earns at least 1 of the 2 antiderivative points, then the response is eligible for the fourth point.

Total for part (c) 5 points

Total for question 3 9 points

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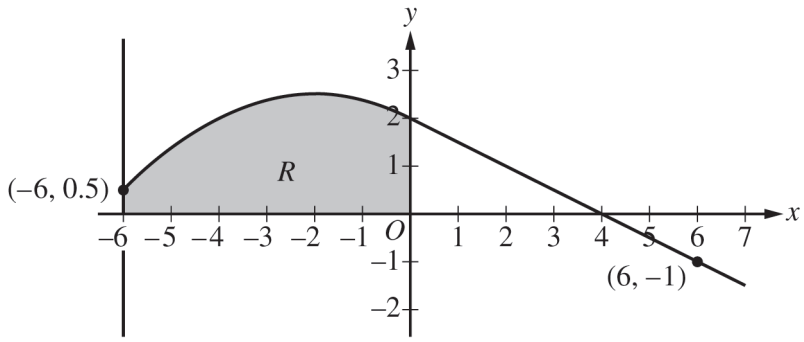


Graph of f

4. The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

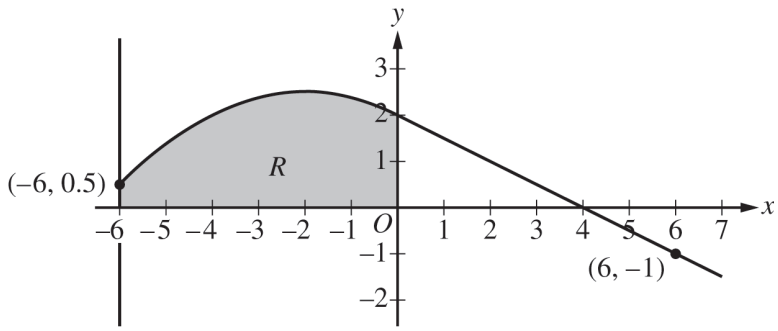
   (a) The function g is defined by $g(x) = \int_0^x f(t)\,dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.

   (b) For the function g defined in part (a), find all values of x in the interval $0 \le x \le 6$ at which the graph of g has a critical point. Give a reason for your answer.

   (c) The function h is defined by $h(x) = \int_{-6}^x f'(t)\,dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

NO CALCULATOR ALLOWED



Graph of f

4. The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

(a) The function g is defined by $g(x) = \int_0^x f(t)\,dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.

(b) For the function g defined in part (a), find all values of x in the interval $0 \le x \le 6$ at which the graph of g has a critical point. Give a reason for your answer.

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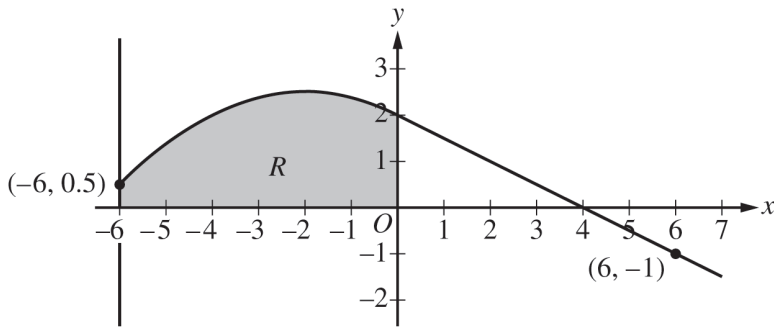
NO CALCULATOR ALLOWED



Graph of f

(c) The function h is defined by $h(x) = \int_{-6}^{x} f'(t)\,dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

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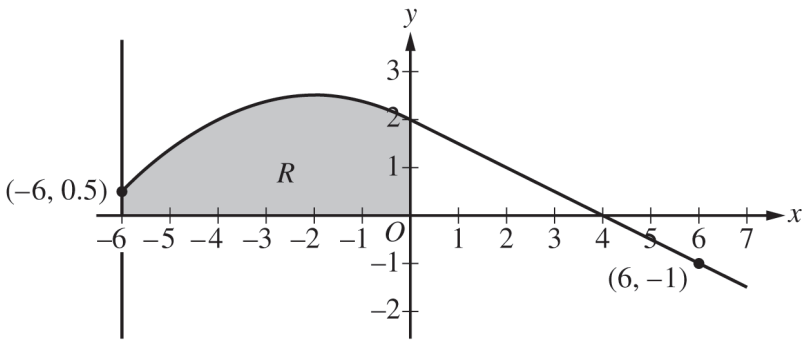


Graph of f

4. The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

   (a) The function g is defined by $g(x) = \int_0^x f(t)\,dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.

   (b) For the function g defined in part (a), find all values of x in the interval $0 \le x \le 6$ at which the graph of g has a critical point. Give a reason for your answer.

   (c) The function h is defined by $h(x) = \int_{-6}^x f'(t)\,dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

(a) $g(-6) = \int_0^{-6} f(t)dt = -\int_{-6}^0 f(t)dt = -12$

$g(4) = \int_0^4 f(t)dt = \frac{1}{2}(2)(4) = 4$

$g(6) = g(4) + \int_4^6 f(t)dt = 4 + \frac{1}{2}(-1)(2) = 3$

3: $\begin{cases} 1: g(-6) \\ 1: g(4) \\ 1: g(6) \end{cases}$

(b) $g'(x) = f(x) = 0 \rightarrow x = 4$

2: $\begin{cases} 1: \text{Uses Fundamental Theorem} \\ 1: \text{Answer with reason} \end{cases}$

(c) $h(6) = \int_{-6}^6 f'^{(t)}dt = f(6) - f(-6) = -1 - 0.5 = -1.5$

$h'(6) = f'(6) = -\frac{1}{2}$

$h''(6) = f''(6) = 0$ because f is linear on $(0,7)$ and $h' = f'$ is a constant.

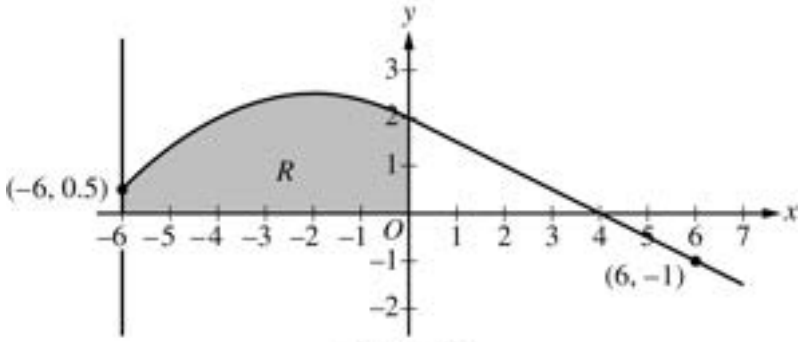
4: $\begin{cases} 1: \text{Uses Fundamental Theorem} \\ 1: h(6) \text{ with supporting work} \\ 1: h'(6) \\ 1: h''(6) \end{cases}$

Part B (AB or BC): Graphing calculator not allowed
Question 4 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Graph of f

The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

	Model Solution	Scoring	
(a)	The function g is defined by $g(x) = \int_0^x f(t)\,dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.		
	$g(-6) = \int_0^{-6} f(t)\,dt = -\int_{-6}^0 f(t)\,dt = -12$	$g(-6)$	**1 point**
	$g(4) = \int_0^4 f(t)\,dt = \frac{1}{2}\cdot 4\cdot 2 = 4$	$g(4)$	**1 point**
	$g(6) = \int_0^6 f(t)\,dt = \frac{1}{2}\cdot 4\cdot 2 - \frac{1}{2}\cdot 2\cdot 1 = 3$	$g(6)$	**1 point**

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Scoring notes:

- Supporting work is not required for any of these values. However, any supporting work that is shown must be correct to earn the corresponding point.
- Special case: A response that explicitly presents $g(x) = \int_{-6}^{x} f(t)\,dt$ does not earn the first point it would have otherwise earned. The response is eligible for all subsequent points for correct answers, or for consistent answers with supporting work.
  - Note: $\int_{-6}^{-6} f(t)\,dt = 0,\ \int_{-6}^{4} f(t)\,dt = 16,\ \int_{-6}^{6} f(t)\,dt = 15$
- Labeled values may be presented in any order. Unlabeled values are read from left to right and from top to bottom as $g(-6),\ g(4),$ and $g(6),$ respectively. A response that presents only 1 or 2 values must label them to earn any points.

Total for part (a) 3 points

(b) For the function g defined in part (a), find all values of x in the interval $0 \le x \le 6$ at which the graph of g has a critical point. Give a reason for your answer.

$g'(x) = f(x)$	Fundamental Theorem of Calculus **1 point**
$g'(x) = f(x) = 0 \Rightarrow x = 4$ Therefore, the graph of g has a critical point at $x = 4$.	Answer with reason **1 point**

Scoring notes:

- The first point is earned for explicitly making the connection $g' = f$ in this part.
  - A response that writes $g'' = f'$ earns the first point but can only earn the second point by reasoning from $f = 0$.
- A response that does not earn the first point is eligible to earn the second point with an implied application of the FTC (e.g., "Because $g'(4) = 0,\ x = 4$ is a critical point").
- A response that reports any additional critical points in $0 < x < 6$ does not earn the second point.
  - Any presented critical point outside the interval $0 < x < 6$ will not affect scoring.

Total for part (b) 2 points

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(c) The function h is defined by $h(x) = \int_{-6}^{x} f'(t)\,dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

$h(6) = \int_{-6}^{6} f'(t)\,dt = f(6) - f(-6) = -1 - 0.5 = -1.5$	Uses Fundamental Theorem of Calculus	**1 point**
	$h(6)$ with supporting work	**1 point**
$h'(x) = f'(x)$, so $h'(6) = f'(6) = -\frac{1}{2}$.	$h'(6)$	**1 point**
$h''(x) = f''(x)$, so $h''(6) = f''(6) = 0$.	$h''(6)$	**1 point**

Scoring notes:

- Labeled values may be presented in any order.
- Unlabeled values are read from left to right and from top to bottom as $h(6)$, $h'(6)$, and $h''(6)$, respectively. A response that presents only 1 or 2 values must label them in order to earn any points.
- A response of $h(6) = -1.5$ does not earn either of the first 2 points. A response of $h(6) = f(6) - f(-6)$ earns the first point but not yet the second point.
- A response of $h(6) = -1 - 0.5$ is the minimum work required to earn both of the first 2 points.
- To earn the third point a response must state either $h'(x) = f'(x)$ or $h'(6) = f'(6)$, and provide an answer of $-\frac{1}{2}$.
- The fourth point is earned for a response of $h''(6) = 0$, with or without supporting work.
- A response that has one or more linkage errors does not earn the first point it would have otherwise earned. For example, $h'(x) = f'(6) = -\frac{1}{2}$ does not earn the third point but is eligible for the fourth point even in the presence of another linkage error, such as $h''(x) = f''(6) = 0$.

Total for part (c) 4 points

Total for question 4 9 points

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5. Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

   (a) There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.

   (b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

   (c) The curve intersects the positive x-axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

   (d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

NO CALCULATOR ALLOWED

5. Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

(a) There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.

(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

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NO CALCULATOR ALLOWED

(c) The curve intersects the positive x-axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

(d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

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5. Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

(a) There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.

(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

(c) The curve intersects the positive x-axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

(d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

(a) $\left.\frac{dy}{dx}\right|_{(2,4)} = \frac{-2 \cdot 2}{3 + 4 \cdot 4} = -\frac{4}{19}$ tangent line: $y = 4 - \frac{4}{19}(x - 2)$

$x = 3 \rightarrow y = 4 - \frac{4}{19}(3 - 2) = \frac{72}{19}$

2: {1: slope of tangent line; 1: approximation}

(b) $\frac{dy}{dx} = 0 \rightarrow -2x = 0 \rightarrow x = 0$

$x = 0, y = 1 \quad 0^2 + 3 \cdot 1 + 2 \cdot 1^2 = 5 \neq 48$

The tangent line is not horizontal because the point $(0,1)$ is not on the curve.

2: {1: sets $\frac{dy}{dx} = 0$; 1: answer with reason}

(c) At the point $\left(\sqrt{48}, 0\right) \rightarrow 3 + 4 \cdot 0 = 3 \neq 0$

$\frac{dy}{dx}$ is defined at $\left(\sqrt{48}, 0\right)$ so the tangent line is not vertical.

1: Answer with reason

(d) $3y^2 \frac{dy}{dt} + 2\left(y\frac{dx}{dt} + x\frac{dy}{dt}\right) = 0$

At $(4,2), \frac{dy}{dt} = -2 \rightarrow 3 \cdot 2^2 \cdot (-2) + 2\left(2 \cdot \frac{dx}{dt} + 4 \cdot (-2)\right) = 0$

$\rightarrow -24 + 2\left(2 \cdot \frac{dx}{dt} - 8\right) = 0 \rightarrow 2 \cdot \frac{dx}{dt} - 8 = \frac{24}{2} = 12$

$\frac{dx}{dt} = \frac{20}{2} = 10$ units per second.

4: {1: Attempts implicit differentiation; 1: $3y^2 \frac{dy}{dt} + 2x\frac{dy}{dt} + 2y\frac{dx}{dt} = 0$; 1: uses $\frac{dy}{dt} = \pm 2$; 1: Answer}

Part B (AB): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

	Model Solution	Scoring
(a)	There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.	
	$\left.\frac{dy}{dx}\right\|_{(x,\,y)=(2,\,4)} = \frac{-2(2)}{3 + 4(4)} = -\frac{4}{19}$	Slope of tangent line **1 point**
	$y \approx 4 - \frac{4}{19}(3 - 2) = \frac{72}{19}$	Approximation **1 point**

Scoring notes:

- A response earns the first point for finding $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,4)} = -\frac{4}{19}$, even if this is not labeled or used as the slope of a tangent line.
- A response that does not explicitly find the value of $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,4)}$ but uses a slope of $-\frac{4}{19}$ in a linear approximation also earns the first point.
- A response that declares $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,4)}$ equal to any nonzero value other than $-\frac{4}{19}$ does not earn the first point but is eligible for the second point for a linear approximation at $x = 3$ through the point $(2, 4)$ with a slope of the declared value.
  - The second point cannot be earned with a linear approximation using a slope other than $-\frac{4}{19}$ if that slope has not been declared to be the value of $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,4)}$.
- The second point cannot be earned for an approximation at any value of x other than 3.
- A response does not have to write the tangent line equation, but must clearly demonstrate its use at $x = 3$ in finding the requested approximation in order to earn both points.
- The minimal work required to earn both points is $4 - \frac{4}{19}(3 - 2)$.

Total for part (a) 2 points

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(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

$\frac{dy}{dx} = \frac{-2x}{3+4y} = 0 \Rightarrow x = 0$	Considers $\frac{dy}{dx} = 0$	**1 point**
And so, if the horizontal line $y = 1$ is tangent to the curve, the point of tangency must be $(0, 1)$. However, the point $(0, 1)$ is not on the curve, because $0^2 + 3 \cdot 1 + 2 \cdot 1^2 = 5 \neq 48$. Therefore, the horizontal line $y = 1$ is not tangent to the curve.	Answer with reason	**1 point**

Scoring notes:

- The first point can be earned with a response of $\frac{dy}{dx} = 0,\ -2x = 0,$ or $x = 0,$ OR by identifying the point $(0, 1)$.
- To earn the second point a response must provide a reason that the line $y = 1$ is not tangent to the curve. Merely stating "$(0, 1)$ does not lie on the curve" is not sufficient to earn the second point.
- Alternate solution:

  If $y = 1,$ then $x^2 + 3 \cdot 1 + 2 \cdot 1^2 = 48 \Rightarrow x = \pm\sqrt{43}$.

  $\left.\frac{dy}{dx}\right|_{(x,\,y)=(\pm\sqrt{43},\,1)} = \frac{\pm 2\sqrt{43}}{7} \neq 0$

  Therefore, the horizontal line $y = 1$ is not tangent to the curve.

  - A response that uses this method earns the first point by using $y = 1$ in $x^2 + 3y + 2y^2 = 48$.
  - A response that fails to consider both $x = +\sqrt{43}$ and $x = -\sqrt{43}$ does not earn the second point.

Total for part (b) 2 points

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(c) The curve intersects the positive x-axis at the point $\left(\sqrt{48}, 0\right)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

At the point $\left(\sqrt{48}, 0\right)$, the slope of the line tangent to the curve is $\frac{dy}{dx} = \frac{-2\sqrt{48}}{3 + 4(0)}$. The denominator of $\frac{dy}{dx}$ is $3 + 4(0)$, which does not equal 0. Therefore, the line tangent to the curve at this point is not vertical.	Answer with reason	**1 point**

Scoring notes:

- A response does not need to consider the numerator of $\left.\frac{dy}{dx}\right|_{(x, y)=(\sqrt{48}, 0)}$ in order to earn this point; considering the denominator is sufficient.
- To earn this point a response must clearly demonstrate that the slope of the tangent line at the point $\left(\sqrt{48}, 0\right)$ is defined and answer "no."
  - Such demonstrations include, but are not limited to, the following:
    - $3 + 4(0) \neq 0$
    - At $\left(\sqrt{48}, 0\right)$, $\frac{dy}{dx} = \frac{-2\sqrt{48}}{3}$.
    - $\frac{-2\sqrt{48}}{3}$
    - When $3 + 4y = 0$, $y \neq 0$.

Total for part (c) 1 point

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(d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

$3y^2 \frac{dy}{dt} + 2x\frac{dy}{dt} + 2y\frac{dx}{dt} = 0$	Attempts implicit differentiation	**1 point**
	$3y^2 \frac{dy}{dt} + 2x\frac{dy}{dt} + 2y\frac{dx}{dt} = 0$	**1 point**
$\frac{dy}{dt} = -2$	Uses $\frac{dy}{dt} = \pm 2$	**1 point**
$3(2)^2(-2) + 2(4)(-2) + 2(2)\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = \frac{40}{4} = 10$	Answer	**1 point**
The rate of change with respect to time in the x-coordinate is 10 units per second.		

Scoring notes:

- The first point is earned for implicitly differentiating $y^3 + 2xy = 24$ with respect to t with at most one error.
  - The first point can also be earned by correctly differentiating with respect to x: $3y^2 \frac{dy}{dx} + 2x\frac{dy}{dx} + 2y = 0.$
- The second point is earned for an equation equivalent to $3y^2 \frac{dy}{dt} + 2x\frac{dy}{dt} + 2y\frac{dx}{dt} = 0.$
- A response does not need to explicitly declare $\frac{dy}{dt} = -2$ or $\frac{dy}{dt} = 2$ in order to earn the third point; this point may be earned by correctly substituting $\frac{dy}{dt} = -2$ or $\frac{dy}{dt} = 2$ in the implicitly differentiated equation. However, a response that uses both $\frac{dy}{dt} = -2$ and $\frac{dy}{dt} = 2$ in the implicitly differentiated equation does not earn the third point.
- The fourth point cannot be earned without the first 3 points. The fourth point is earned only for the value of 10 with no mistakes in supporting work.
  - Note that a response that uses $\frac{dy}{dt} = 2$ and then mishandles subtracting 40 from both sides of the equation, e.g., $3(2)^2(2) + 2(4)(2) + 2(2)\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = \frac{40}{4}$, does not earn the fourth point.

Total for part (d) 4 points

Total for question 5 9 points

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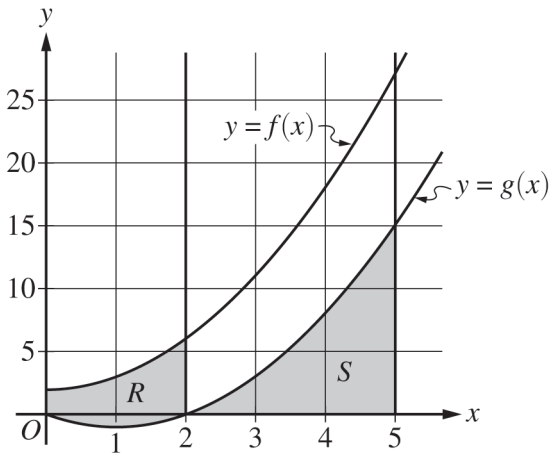


6. The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

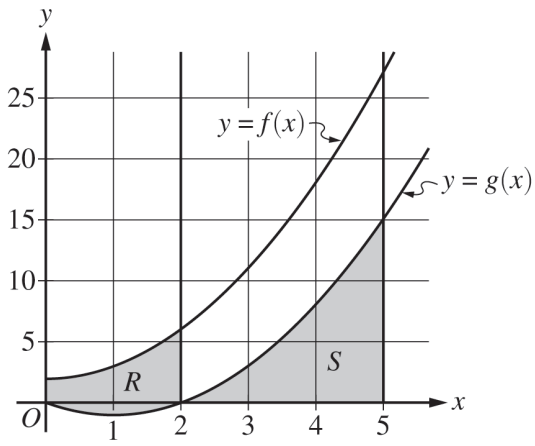
   (a) Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.

   (b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

NO CALCULATOR ALLOWED



6. The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

(a) Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.

(b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

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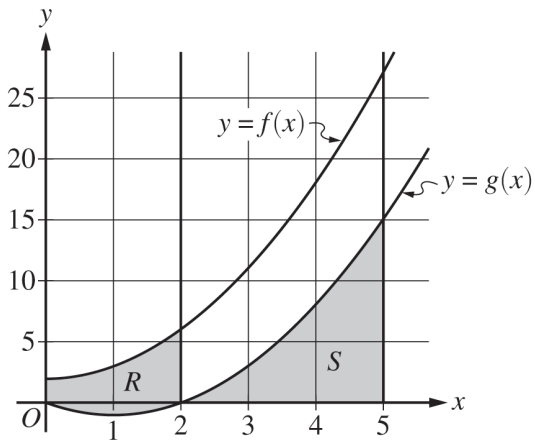

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

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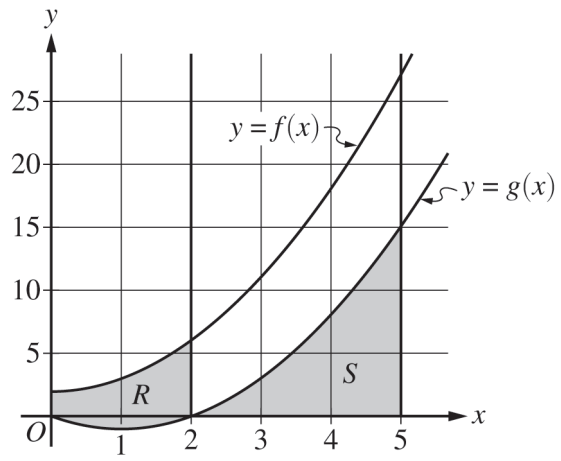


6. The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

   (a) Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.

   (b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

(a) Area of the region $R = \displaystyle\int_0^2 [f(x) - g(x)]\,dx$

2: {1: integrand; 1: expression}

(b) Volume $= \displaystyle\int_2^5 \left[\frac{1}{2}g(x) \cdot g(x)\right] dx = \frac{1}{2}\int_2^5 [x^4 - 4x^3 + 4x^2]\,dx$

$$= \frac{1}{2}\left(\frac{1}{5}x^5 - x^4 + \frac{4}{3}x^3\right)\Big|_2^5$$

$$= \frac{1}{2}\left[\left(\frac{1}{5}5^5 - 5^4 + \frac{4}{3}5^3\right) - \left(\frac{1}{5}2^5 - 2^4 + \frac{4}{3}2^3\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{4}{3}5^3\right) - \left(\frac{32}{5} - 16 + \frac{32}{3}\right)\right] = \frac{1}{2}\left[(156) - \left(-\frac{48}{5}\right)\right] = \frac{414}{5}$$

4: {1: integrand; 1: limits; 1: antiderivative; 1: answer}

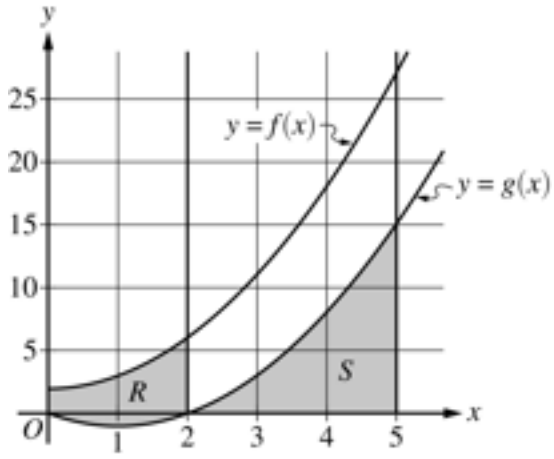
(c) Volume $= \pi \displaystyle\int_2^5 \left[20^2 - \left(20 - g(x)\right)^2\right] dx$

3: {1: form of integrand; 1: integrand; 1: Limits and constant}

Part B (AB): Graphing calculator not allowed
Question 6 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

	Model Solution	Scoring	
(a)	Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.		
	Area $= \int_0^2 (f(x) - g(x))\,dx$	Integrand	**1 point**
		Answer	**1 point**

Scoring notes:

- The first point is earned for a response that presents an integrand of $f(x) - g(x)$, $|f(x) - g(x)|$, $g(x) - f(x)$, or $|g(x) - f(x)|$ in one or more definite integrals.
- The first point could also be earned for a difference of definite integrals with integrands $f(x)$ and $g(x)$.
- The second point is earned only for one or more integrals equivalent to $\int_0^2 (f(x) - g(x))\,dx$, such as $\int_0^2 f(x)\,dx - \int_0^2 g(x)\,dx$, $\int_0^2 |f(x) - g(x)|\,dx$, $-\int_0^2 (g(x) - f(x))\,dx$, or $\int_0^2 |g(x) - f(x)|\,dx$.
  - Note: $\int_0^2 f(x)\,dx + \left|\int_0^2 g(x)\,dx\right|$ would earn both points.

Total for part (a) 2 points

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(b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

$\text{Volume} = \int_2^5 \frac{1}{2}(g(x))^2\,dx = \int_2^5 \frac{1}{2}\left(x^2 - 2x\right)^2\,dx$	Integrand	**1 point**
	Limits	**1 point**
$= \frac{1}{2}\int_2^5 \left(x^4 - 4x^3 + 4x^2\right)\,dx$ $= \frac{1}{2}\left[\frac{x^5}{5} - x^4 + \frac{4x^3}{3}\right]_2^5$	Antiderivative	**1 point**
$= \frac{1}{2}\left[\left(\frac{5^5}{5} - 5^4 + \frac{500}{3}\right) - \left(\frac{32}{5} - 16 + \frac{32}{3}\right)\right]$ $= \frac{1}{2}\left(\frac{500}{3} - \frac{16}{15}\right) = \frac{414}{5}$	Answer	**1 point**

Scoring notes:

- The first point is earned only by a response with an integrand of the form $k(g(x))^2$ in a definite integral, where k is any nonzero constant.
- The second point is earned for limits of $x = 2$ and $x = 5$ in a definite integral with an integrand of the form $a(x) \cdot g(x)$ for any nonzero factor $a(x)$.
- To earn the third point a response must provide a correct antiderivative of $k\left(x^2 - 2x\right)^n$ for some integer $n \geq 2$.
- The fourth point is earned only for a numeric answer equivalent to $\frac{1}{2}\left(\frac{500}{3} - \frac{16}{15}\right)$.
- Special case: A response of $\text{Volume} = \int_2^5 \frac{1}{2}(f(x))^2\,dx$ or $\text{Volume} = \int_2^5 \frac{1}{2}\left(x^2 + 2\right)^2\,dx$ earns the first 2 points and is not eligible for the last 2 points.

Total for part (b) 4 points

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(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

$\text{Volume} = \pi\int_2^5\left[\left(20^2\right) - (20 - g(x))^2\right] dx$ $= \pi\int_2^5\left[400 - (20 - g(x))^2\right] dx$ $= \pi\int_2^5\left[400 - \left(20 - \left(x^2 - 2x\right)\right)^2\right] dx$	Form of integrand	**1 point**
	Integrand	**1 point**
	Limits and constant	**1 point**

Scoring notes:

- The first point is earned for a response that presents an integrand of $20^2 - (20 - g(x))^2$, $20^2 - (g(x) - 20)^2$, $(20 - g(x))^2 - 20^2$, $(g(x) - 20)^2 - 20^2$, or any mathematically equivalent expression, in one or more definite integrals.
- The second point is earned only for an integrand mathematically equivalent to $20^2 - (20 - g(x))^2$ or $20^2 - (g(x) - 20)^2$ in a definite integral. Note that $\int_a^b\left|(20 - g(x))^2 - 400\right| dx$ or $\left|\int_a^b\left((20 - g(x))^2 - 400\right) dx\right|$ earns the first 2 points.
- The integral may be split into two integrals.
  - For example, $\text{Volume} = \pi\int_2^5\left(20^2\right) dx - \pi\int_2^5(20 - g(x))^2\, dx$ or $\text{Volume} = \pi \cdot 20^2 \cdot 3 - \pi\int_2^5(20 - g(x))^2\, dx$.
- A response that presents an allowable integrand involving $g(x)$, but continues and makes an error in using the expression for $g(x)$, does not earn the second point.
  - For example, $\pi\int_2^5\left[20^2 - (20 - g(x))^2\right] dx = \pi\int_2^5\left[20^2 - \left(20 - x^2 - 2x\right)^2\right] dx$ earns the first point but does not earn the second point.
- To be eligible for the third point a response must have earned at least 1 of the first 2 points or must have presented an integrand involving $g(x)$ of the form $R^2 - r^2$ in a definite integral.
- The third point is earned only by a definite integral including the constant π and limits $x = 2$ to $x = 5$. A response that presents any other constant or limits (including $x = 5$ to $x = 2$, except in the note below) does not earn the third point.
  - Note: $\pi\int_5^2\left[(20 - g(x))^2 - 400\right] dx$ would earn all three points.
- Special case: A response of $\pi\int_2^5\left[400 - (20 - f(x))^2\right] dx$ or equivalent earns 2 of the 3 points.

Total for part (c) 3 points

Total for question 6 9 points

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1. An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

A. Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

B. Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \le t \le 4$. Show the setup for your calculations.

C. Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

D. At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

Part A (AB or BC): Graphing calculator required
Question 1 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

	Model Solution	Scoring	
A	Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.		
	$\frac{1}{4-0}\int_0^4 C(t)\,dt$	Average value formula	**Point 1 (P1)**
	$= \frac{1}{4}(11.112896) = 2.778224$ From time $t = 0$ to $t = 4$ weeks, the average number of acres affected by the invasive species was 2.778 acres.	Answer	**Point 2 (P2)**

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Scoring Notes for Part A

- **P1** is earned for the correct integral, with or without the differential, along with evidence of division by 4. In the presence of the correct integral, the correct answer will suffice as evidence of division by 4. These may appear all in one step, as in the model solution, or in multiple steps.
- **P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect or unclear communication between the correct integral and the correct answer is treated as scratch work and is not considered in scoring. For example:
  - $\int_0^4 C(t)\,dt = 11.112896$ so the average velocity is 2.778224.

    Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer.
  - $\int_0^4 C(t)\,dt = \frac{11.112896}{4} = 2.778224$

    Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
  - $\int_0^4 C(t)\,dt = 2.778224$

    Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- Note that the values $\frac{1}{4}(11.112)$ and $\frac{1}{4}(11.113)$ are accurate to three digits after the decimal and therefore earn **P2**.

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B Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \le t \le 4$. Show the setup for your calculations.

$\frac{C(4)-C(0)}{4-0} = 1.282008$	Uses average rate of change	**Point 3 (P3)**
$C'(t) = \frac{38}{25+t^2} = 1.282008 \Rightarrow t = 2.154298$ The instantaneous rate of change of C equals the average rate of change of C over the interval $0 \le t \le 4$ at time $t = 2.154$.	Answer with supporting work	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** may be earned by presenting the expression or value for the average rate of change. Note that because $C(0) = 0$ and the interval is $0 \le t \le 4$, any of the following will earn **P3**: $\frac{\int_0^4 C'(t)\,dt}{4}$, $\frac{C(4)-C(0)}{4-0}$, $\frac{C(4)}{4}$, $\frac{5.128-0}{4-0}$, $\frac{5.128}{4}$, or 1.282. However, neither **P3** nor **P4** is earned by just presenting $t = 1.282$.
- **P4** is earned for the correct answer supported by the appropriate equation. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
  - The following response, for example, earns both **P3** and **P4**: $C'(t) = \frac{C(4)-C(0)}{4}$ when $t = 2.154$.

C Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

$\lim_{t\to\infty} C'(t) = \lim_{t\to\infty} \frac{38}{25+t^2}$	Limit expression	**Point 5 (P5)**
$= 0$	Value	**Point 6 (P6)**

Scoring Notes for Part C

- **P5** can be earned for either $\lim_{t\to\infty} C'(t)$ or $\lim_{t\to\infty} C(t)$.
- A response that includes $\lim_{t\to\infty} C(t)$ is not eligible to earn **P6**.
- For **P6**, arithmetic with infinity, e.g., $\frac{38}{25+\infty^2} = 0$, will be considered as scratch work and will not be considered in scoring.

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AP Calculus AB 2025 Free-Response Questions Scoring Guides

1. An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

A. Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

B. Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \le t \le 4$. Show the setup for your calculations.

C. Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

D. At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

(a) $\dfrac{1}{4-0}\displaystyle\int_0^4 C(t)dt = 2.778$ is the average number of acres affected from $t = 0$ to $t = 4$ weeks.

2: {P1: Average value formula; P2: Answer}

(b) Average rate of change $Q = \dfrac{C(4) - C(0)}{4 - 0} = 1.282$

$C'(t) = Q \rightarrow t = 2.154$

2: {P3: Uses average rate of change; P4: Answer with supporting work}

(c) $\lim\limits_{t \to \infty} \dfrac{38}{25 + t^2} = 0$ because the limit becomes the form $\dfrac{\text{constant}}{\infty}$

2: {P5: Limit expression; P6: Value}

(d) $A'(t) = C'(t) - 0.1 \ln(t) = 0 \rightarrow t = 11.441 \ldots = p$

t	$A(t)$
4	5.128...
p	7.316 ...
36	1.743 ...

$A(t)$ attains its maximum value at $t = p = 11.441 \ldots$

3: {P7: Considers $A'(t) = 0$; P8: Justification; P9: Answer with supporting work}

D At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\, dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

$A'(t) = C'(t) - 0.1 \cdot \ln t$ For $4 \le t \le 36$, the maximum value of $A(t)$ occurs when $A'(t) = 0$ or at an endpoint. $A'(t) = C'(t) - 0.1 \cdot \ln t = 0 \Rightarrow C'(t) = 0.1 \cdot \ln t$	Considers $A'(t) = 0$	**Point 7 (P7)**
$\Rightarrow t = 11.441700$ t | $A(t)$ 4 | 5.128031 11.441700 | 7.316978 36 | 1.743056	Justification	**Point 8 (P8)**
Therefore, the number of acres affected by the species is a maximum at time $t = 11.442$ (or 11.441) weeks.	Answer with supporting work	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for considering $A'(t) = 0$, $C'(t) - 0.1 \cdot \ln t = 0$, or $C'(t) = 0.1 \cdot \ln t$. **P7** is not earned by just presenting $t = 11.441700$.
  - A response that discusses the sign of $A'(t)$ changing or uses the phrase "critical points of A" also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by correctly evaluating $A(t)$ at $t = 4$, $t = 11.441700$, and $t = 36$. The evaluations must be correct to the first digit after the decimal (rounded or truncated).
- Alternate justifications:
  - $A'(t) > 0$ for $4 < t < 11.442$, and $A'(t) < 0$ for $11.442 < t < 36$. Therefore, $t = 11.442$ is the location of the absolute maximum for A on the interval $4 \le t \le 36$.
  - Because $A'(t)$ changes sign from positive to negative at $t = 11.442$ (this might be presented as "$A'(t) > 0$ for $t < 11.442$, and $A'(t) < 0$ for $t > 11.442$"), it is the location of a relative maximum for A. And because $t = 11.442$ is the only critical point of A in the interval $4 \le t \le 36$, it is the location of the absolute maximum for A on the interval.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

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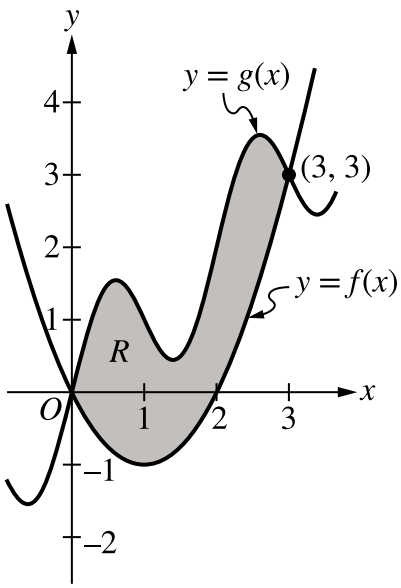
2. The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.



(Note: Your calculator should be in radian mode.)

A. Find the area of R. Show the setup for your calculations.

B. Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.

C. Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.

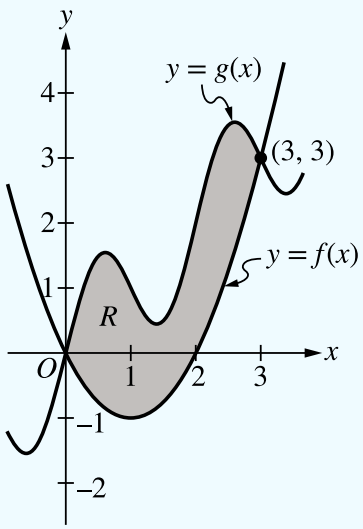
D. It can be shown that $g'(x) = 1 + \pi\cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.

Part A (AB): Graphing calculator required
Question 2 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.



(Note: Your calculator should be in radian mode.)

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	Model Solution	Scoring	
A	Find the area of R. Show the setup for your calculations.		
	$\int_0^3 (g(x) - f(x))\, dx$	Form of integrand	**Point 1 (P1)**
	$= 5.136620$ The area is 5.137 (or 5.136).	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned for a response that presents an integrand of $g(x) - f(x)$, $|g(x) - f(x)|$, $f(x) - g(x)$, or $|f(x) - g(x)|$ in a definite integral, with or without the differential dx.
- **P1** could also be earned for a difference of definite integrals with integrands $g(x)$ and $f(x)$.
- **P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect communication between the integral and the correct answer is treated as scratch work and is not considered in scoring.
  - $\int_0^3 (f(x) - g(x))\, dx = -5.137$ so the area is 5.137.

    Note: This response earns **P1** for the integral. It also earns **P2** for the correct answer.
  - $\int_0^3 (f(x) - g(x))\, dx = 5.137$

    Note: This response earns **P1** for the integral. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- The exact answer is $\frac{4 + 9\pi}{2\pi}$.

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B Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.

$\int_0^3 x(g(x) - f(x))\,dx$	Form of integrand	**Point 3 (P3)**
$= 7.704930$ The volume of the solid is 7.705 (or 7.704).	Answer	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned for a definite integral with an integrand presented as a product of two nonconstant factors, with one of the factors equal to x, $g(x) - f(x)$, or $f(x) - g(x)$.
- The presence or absence of the differential dx will not be considered in scoring **P3** or **P4**.
- **P4** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
- Incorrect or unclear communication between the integral and the correct answer is treated as scratch work and is not considered in scoring. For example:
  - $\int_0^3 x(f(x) - g(x))\,dx = -7.705$ so the volume is 7.705.

    Note: This response earns **P3** for the integral. It also earns **P4** for the correct answer.
  - $\int_0^3 x(f(x) - g(x))\,dx = 7.705$

    Note: This response earns **P3** for the integral. It also earns **P4** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- The exact answer is $\frac{12 + 27\pi}{4\pi}$.

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C Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.

Volume $= \pi\int_0^3 \left((g(x) - (-2))^2 - (f(x) - (-2))^2 \right) dx$	Form of integrand	**Point 5 (P5)**
	Integrand	**Point 6 (P6)**
	Limits, constant, and differential	**Point 7 (P7)**

Scoring Notes for Part C

- **P5** is earned for a definite integral with an integrand of $R^2 - r^2$ or $\left|R^2 - r^2\right|$, where one of $\{R, r\}$ is correct or a difference between g and a nonzero constant, and the other is correct or a difference between f and a nonzero constant.
- **P6** is earned for the integral $\int_0^3 \left((g(x) + 2)^2 - (f(x) + 2)^2 \right) dx$, $\int_0^3 \left((f(x) + 2)^2 - (g(x) + 2)^2 \right) dx$, or a mathematically equivalent expression.
- Note **P5** and **P6** could be earned for a difference of definite integrals.
- A response that presents an integral expression that does not include the constant π is eligible for **P5** and **P6** but does not earn **P7**.
- To be eligible for **P7**, a response must have earned **P5**.
- A response that reverses the difference of squares must resolve the reversal with either the constant or the limits of integration AND include the differential to earn **P7**. For example:
  - A response of $-\pi\int_0^3 \left((f(x) + 2)^2 - (g(x) + 2)^2 \right) dx$ or $\pi\int_3^0 \left((f(x) + 2)^2 - (g(x) + 2)^2 \right) dx$ earns **P5**, **P6**, and **P7**.
  - A response of $\pi\int_0^3 \left((f(x) + 2)^2 - (g(x) + 2)^2 \right) dx$ earns **P5** and **P6** but does not earn **P7**.
- Note: A response of only $\pi\int_0^3 \left((g(x) - (-2))^2 - (f(x) - (-2))^2 \right) dx$ earns **P5**, **P6**, and **P7**.

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D It can be shown that $g'(x) = 1 + \pi\cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.

$f'(x) = g'(x) \Rightarrow 2x - 2 = 1 + \pi\cos(\pi x)$	$f'(x) = g'(x)$	**Point 8 (P8)**
$\Rightarrow x = 0.675819$ The lines tangent to the graphs of f and g are parallel at $x = 0.676$ (or 0.675).	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P8** is earned for a general statement, such as $f'(x) = g'(x)$, or for any correct equation formed by substituting $2x - 2$ for $f'(x)$, $1 + \pi\cos(\pi x)$ for $g'(x)$, or both.
- **P9** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

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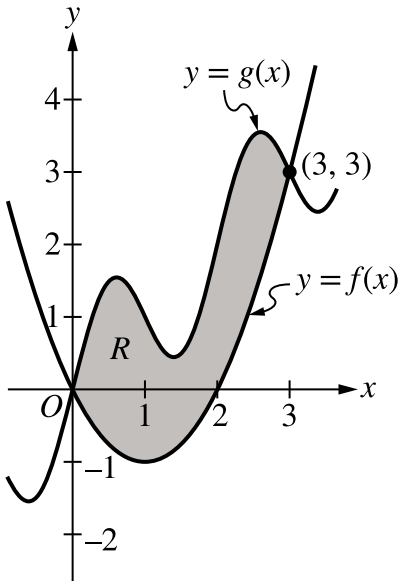
2. The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.



(Note: Your calculator should be in radian mode.)

A. Find the area of R. Show the setup for your calculations.

B. Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.

C. Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.

D. It can be shown that $g'(x) = 1 + \pi\cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.

(a) area of $R = \int_0^3 [g(x) - f(x)]dx = 5.136$ (or 5.137)	2: {P1: Form of integrand; P2: Answer}
(b) volume $= \int_0^3 x[g(x) - f(x)]dx = 7.704$ (or 7.705)	2: {P3: Form of integrand; P4: Answer}
(c) volume $= \pi\int_0^3 [(g(x) + 2)^2 - (f(x) + 2)^2]dx$	3: {P5: form of integrand; P6: integrand; P7: Limits, constant, and differential}
(d) $f'(x) = g'(x) \rightarrow x = 0.675$ (or 0.676)	2: {P8: $f'(x) = g'(x)$; P9: Answer}

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

Part B (AB or BC): Graphing calculator not allowed
Question 3 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

	Model Solution	Scoring	
A	Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.		
	$R'(1) \approx \dfrac{R(2) - R(0)}{2 - 0}$ $= \dfrac{100 - 90}{2} = \dfrac{10}{2} = 5$ words per minute per minute	Answer with setup Units	**Point 1 (P1)** **Point 2 (P2)**

Scoring Notes for Part A

- To earn **P1**, a response must present the answer along with the supporting work of a difference and a quotient using values from the table.
  - $\dfrac{100 - 90}{2 - 0}$, $\dfrac{10}{2 - 0}$, $\dfrac{100 - 90}{2}$, or $\dfrac{R(2) - R(0)}{2 - 0} = 5$ is sufficient to earn **P1**.
  - $\dfrac{R(2) - R(0)}{2 - 0}$ by itself is not sufficient to earn **P1**.
- **P2** is earned for correct units, whether or not they are attached to a numerical value for the average rate of change.
- **P2** is also earned for the units "words/minute^2."

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B Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

R is differentiable implies R is continuous.	Differentiable implies continuous	**Point 3 (P3)**
$R(0) = 90 < 155 < R(10) = 162$ Therefore, by the Intermediate Value Theorem, there must be a value c, with $0 < c < 10$, such that $R(c) = 155$.	Answer with justification	**Point 4 (P4)**

Scoring Notes for Part B

- To earn **P3**, a response must state that R is continuous because R is differentiable (or equivalent). A response that simply states "R is continuous" without justification does not earn **P3**.
- A response does not need to earn **P3** to be eligible for **P4**.
- To earn **P4**, a response must indicate that $R(0) < 155$ (or $R(2) < 155$ or $R(8) < 155$) and $R(10) > 155$, state that "R is continuous," and answer "yes" in some way.
- To earn **P4**, a response need not explicitly name the Intermediate Value Theorem, but if a theorem is named, it must be correct.

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C Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

$\int_0^{10} R(t)\,dt$ $\approx \frac{R(0)+R(2)}{2}(2-0)+\frac{R(2)+R(8)}{2}(8-2)$ $+\frac{R(8)+R(10)}{2}(10-8)$	Form of trapezoidal sum	**Point 5 (P5)**
$=\frac{90+100}{2}(2-0)+\frac{100+150}{2}(8-2)+\frac{150+162}{2}(10-8)$ $=\frac{190}{2}(2)+\frac{250}{2}(6)+\frac{312}{2}(2)=190+750+312=1252$	Answer with supporting work	**Point 6 (P6)**

Scoring Notes for Part C

- Read "=" as "$\approx$" for **P5**.
- The form of a trapezoidal sum includes three terms, each of which includes a product of two factors, where one of the factors incorporates the $\frac{1}{2}$ as part of the product. To earn **P5**, at least five of the six factors must be correct. If any of the six factors is incorrect, the response does not earn **P6**. Consider the following examples:
  - $\frac{90+100}{2}(2-0)+\frac{100+150}{2}(8-2)+\frac{150+162}{2}(10-8)$ earns **P5** and is sufficient to earn **P6**.
  - $\frac{190}{2}(2)+\frac{250}{2}(6)+\frac{312}{2}(2)$ earns **P5** and is sufficient to earn **P6**.
  - $\frac{1}{2}((R(0)+R(2))(2)+(R(2)+R(8))(6)+(R(8)+R(10))(2))$ earns **P5** and is eligible for **P6**.
  - $\frac{90+100}{2}(2)+\frac{100+150}{2}(2)+\frac{150+162}{2}(2)$ earns **P5** but is not eligible for **P6**.
    (Note that the factor of 2 in the second term of this expression is incorrect.)
- **Special case:** A response of $(90+100)+(100+150)3+(150+162)$ earns both **P5** and **P6**.
- To be eligible for **P6**, a response must have earned **P5**.
  - **Special case:** A response of $95\cdot 2+125\cdot 6+156\cdot 2$ earns **P6** but does not earn **P5**.
- A response of $\frac{90+100}{2}(2-0)+\frac{100+150}{2}(8-2)+\frac{150+162}{2}(10-8)$ or equivalent banks **P6** (i.e., subsequent errors in simplification will not be considered in scoring for **P6)**.
- A response of $\frac{(90\cdot 2+100\cdot 6+150\cdot 2)+(100\cdot 2+150\cdot 6+162\cdot 2)}{2}$ or equivalent earns both **P5** and **P6**. (Note that the average of the left Riemann sum and right Riemann sum is equivalent to the trapezoidal sum.)
- A completely correct left Riemann sum (e.g., $90\cdot 2+100\cdot 6+150\cdot 2=1080$) or a completely correct right Riemann sum (e.g., $100\cdot 2+150\cdot 6+162\cdot 2=1424$) earns **P5** but does not earn **P6**.

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D A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

$\int_0^{10} W(t)\,dt = \int_0^{10}\left(-\frac{3}{10}t^2 + 8t + 100\right)dt$	Integrand	**Point 7 (P7)**
$= \left(-\frac{1}{10}t^3 + 4t^2 + 100t\right)\Big\|_0^{10}$	Antiderivative	**Point 8 (P8)**
$= \left(-\frac{1}{10}\cdot 1000 + 4\cdot 100 + 100\cdot 10\right) - \left(-\frac{1}{10}\cdot 0 + 4\cdot 0 + 100\cdot 0\right)$ $= 1300$ Based on the model, the teacher has read 1300 words by the end of the 10 minutes.	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for an indefinite or definite integral with integrand $W(t)$, with or without the differential dt.
- **P8** is earned for the correct antiderivative, with or without the constant of integration.
- To be eligible for **P9**, a response must have earned **P8**.
- A response of $\left(-\frac{1}{10}\cdot 1000 + 4\cdot 100 + 100\cdot 10\right) - \left(-\frac{1}{10}\cdot 0 + 4\cdot 0 + 100\cdot 0\right)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

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3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

(a) $R'(1) \approx \dfrac{R(2) - R(0)}{2 - 0} = \dfrac{100 - 90}{2} = \dfrac{10}{2} = 5$

words per minute per minute.

2: {P1: Answer with set up; P2: Units}

(b) Yes there must be a value c, for $0 < c < 10$, such that $R(c) = 155$. R is differentiable so R is continuous and $R(0) = 90 < 155 < 162 = R(10)$. The Intermediate Value Theorem guarantees there is at least one value c between 8 and 10 where $R(c) = 155$.

2: {P3: Differentiable implies continuous; P4: Answer with justification}

(c) $\displaystyle\int_0^{10} R(t)dt \approx \left(\frac{90 + 100}{2}\right)(2) + \left(\frac{100 + 150}{2}\right)(6) + \left(\frac{150 + 162}{2}\right)(2) = \left(\frac{190}{2}\right)(2) + \left(\frac{250}{2}\right)(6) + \left(\frac{312}{2}\right)(2)$

$= 190 + 750 + 312 = 1252$

2: {P5: Form of Trapezoidal sum; P6: Answer with supporting work}

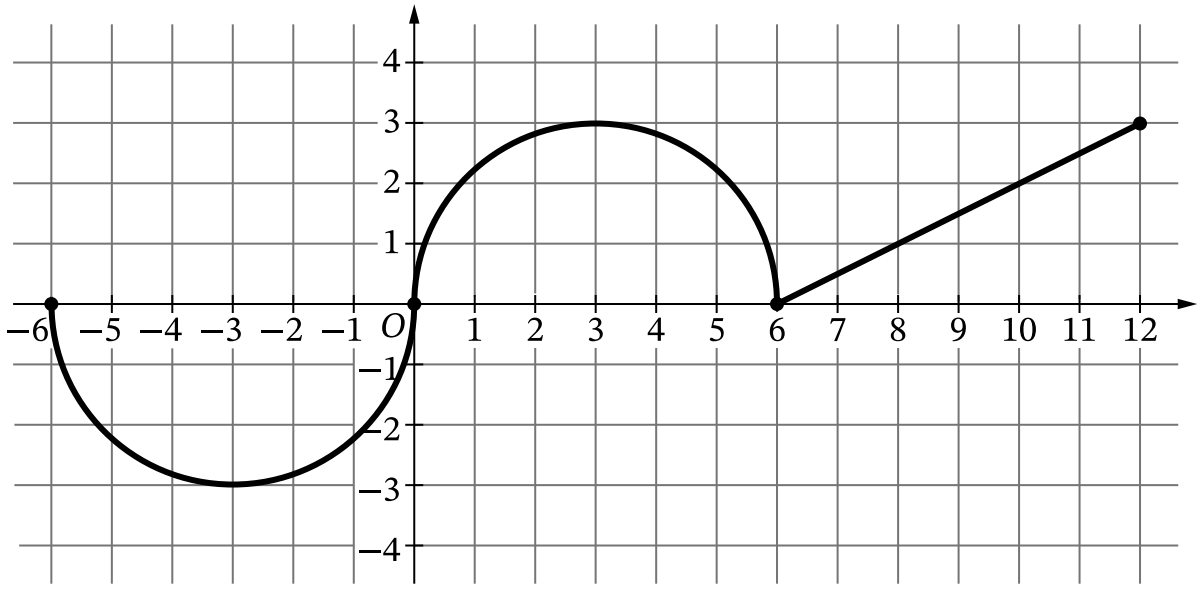
(d) $\displaystyle\int_0^{10} W(t)dt = \left[-\frac{3}{10} \cdot \frac{1}{3}t^3 + 8 \cdot \frac{1}{2}t^2 + 100t\right]_0^{10}$

$\displaystyle= -\frac{1}{10}(10)^3 + 4(10)^2 + 100(10)$

$= -100 + 400 + 1000 = 1300$

3: {P7: Integrand; P8: Antiderivative; P9: Answer}

4. The continuous function f is defined on the closed interval $-6 \le x \le 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t)\,dt$.

A. Find $g'(8)$. Give a reason for your answer.

B. Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

C. Find $g(12)$ and $g(0)$. Label your answers.

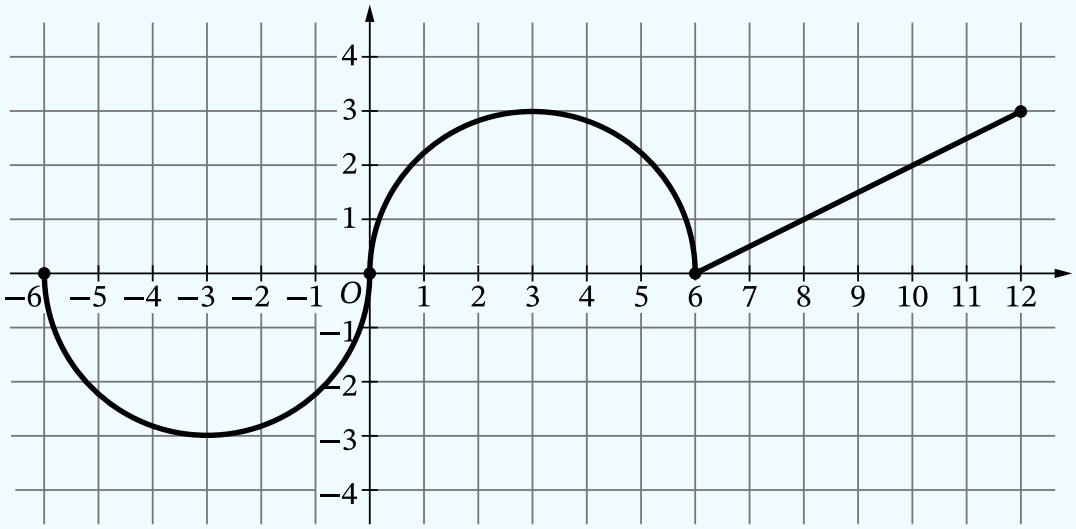
D. Find the value of x at which g attains an absolute minimum on the closed interval $-6 \le x \le 12$. Justify your answer.

Part A (AB or BC): Graphing calculator not allowed
Question 4 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The continuous function f is defined on the closed interval $-6 \le x \le 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t)\,dt$.

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	Model Solution	Scoring	
A	Find $g'(8)$. Give a reason for your answer.		
	$g'(x) = f(x)$	Considers $g'(x) = f(x)$	**Point 1 (P1)**
	$g'(8) = f(8) = 1$	Answer	**Point 2 (P2)**
	Scoring Notes for Part A		

- **P1** is earned for $g' = f$, $g'(x) = f(x)$, or $g'(8) = f(8)$ in part A.
- A response of $g'(8) = f(8) = 1$ earns both **P1** and **P2**.
- A response that does not earn **P1** can earn **P2** with an implied application of the Fundamental Theorem of Calculus, e.g., $g'(8) = 1$ or $f(8) = 1$.
- A response of $g'(8) = f(8) - f(6) = 1$ earns **P2** but not **P1**.

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B Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

The graph of g has a point of inflection where $g'' = f'$ changes sign, which is where $g' = f$ changes from decreasing to increasing or vice versa. The graph of g has points of inflection at $x = -3$ and $x = 6$ because f changes from decreasing to increasing there. The graph of g also has a point of inflection at $x = 3$ because f changes from increasing to decreasing there.	Answer	**Point 3 (P3)**
	Reason	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned only for an answer of $x = -3$, $x = 3$, and $x = 6$. If any other/additional values of x in $-6 < x < 12$ are declared to be points of inflection, the response does not earn either **P3** or **P4**. Consideration of $x = -6$ or of $x = 12$ does not impact scoring.
- To earn **P4**, a response must tie the reason to the given graph of f.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f changes from increasing to decreasing or decreasing to increasing there" earns both **P3** and **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because the slope of f changes sign there" earns both **P3** and **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f attains relative extrema there" earns both **P3** and **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because g changes concavity there" earns **P3** but not **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because $g'' = f'$ changes sign there" earns **P3** but not **P4**.
  - A response that relies upon an ambiguous term such as "the function" or "the graph" does not earn **P4**.
- **Special case:** A response with two of the three correct x-values with correct reasoning, and no other/additional values of x declared to be points of inflection, earns **P4** but not **P3**.

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C Find $g(12)$ and $g(0)$. Label your answers.

$g(12) = \int_6^{12} f(t)\,dt = \frac{1}{2} \cdot 6 \cdot 3 = 9$	$g(12)$	**Point 5 (P5)**
$g(0) = \int_6^0 f(x)\,dx = -\int_0^6 f(x)\,dx = -\frac{\pi}{2}3^2 = -\frac{9\pi}{2}$	$g(0)$	**Point 6 (P6)**

Scoring Notes for Part C

- Unlabeled values do not earn either **P5** or **P6**.
- **P5** is earned for a response of $g(12) = 9$, with or without supporting work.
- **P6** is earned for a response of $g(0) = -\frac{9\pi}{2}$, with or without supporting work.
  - Incorrect communication between the label "$g(0)$" and the answer will be treated as scratch work and will not impact scoring. For example, $g(0) = \int_0^6 f(x)\,dx = -\frac{9\pi}{2}$ earns **P6**.

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D Find the value of x at which g attains an absolute minimum on the closed interval $-6 \le x \le 12$. Justify your answer.

For $-6 \le x \le 12$, g attains a minimum either when $g'(x) = f(x) = 0$ or at an endpoint. $g'(x) = f(x) = 0$	Considers $g'(x) = 0$	**Point 7 (P7)**
$\Rightarrow \quad x = 0, \ x = 6$ $\begin{array}{c\|c} x & g(x) \\ \hline -6 & 0 \\ 0 & -\frac{9\pi}{2} \\ 6 & 0 \\ 12 & 9 \end{array}$	Justification	**Point 8 (P8)**
Therefore, on the closed interval $-6 \le x \le 12$, g attains an absolute minimum value at $x = 0$.	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for considering $g'(x) = 0$ or $f(x) = 0$. **P7** is not earned by just presenting $x = 0$ and $x = 6$.
  - A response that discusses the sign of $g'(x)$ or $f(x)$ changing OR uses the phrase "critical points of g" also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by providing evaluations or reasoning for each of $g(-6)$, $g(0)$, $g(6)$, and $g(12)$ (and no other x-values).
- Alternate justification and answer:

  Because $g'(x) \le 0$ (or $f(x) \le 0$) for $-6 \le x < 0$ and $g'(x) \ge 0$ (or $f(x) \ge 0$) for $0 < x \le 12$, the absolute minimum of g occurs at $x = 0$.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer of $x = 0$.
- For **P8**, values of $g(0)$ and $g(12)$ can be imported from part C. A response can earn **P9** with an answer that is consistent with the imported values.

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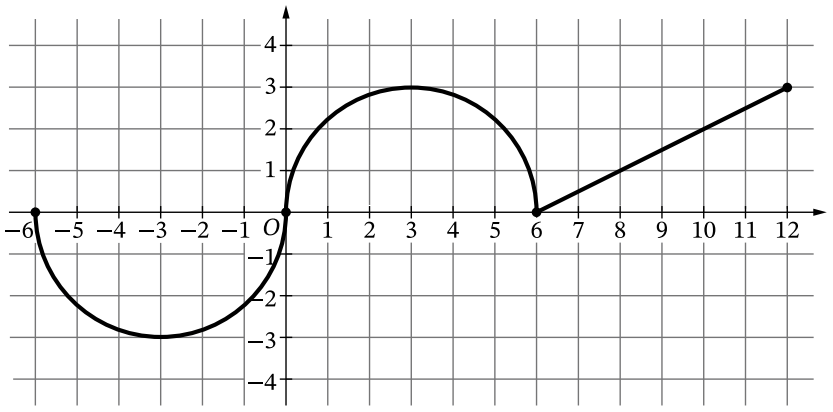
4. The continuous function f is defined on the closed interval $-6 \le x \le 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t)dt$.

A. Find $g'(8)$. Give a reason for your answer.

B. Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

C. Find $g(12)$ and $g(0)$. Label your answers.

D. Find the value of x at which g attains an absolute minimum on the closed interval $-6 \le x \le 12$. Justify your answer.

(a) $g'(x) = f(x) \rightarrow g'(8) = f(8) = 1$

2: {P1: Considers $g'(x) = f(x)$; P2: Answer}

(b) g has points of inflections at $x = -3, 3,$ and 6 because $g'(x) = f(x)$ changes from decreasing to increasing or vice versa at these values of x.

2: {P3: Answer; P4: Reason}

(c) $g(12) = \int_6^{12} f(t)dt = \frac{1}{2}(6)(3) = 9$

$g(0) = \int_6^0 f(t)dt = -\int_0^6 f(t)dt = -\frac{1}{2}\pi(3)^2 = -\frac{9}{2}\pi$

2: {P5: $g(12)$; P6: $g(0)$}

(d) $g'(x) = f(x) = 0 \rightarrow x = 0,6$

x	$g(x) = \int_6^x f(t)dt$
-6	$-\int_{-6}^6 f(t)dt = -\left(-\frac{9}{2}\pi + \frac{9}{2}\pi\right) = 0$
0	$-\frac{9}{2}\pi$
6	0
12	9

g attains an absolute minimum at $x = 0$.

3: {P7: Considers $g'(x) = 0$; P8: Justification; P9: Answer}

5. Two particles, *H* and *J*, are moving along the *x*-axis. For $0 \le t \le 5$, the position of particle *H* at time *t* is given by $x_H(t) = e^{t^2 - 4t}$ and the velocity of particle *J* at time *t* is given by $v_J(t) = 2t(t^2 - 1)^3$.

A. Find the velocity of particle *H* at time $t = 1$. Show the work that leads to your answer.

B. During what open intervals of time *t*, for $0 < t < 5$, are particles *H* and *J* moving in opposite directions? Give a reason for your answer.

C. It can be shown that $v_J'(2) > 0$. Is the speed of particle *J* increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

D. Particle *J* is at position $x = 7$ at time $t = 0$. Find the position of particle *J* at time $t = 2$. Show the work that leads to your answer.

Part B (AB): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Two particles, H and J, are moving along the x-axis. For $0 \le t \le 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2-4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t\left(t^2-1\right)^3$.

	Model Solution	Scoring	
A	Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.		
	$x_H'(t) = v_H(t) = (2t-4)e^{t^2-4t}$ $x_H'(1) = v_H(1) = -2e^{-3}$	Considers x_H'	**Point 1 (P1)**
		Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** can be earned by presenting $x_H'(t)$, $x_H'(1)$, $x'(t)$, $x'(1)$, $(2t-4)e^{t^2-4t}$, or $(2\cdot 1-4)e^{1^2-4\cdot 1}$.
- An unsupported answer of $-2e^{-3}$ earns **P2** but not **P1**.

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B During what open intervals of time t, for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.

From part A, $x_H'(t) = v_H(t) = (2t - 4)e^{t^2-4t}$.

$x_H'(t) = (2t - 4)e^{t^2-4t} = 0 \Rightarrow t = 2$

$x_H'(t) < 0$ for $0 < t < 2$, and $x_H'(t) > 0$ for $2 < t < 5$.

Thus, particle H is moving to the left for $0 < t < 2$ and moving to the right for $2 < t < 5$.

$v_J(t) = 2t\left(t^2 - 1\right)^3 = 0$ for $0 < t < 5 \Rightarrow t = 1$

$v_J(t) < 0$ for $0 < t < 1$, and $v_J(t) > 0$ for $1 < t < 5$.

Thus, particle J is moving to the left for $0 < t < 1$ and moving to the right for $1 < t < 5$.

Therefore, particles H and J are moving in opposite directions for $1 < t < 2$.

Considers sign of $x_H'(t)$ or $v_J(t)$	**Point 3 (P3)**
Analysis for one particle	**Point 4 (P4)**
Answer with reason	**Point 5 (P5)**

Scoring Notes for Part B

- To earn **P3**, a response can do one of the following:
  - Set $x_H'(t) = 0$, $v_H(t) = 0$, or $(2t - 4)e^{t^2-4t} = 0$
  - Set $v_J(t) = 0$ or $2t\left(t^2 - 1\right)^3 = 0$
  - Identify $t = 2$ for particle H and no other values in the interval $0 < t < 5$
  - Identify $t = 1$ for particle J and no other values in the interval $0 < t < 5$
  - Identify the interval $1 < t < 2$
- To earn **P4**, a response can provide an analysis of signs of velocity or direction of motion on the interval $0 < t < 5$ for either particle H or particle J.
- To be eligible for **P5**, a response must provide correct analyses of signs of velocity or direction of motion on the interval $0 < t < 5$ for both particles.
- Only analysis within the interval $0 < t < 5$ will be considered in scoring.

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C It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

$v_J(2) > 0$ and $v_J'(2) > 0$. Because $v_J(2)$ and $v_J'(2)$ have the same sign, the speed of particle J is increasing at $t = 2$.	Answer with reason	**Point 6 (P6)**

Scoring Notes for Part C

- An evaluation of $v_J(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J(2) = 108$.
- An evaluation of $v_J'(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J'(2) = 486$.
- A response can either import the analysis for the sign of $v_J(2)$ from part B or restart.
- A response that stated "$v_J(t) > 0$ for $1 < t < 5$" in part B does not need to restate $v_J(2) > 0$ and earns **P6** for "$v_J(2)$ and $v_J'(2)$ have the same sign, so the speed is increasing."

D Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

$x_J(2) = x_J(0) + \int_0^2 v_J(t)\,dt = 7 + \int_0^2 2t\left(t^2 - 1\right)^3\,dt$ $= 7 + \left[\frac{1}{4}\left(t^2 - 1\right)^4\right]_0^2$ $= 7 + \frac{1}{4}\left((3)^4 - (-1)^4\right) = 7 + \frac{1}{4}(80) = 27$	Integrand	**Point 7 (P7)**
	Antiderivative	**Point 8 (P8)**
	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- To earn **P7**, a response must present an indefinite or definite integral with an integrand of $v_J(t)$ or $2t\left(t^2 - 1\right)^3$. (See below for notes on how to handle a missing differential dt.)
- **P8** is earned for an antiderivative of the form $k\left(t^2 - 1\right)^4$ or equivalent, for $k > 0$. If $k \neq \frac{1}{4}$, then the response is not eligible to earn **P9**.
- A response of $7 + \frac{1}{4}\left((3)^4 - (-1)^4\right)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).
  - Note: An ambiguous response, such as $7 + \frac{1}{4}\left((3)^4 - (-1)^4$, does not bank **P9** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., $7 + \frac{1}{4}(80)$ or 27) to earn **P9**.

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- If the differential dt is missing:
  - Writing $\int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9.**
  - Writing $7 + \int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9.**
  - Writing $\int_0^2 v_J(t) + 7$ introduces an ambiguity for the intended integrand.
    - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}\left(t^2 - 1\right)^4\right]_0^2 + 7$ resolves the ambiguity.

      Therefore, this earns **P7** and **P8** and is eligible for **P9**.
    - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}\left(t^2 - 1\right)^4 + 7t\right]_0^2$ confirms that an incorrect integrand was used.

      Therefore, this does not earn **P7**, earns **P8**, and is not eligible for **P9**.
    - If the ambiguity is not resolved, this does not earn **P7**, **P8**, or **P9**.
- Alternate solution using u-substitution:

  Let $u = t^2 - 1$, then $du = 2t\,dt$.

  $t = 0 \Rightarrow u = -1$

  $t = 2 \Rightarrow u = 3$

  $x_J(2) = x_J(0) + \int_0^2 v_J(t)\,dt = 7 + \int_0^2 2t\left(t^2 - 1\right)^3 dt$

  $= 7 + \int_{-1}^3 u^3\,du = 7 + \left[\frac{1}{4}u^4\right]_{-1}^3$

  $= 7 + \frac{1}{4}\left((3)^4 - (-1)^4\right) = 7 + \frac{1}{4}(80) = 27$
- Alternate solution using indefinite integral:

  $\int 2t\left(t^2 - 1\right)^3 dt = \frac{1}{4}\left(t^2 - 1\right)^4 + C$

  $x_J(0) = 7 = \frac{1}{4}\left(0^2 - 1\right)^4 + C \Rightarrow C = \frac{27}{4}$

  $x_J(t) = \frac{1}{4}\left(t^2 - 1\right)^4 + \frac{27}{4}$

  $x_J(2) = \frac{1}{4}\left(2^2 - 1\right)^4 + \frac{27}{4} = \frac{108}{4} = 27$

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5. Two particles, H and J, are moving along the x-axis. For $0 \le t \le 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2 - 4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2 - 1)^3$.

A. Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.

B. During what open intervals of time t, for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.

C. It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

D. Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

(a) $v_H(t) = x_H'(t) = e^{t^2-4t}(2t - 4)$

$v_H(1) = e^{1^2-4(1)}(2 \cdot 1 - 4) = -2\,e^{-3}$

2: $\begin{cases} \text{P1: Considers } x_H' \\ \text{P2: Answer} \end{cases}$

(b) The particles H and J are moving in opposite directions on $(1,2)$.

$$v_H(t)\ \underbrace{\overbrace{+ - - + -}^{\substack{(+)(-)\\ -}}}_{0\ \ 1\ \ 2}\ \underbrace{+\ \overbrace{- - +}^{\substack{(+)(+)\\ +}}}_{2\ \ 5} \qquad v_J(t)\ \underbrace{\overbrace{+ - -}^{\substack{(+)(-)\\ -}}}_{0\ \ 1}\ \underbrace{+\ \overbrace{- + - - +}^{\substack{(+)(+)\\ +}}}_{2\ \ 5}$$

3: $\begin{cases} \text{P3: Considers sign of } x_H'(t) \text{ or } v_j(t) \\ \text{P4: Analysis for one particle} \\ \text{P5: Answer with reason} \end{cases}$

(c) The speed of particle J is increasing because $v_J(2) > 0$ and $v_J'(2) > 0$ have the same sign.

1: {P6: Answer with reason

(d) $x_j(2) = 7 + \int_0^2 [2t(t^2 - 1)^3]dt = 7 + \int_0^2 \left[\left(\underbrace{t^2 - 1}_{u}\right)^3\right]\underbrace{(2tdt)}_{du}$

$= 7 + \int_{-1}^{3} [u^3]du = 7 + \left[\frac{1}{4}u^4\right]_{-1}^{3} = 7 + \left[\left(\frac{81}{4}\right) - \left(\frac{1}{4}\right)\right]$

$= 7 + 20 = 27$

3: $\begin{cases} \text{P7: Integrand} \\ \text{P8: Antiderivative} \\ \text{P9: Answer} \end{cases}$

6. Consider the curve *G* defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

A. Show that $\frac{dy}{dx} = \frac{-x}{2\left(3y^2 - 2y - 1\right)}$.

B. There is a point *P* on the curve *G* near $(2, -1)$ with *x*-coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the *y*-coordinate of point *P*.

C. For $x > 0$ and $y > 0$, there is a point *S* on the curve *G* at which the line tangent to the curve at that point is vertical. Find the *y*-coordinate of point *S*. Show the work that leads to your answer.

D. A particle moves along the curve *H* defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.

Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

	Model Solution	Scoring	
A	Show that $\frac{dy}{dx} = \frac{-x}{2\left(3y^2 - 2y - 1\right)}$.		
	$\frac{d}{dx}\left(y^3 - y^2 - y + \frac{1}{4}x^2\right) = \frac{d}{dx}(0)$ $\Rightarrow \; 3y^2 \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} - \frac{dy}{dx} + \frac{x}{2} = 0$	Implicit differentiation	**Point 1 (P1)**
	$\Rightarrow \; \left(3y^2 - 2y - 1\right)\frac{dy}{dx} = -\frac{x}{2}$ $\Rightarrow \; \frac{dy}{dx} = \frac{-x}{2\left(3y^2 - 2y - 1\right)}$	Verification	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned only for the correct implicit differentiation of $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y'.
- To be eligible for **P2**, a response must have earned **P1**.
- It is sufficient to present $\left(3y^2 - 2y - 1\right)\frac{dy}{dx} = -\frac{x}{2}$ to earn **P2**, provided there are no subsequent errors.

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B There is a point P on the curve G near $(2, -1)$ with x-coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y-coordinate of point P.

$\left.\frac{dy}{dx}\right\|_{(x, y)=(2, -1)} = \frac{-2}{2(3+2-1)} = -\frac{1}{4}$	Slope of tangent line	**Point 3 (P3)**
$y \approx -1 - \frac{1}{4}(1.6 - 2) = -0.9$	Tangent line approximation	**Point 4 (P4)**

Scoring Notes for Part B

- A response can earn **P3** with $\left.\frac{dy}{dx}\right|_{(x, y)=(2, -1)} = -\frac{1}{4}$, $\frac{dy}{dx} = -\frac{1}{4}$, "slope is $-\frac{1}{4}$," or equivalent.
- A response that presents a linear approximation with a slope of $-\frac{1}{4}$ also earns **P3**.
- A response that declares $\left.\frac{dy}{dx}\right|_{(x, y)=(2, -1)}$ (or the slope) equal to any nonzero value $k \neq -\frac{1}{4}$ does not earn **P3**. Such a response earns **P4** for a presented approximation mathematically equivalent to $-1 + k(-0.4)$.
- **P4** cannot be earned with a linear approximation using a slope other than $-\frac{1}{4}$ if that slope has not been declared to be the value of $\left.\frac{dy}{dx}\right|_{(x, y)=(2, -1)}$.
- A response does not have to present the tangent line equation but must clearly demonstrate its use at $x = 1.6$ in finding the requested approximation in order to be eligible for **P4**.
- A response of $-1 - \frac{1}{4}(-0.4)$ earns both **P3** and **P4**.
- A response of $-1 - \frac{1}{4}(1.6 - 2)$ or equivalent banks **P4** (i.e., subsequent errors in simplification will not be considered in scoring for **P4)**.
  - Note: An ambiguous response, such as $-1 - \frac{1}{4}(1.6 - 2$, does not bank **P4** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., -0.9) to earn **P4**.

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C For $x > 0$ and $y > 0$, there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y-coordinate of point S. Show the work that leads to your answer.

For $x > 0$, the curve G has a vertical tangent line when $2\left(3y^2 - 2y - 1\right) = 0$.	Sets denominator equal to 0 **Point 5 (P5)**
$2\left(3y^2 - 2y - 1\right) = 0 \Rightarrow 2(3y + 1)(y - 1) = 0$ Because $y > 0$, it follows that $y = 1$. The line tangent to the curve is vertical at the point on the curve where $y = 1$.	Answer **Point 6 (P6)**

Scoring Notes for Part C

- **P5** is earned with any of $2\left(3y^2 - 2y - 1\right) = 0$, $3y^2 - 2y - 1 = 0$, $2(3y \pm 1)(y \pm 1) = 0$, or $(3y \pm 1)(y \pm 1) = 0$.
- To be eligible for **P6**, a response must have earned **P5**.
- A response does not need to consider the numerator of $\frac{dy}{dx} = \frac{-x}{2\left(3y^2 - 2y - 1\right)}$ in order to earn **P5** or **P6**; considering the denominator is sufficient.
- A response that states solutions of $y = -\frac{1}{3}$ and $y = 1$, but does not clearly identify that $y = 1$ is the only solution to the prompt, does not earn **P6**.
- In the presence of algebraic work to find the value of y, **P6** is earned only if the algebraic work is correct.
- A response of "$2\left(3y^2 - 2y - 1\right) = 0$, $y = 1$" earns both **P5** and **P6.**

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D A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.

$\frac{d}{dt}(2xy + \ln y) = \frac{d}{dt}(8)$ $2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$	Attempts implicit differentiation with respect to t	**Point 7 (P7)**
	$2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$	**Point 8 (P8)**
$2(3)(1) + 2(4)\frac{dy}{dt} + \frac{1}{1}\frac{dy}{dt} = 0$ $\Rightarrow \ 6 + 9\frac{dy}{dt} = 0 \ \Rightarrow \ \frac{dy}{dt} = -\frac{2}{3}$	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for implicitly differentiating $2xy + \ln y = 8$ with respect to t with at most one error.
- **P8** is earned for an equation equivalent to $2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$.
- To be eligible for **P9**, a response must have earned **P7** and **P8**, with no errors in implicit differentiation.
- **P9** is earned only for the value of $-\frac{2}{3}$.
- Alternate solution:

  $$\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$$

  $$\frac{d}{dx}(2xy + \ln y) = \frac{d}{dx}(8) \ \Rightarrow \ 2y + 2x\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = 0 \ \Rightarrow \ \frac{dy}{dx} = \frac{-2y}{2x + \frac{1}{y}}$$

  $$\left.\frac{dy}{dx}\right|_{(x,\,y)=(4,\,1)} = \frac{-2(1)}{2(4) + \frac{1}{1}} = -\frac{2}{9}$$

  $$\left.\frac{dy}{dt}\right|_{(x,\,y)=(4,\,1)} = \left.\frac{dy}{dx}\cdot\frac{dx}{dt}\right|_{(x,\,y)=(4,\,1)} = -\frac{2}{9}\cdot 3 = -\frac{2}{3}$$

  - **P7** is earned for an implicit differentiation with respect to x with at most one error, as long as $\frac{dy}{dx}$ is eventually correctly linked to $\frac{dx}{dt}$.
  - **P8** is earned for finding $\frac{dy}{dx}$ and multiplying the result by $\frac{dx}{dt} = 3$.
  - **P8** can be earned for a stated incorrect $\frac{dy}{dx}$, as long as it is multiplied by 3.
  - **P9** is only earned for a correct value of $-\frac{2}{3}$ or equivalent.
  - Stating $\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$ alone does not earn any points.

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6. Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

A. Show that $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$.

B. There is a point P on the curve G near $(2, -1)$ with x-coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y-coordinate of point P.

C. For $x > 0$ and $y > 0$, there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y-coordinate of point S. Show the work that leads to your answer.

D. A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.

(a) $(3y^2 - 2y - 1)\frac{dy}{dx} + \frac{2}{4}x = 0 \rightarrow (3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{1}{2}x$

$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$

2: {1: Implicit Differentiation; 1: Verification}

(b) $\left.\frac{dy}{dx}\right|_{(2,-1)} = \frac{-2}{2(3(-1)^2 - 2(-1) - 1)} = -\frac{1}{4}$

Tangent: $y = -1 + \left(-\frac{1}{4}\right)(x - 2)$

$y(1.6) = -1 - \frac{1}{4}(1.6 - 2) = -1 - \frac{1}{4}(-0.4) = -1 + 0.1 = -0.9$

$P(1.6) \approx -0.9$

2: {1: slope of tangent line; 1: approximation}

(c) $\frac{dy}{dx} = \infty \rightarrow (3y^2 - 2y - 1) = 0 \rightarrow (3y + 1)(y - 1) = 0 \rightarrow y = 1$

2: {1: $(3y^2 - 2y - 1) = 0$; 1: Answer with reason}

(d) $2\left(\frac{dx}{dt}y + x\frac{dy}{dt}\right) + \frac{1}{y}\frac{dy}{dt} = 0$

$(4,1) \rightarrow 2\left(\left.\frac{dx}{dt}\right|_{(4,1)}(1) + (4)\left.\frac{dy}{dt}\right|_{(4,1)}\right) + \frac{1}{1}\left.\frac{dy}{dt}\right|_{(4,1)} = 0$

$2\left((3)(1) + (4)\left.\frac{dy}{dt}\right|_{(4,1)}\right) + \left.\frac{dy}{dt}\right|_{(4,1)} = 6 + 8\left.\frac{dy}{dt}\right|_{(4,1)} + \left.\frac{dy}{dt}\right|_{(4,1)} = 0$

$6 + 9\left.\frac{dy}{dt}\right|_{(4,1)} = 0 \rightarrow \left.\frac{dy}{dt}\right|_{(4,1)} = -\frac{6}{9} = -\frac{2}{3}$

3: {1: $2\left(\frac{dx}{dt}y + x\frac{dy}{dt}\right) + \frac{1}{y}\frac{dy}{dt} = 0$; 1: Uses $\left.\frac{dx}{dt}\right|_{(4,1)} = 3$; 1: Answer}